Chapter Index:

Notes for Students

1. Read each and every word; use Simple English; if some tough (= difficult, not easy) word comes, look for meaning then and there.

2. Every section has worked examples. Immediately below (=there only) exercises (= problems) are given; please do (= write, solve, work out) each and every problem.

3. Mathematics is not difficult; it is made easier here. Be confident; go step by step; you will understand.

4. Always try first yourself; do something right or wrong; show to elders and teachers; then you will learn.

5. Do not read for exam or for marks; do for learning and understanding. If you do so, you will see Maths everywhere around you.

6. Our aim is to make you at ease with Maths. We try to make mathematics easy; you try to make friends with it.

7. Feedback is welcome.

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Notes for Teachers

1. This is student's version of Functional Mathematics. We have tried to make students version so user friendly that a student can learn on his own. For this reason very simple language is used (sometimes at the cost of accuracy, please excuse). Every slightly tough or uncommon usage of English word is explained. We have tried to describe, define and explain almost all technical terms. As a teacher, you could help here.

2. Subjects handled range from the primary to middle school level. No previous knowledge on the part of the student is assumed. We have tried to make this whole version understandable independently, easily and logically. To facilitate the learning process practically, we have included (worked) examples immediately followed by problems and exercises. These are so easy that any new student can do them. Problems are meant for practice; exercises are meant for education not for examination.

3. Assorted assessment exercise options are also given. Please use them not for grading but for gradual guiding. In these aspects, you as a teacher will be quite valuable.

4. Many activities are suggested. Many games are imagined. These have the central theme of understanding and at the same time have some fun. Since this writer likes 'fun-for-all', these activities need everybody's participation. You can adopt some for individuals or invent new ones. In any case some materials ("aids") will help. Hope you could help in this area.

5. The teacher must be a facilitator to help the student finish each (small) section (along with problems) before going further. Please do not wait for the full chapter (or even the book) to be finished.

6. Feedback is welcome.

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Chapter - 1 Numbers and Addition

1.1 Number system (0 to 999)

1.1.1 Let students write 1 to 10 neatly

- Spelling in English ONE, TWO, THREE etc., is optional but not very important. **Speaking and reading** out in English is important.
- Do not go to 11, 12 ….

1.1.2 Go to 10, 20, …….. 100

1.1.3 Then go to 100, 200 ….. 900

1.1.4 Write 11 to 19.

- No spelling
- Show that 10 + 1 = 11, 10 + 5 = 15 etc.
- 11, 12 are special (i.e. exceptions). Others are teens.
- Instantly test the grasp by asking the age of the student and the age of his/her elder/younger brother/sister.

1.1.5 The system of 2 digits in English is very easy
E.g.: Twenty + one = ……, 20+ 9 = …… (speak this out).

1.1.6 0 – 9 is the basic number system, all the others follow.
0 = 00
1 = 01
9 = 09
But 19 = 10 + 9

Similarly 101 = 100 + 1
123 = 100 + 20 + 3

1.2 Single digit addition (+)

1.2.1 Play a game of adding any number (between 1 to 10) and another one digit number.

1.2.2 Each student should write and fill up the following

1 + □ = 10
2 + □ = 10
3 + □ = 10
4 + □ = 10
5 + □ = 10
6 + □ = 10
7 + □ = 10
8 + □ = 10
9 + □ = 10
10 + □ = 10

1.2.3 Cut and make ‘bricks’ (i.e. thick squares or rectangles cardboard)
0 to 9: 3 sets
10 to 20: 2 sets.

- Play single digit addition game.

This could be $\boxed{A} + \boxed{B} = \boxed{?}$

or $\boxed{A} + \boxed{?} = \boxed{C}$

or $\boxed{?} + \boxed{B} = \boxed{C}$

A, B come from (0 – 9) bricks, C comes from A or B or $\boxed{10 – 20}$ set

1.2.4 Simplified addition grid as a Frame is given in sec 1.2.7. Let the students fill it up at home.

1.2.5 Adding tips
- Do not allow the primary school method of 3 + 2 = (***) + (**) = …….5
- If some persons do it ask them to add 2 + 9.
- If they start making 9 dots or counting 9 fingers, stop it.
- Say 2 + 9 = 9 + 2 = 9 +1 + 1 = 10 + 1 = 11
  i.e. take the bigger and start counting from that stage.

1.2.6 Use the method of (1.2.5) for bigger numbers (All Oral Only):
9 + 3 =?
39 + 3 =?
89 + 3 =?
109 + 3 =?

- How to add 1234 + 4

1234 +  = 1235 +  = 1236 +  = 1237 +  = 1238

1.2.7 Simplified Addition Grid: Students should fill this.

+	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	2	3							10
2	3	4							
3									
4									
5									
6									
7									
8									
9	10								18

1.3 Addition – 2 digits

1.3.1 Before addition of two digits, start with:
20 + 4 = 24;
30 + 7 = 37 etc.
Make sure you know 37 has 30 + 7.

1.3.2 Let addition be done orally:
E.g.: 30 + 4 = 34
34 + 5 = 30 + 4 + 5
Now 4 + 5 = 9.
Therefore 30 + (4 + 5) = 39
E.g.: 44 + 6 = 40 + 4 + 6
Now 4 + 6 = 10.
Therefore 40 + (4 + 6) = 40 + 10 = 50
E.g.: 59 + 7 = 50 + (9 + 7)
= 50 + (16)
= 50 + (10) + (6) = 60 + 6 = 66
Even if it appears long, let it be.

Now write these sums:

3 4	4 4	5 9
+ 5	+ 6	+ 7
----	-----	-----
3 9	4 0	5 0
----	+ 1 0	+ 1 6
	------	-----
	------	-----

1.3.3 Do:

5 0	5 1	5 4	5 6	5 9	5 0
+ 5	+ 5	+ 5	+ 5	+ 5	+1 5
-----	-----	-----	-----	-----	-----
-----	-----	-----	-----	-----	-----

5 1	5 4	5 6	5 9
+2 5	+3 5	+3 5	+4 5
-----	-----	-----	-----
-----	-----	-----	-----

1.3.4 Do by the long way and by the usual short method:

E.g.: = 99 + 88
= 90 + 9 + 80 + 8
= 90 + 80 + 9 + 8
= 90 + 80 +(17)
= (170) + (17)
= (170) + (10) + 7
= (180) + 7 = 187

1
9 9
+ 8 8

187

1.3.5 Go back to 1.3.4 example and do:

1 5 5	1 5 5	1 5 5	1 5 5
+ 5	+ 1 5	+ 3 5	+ 6 5
------	-------	-------	-------
------	-------	-------	-------

If these are OK, then:

1 5 5	1 5 5	1 5 5	1 5 5
+ 1 0 0	+ 1 0 5	+1 1 5	+ 9 5 5
------	-------	-------	-------
------	-------	-------	-------

Exercises - Chapter I

Ex I.1 Example:

- Write Down:

Nine	**Ans: 9**
Eightynine	**Ans: 89**
Four hundred and Forty four	**Ans: 444**
Ten thousand four hundred forty four	**Ans: 10444**

Now:

a. Seven
b. Seventy Seven
c. Seventy hundred and seventy seven
d. Seven hundred and seven
e. Seven thousand seven
f. Seven thousand seventy seven
g. Seven thousand seven hundred seventy seven
h. Seventy thousand seven
i. Seven lakhs seven thousand

j. Seven crores seven lakhs seven

Ex I.2 See the number and write down in words (and also speak out) (reading the number can be first in mother tongue and soon shift to English).

Example: 122: **Noora ippatheradu – One hundred and twenty two.**

Now:

a. 47
b. 87
c. 107
d. 359
e. 8889
f. 6666689
g. 66689
h. 567891
i. 12345674
j. Your (or any) mobile number

Ex I.3 Example:

Find which is bigger (of the two) 7999, 9007.
Ans: 9007 >7999 (> is greater than)

Do Now:

a. 99, 101
b. 807, 799
c. 49999, 5001
d. 100001, 99899

Ex I.4 Example:

Find which is smaller 89, 98
And: 89 < 98 (< is smaller than)

Do Now:

a. 7444, 6989
b. 609, 906
c. 771, 769
d. 10203040, 9898989

Ex I.5 Example:

Write in ascending order: 98, 101, 9
And" 9, 98, 101 or 9< 98 < 101

Do Now:

a. 121, 112, 201
b. 1001, 9821, 999
c. 110012, 41099, 101012
d. 20202021, 191919191, 7667766

Ex I.6 Example:

Write in descending order: 97, 101, 24
Ans: 101, 97, 24 or 101> 97 > 24

Do: a to d Ex I.5

Ex I.7 Write both on ascending and descending orders:

a. 1, 8, 5, 3, 4
b. 11, 18 , 15, 13, 14
c. 10, 80, 50, 40, 30
d. 125, 85, 325, 65, 5
e. 101, 11, 1, 111, 1001, 100001
f. 989, 998, 889, 898, 900,190,1989
g. 1002, 1020, 1201, 1220, 1120, 1112

Ex I.8 Example:

Add: (12345) + 4
Ans: Orally [(12345), (12346), (12347), (12348), (12349)]
Start one two three four
Stop Answer is 12349

Do Additions Orally:

a. 28 + 4
b. 128 + 3
c. 99 + 5
d. 98 + 6
e. 98 + 7
f. 1238 + 2
h. 98 + 9
i. 12387 + 4

Ex I.9 Example:

Add 98 + 7
Orally (98 + 7) = 108 – 3 = 108, 107, 106, 105
Start one two three
Ans is 105

Do:
a. 98 + 9 c. 178 + 8 e. 12346 + 8
b. 89 + 8 d. 12345 + 9 f. 184 + 7

Ex I.10 Example:

```
(1234) + (25)   Also 1234 = 1000 + 200 + 30 + 4
Ans:   1234              25 =          20 + 5
    +  25           Total = 1259
       -------
       -------
```

Add in both ways:

a. 99 + 1 e. 43 + 36 i. (40065) + 4
b. 87 + 3 f. (10099) + 1 j. (50043) + 36
c. 76 + 3 g. (20087) + 3
d. 65 + 4 h. (30076) + 3

Ex I.11 Example:

```
(1234) + (17)
Ans:   1234         Also 1234 = 1000 + 200 + 30 + 4
    +  17                  17 =            10 + 7
     ------
     ------         Total = 1240 + 11   = 1251
```

Add in both ways:

a. 99 + 3 e. 843 + 38 i. 100465 + 7
b. 87 + 5 f. 10099 + 3 j. 100843 + 38
c. 176 + 6 g. 10087 + 5
d. 465 + 7 h. 100176 + 6

Ex I.12 Add (in the regular method):

a. (11) + (29) + (38) e. (10011) + (10029) + 10038)
b. (12) + (28) +(37) + (55) f. (10012) + (10028) + (10037) + 10055)
c. (14) + (27) + (35) + (55) g. (1014) + (1027) + 1035) + (0055)
d. (17) + (24) + (34) + (56) h. (917) + (924) + (934) + (956)

Chapter - 2 Subtraction

2.1 Go to addition and make:

			Rewrite this as:		
1 + 4	=	□	5 – 4	=	1
2 + □	=	5	5 - □	=	2
3 + □	=	5	5 - □	=	3
4 + □	=	5	5 - □	=	4
5 + □	=	5	5 - □	=	5
			5 - □	=	0

Students should understand that subtraction is the same as **or** part of **or** reverse of addition. These are different ways of saying the same thing.

Note:

- Limit this approach to positive numbers.
- Therefore the first number should be bigger.
- Technical terms like minus numbers etc are not necessary at this stage.
- Let us have only 2 numbers.
- Let different student groups make sets of numbers adding up to 6, 7, 8, 9, 10:

E.g.:	1+5 = 6	6 – 1 = 5

2.2 Subtraction with fingers in two ways.

2.2.1 Start from the bigger (=first number). It is in your mouth. Take out one by one by fingers, until you reach the second number. Your fingers show the result.

E.g.: 10 – 7 =? $\boxed{10} \longrightarrow \boxed{9} + 1 \longrightarrow \boxed{8} +1,1 \longrightarrow \boxed{7} + 1,1,1$

You have reached $\boxed{7}$ Therefore Answer = 1,1,1 = 3

2.2.2 Reverse the process. Put the second number in the mouth. Start counting up until you reach the second number.

E.g.: 10 – 7 = ? $\boxed{7} \longrightarrow \boxed{8} - 1 \longrightarrow \boxed{9} - 1,1 \longrightarrow \boxed{10} - 1, 1,1$

Therefore Answer = 1,1,1 = 3

2.2.3 Write down the above:

8	8	8	8	8	8	8
- 1	- 2	-3	- 4	- 5	- 6	- 7
---	----	----	----	----	----	----
---	----	----	----	----	----	----

2.3 Subtraction without 'borrowing'.

2.3.1 Let the students select numbers from number 'bricks'.

Give one group the set of $\boxed{\text{5 to 9}}$ and the other $\boxed{\text{0 to 4}}$

Let the first group make numbers; Let the second Subtract second from the first.

E.g.:	5	6	7	8	9
	- 0	- 1	-2	- 3	- 4
	---	----	----	----	----
	---	----	----	----	----
E.g.:	5 5	6 6	7 7	8 8	9 9
	- 1 0	- 2 0	-3 0	- 4 2	- 4 4
	----	----	----	----	----
	----	----	----	----	----

This way each digit gives the answer.

2.3.2 After doing the above see how it works:

E.g.: 99 – 44 = (90 – 40), (9 – 4) = 50, 5 = 55

2.3.3 Extend to 3 digits:

E.g.:	1 2 3	1 2 3	2 4 6
-	1 2 2	- 1 1 2	-1 3 5
	-----	-----	------
	-----	-----	------

2.4 Subtraction with 'borrowing'.

2.4.1	2	2	1 2	1 2
	- 1	- 2	- 1	- 2
	---	----	----	----
	---	----	----	----

Now ask	2	
	- 3	

	?	Not possible - is OK as an answer.
	- -----	
	1 2	
	- 3	

	?	

Only $\boxed{\text{ONE}}$ is borrowed from the next digit.

That $\boxed{\text{ONE}}$ = $\boxed{\text{TEN}}$ in the new place

Go to
2 2
\- 3
\----
\----

2.4.2 As in 2.3.1 let two groups make numbers – first 2 digits – after making interchange the unit place – let them do subtraction.

Extend this game to 3 digits. Interchange either the unit place or ten place or both. Now all these require borrowing.

2.5 **Subtraction Grid:** Go back to the addition grid.

Students can convert it to subtraction grid. Skeleton in some places will be empty. (To the teacher: empty spaces can have negative numbers. Those ideas a little later.)

-	**0**	**1**	**2**	**3**	**4**	**5**	**5**	**7**	**8**	**9**
0	0	1	2							
1		0	1							
2										
3										
4										
5										
6										
7										
8										
9										

Exercises - Chapter 2

Ex II.1 Example:
27 – 4 =?
Ans: Orally $\boxed{27} \longrightarrow \boxed{26} \longrightarrow \boxed{25} \longrightarrow \boxed{24} \longrightarrow \boxed{23}$

Start one two three four **(STOP)**
Ans = 23

Do Orally:
a. 25 – 4
b. 16 – 3
c. 17 – 5
d. 29 – 2
e. 30 – 6
f. 10025 – 4
g. 20016 – 3
h. 30017 -5
i. 50029 - 2
j. 99930 - 6

Ex II.2 Example:
26 – 8 =?
Answer: You can do as given in Ex II.1. But better is 26-8 = (26-10)+2=16+ 2

Start, $\boxed{16} \longrightarrow \boxed{17} \quad \boxed{18}$

0ne two **Ans = 18**

Do Orally (Shortcut Method):
a. 25 – 6
b. 16 – 7
c. 17 – 9
d. 29 – 8
e. 30 – 9
f. 10025 – 6
g. 20016 – 7
h. 30017 - 9
i. 50029 - 8
j. 99930 – 9

Ex II.3 Example:

123 – 12 =?

```
Ans:   123
      - 12
       -----
       111      Ans: 111
       -----
```

Do:

a. (987) – (76)
b. (976) – (43)
c. (654) – 42)
d. (109) – (107)
e. (20128) – (10117)
f. (17) – (6)
g. (26) – (3)
h. (34) – (12)
i. (9823109) – (9823107)
j. (138) – (107)

Ex II.4 Example:

(123) – (14) =?

```
         1
Ans:   123
      - 14
       -----
       109      Ans: 109
       -----
```

Do:

a. (987) – (88)
b. (931) – (43)
c. (654) – (65)
d. (107) – (88)
e. (20117) – (10128)
f. (27) – (18)
g. (56731) – (12343)
h. (123654) – (12365)
i. (987107) – (987088)
j. (7117) – (6128)

Ex II.5 Example:

Do the problems of EII.3 by “ULTA” Method (Opposite of “SEEDHA” method)

Ex: (123) – (14) =?

It is the same as

$$\begin{array}{r} 14 \\ + \boxed{} \\ \hline 123 \\ \hline \end{array} \quad \text{Ans: 109}$$

Do (a) to (j) by this method.

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Chapter 3 Addition, Subtraction

3. Addition, Subtraction

3.1 After Chapters 1 & 2, it is time to do addition and subtraction in the regular method. The students start working from the unit place (i.e. right hand side of the number).

3.2

```
   5      1 8     1 5     2 1 5     2 1 5     2 1 5
 + 4    +  4     +1 4       + 4    + 1 4    + 1 1 4
 ---    ----     ----    -------    -----     -----
 ---    ----     ----    -------    -----     -----

 3 2 1 5          3 2 1 5          1 2 3 4 5 0 0 6 7 8
 + 1 1 4        + 2 2 1 5          + 2 3 4 0 0 5 1 2 1
 ---------        --------         ---------------------
 ---------        --------         ---------------------
```

3.3 Now go to the next step:

```
  6       8 6       9 6     8 8 6     9 9 6
+ 4     +  4       + 4      + 4        + 4
 ---     ----      ----    -------    -----
 ---     ----      ----    -------    -----
9 8 8 6        9 8 8 6       9 8 8 6        9 8 8 6
+     4        +   1 4       + 1 0 4        + 1 1 4
---------      --------      ---------      --------
---------      --------      ---------      --------
```

3.4 Understand the term ‘Carry Over”. In these examples that carry over is only 1 ; it stands for ‘10’. Similarly ‘borrow’ is also only 1; but it is = 10.

3.5 See a calculator. Add 2 numbers; subtract the same – now use 3 or more numbers only for addition. See we can also do it by hand.

3.6 **Subtraction:**

3.6.1 Subtraction is the process of ‘removal’. Subtract B from A means, 2 things:

a. A is bigger than B.
b. B is taken away from A.

- The answer can be written as A – B.
- Subtracting bigger number from a smaller number is not possible (in simple arithmetic). At higher level, the answer can be written using negative numbers.

Thus 9 – 8, 9 - 7, 9 – 6, 9 – 1 …… etc., are OK
9 – 9 also is OK
(9 – 10) will require knowledge of negative numbers etc.,

3.6.2

```
   9 9 9 9
-  7 7 7 7   This is without any “Borrowing”
  ----------
   2 2 2 2
  ---------
```

Students can do:

```
1 2 3 4 5      1 2 3 4 5       1 2 3 4 5
- 2 3 4 5      - 2 2 3 4     - 1 1 3 4 5
 ----------    ----------      -----------
 ----------    ----------      -----------
```

3.6.3

```
       1
   9 9 9 2
-  7 7 7 3   This required “Borrowing”
  ----------
   2 2 1 9
  ---------
```

‘1’ shown above is borrowed from 10th place. This is equal to 10unit places, making the unit place as 12. From this 3 removed gives 9.

Now in the 20th place, one less, i.e., (8 only) remains. From this 7 subtracted gives 1.

Students can do:

```
  8 7 6 5      1 2 3 4 5      1 2 3 4
- 7 9 6 7      - 3 4 5 6      -  2 3 5
----------     ----------     -----------
----------     ----------     -----------
```

3.7 The above subtraction can be done by an “ULTA” method. If what was shown above is called “SEEDHA” what will follow can be called “ULTA”.

3.7.1 To Do:

```
   9 9 9 9        Ask what added to 2 will give me 9. This is 7.
 - 8 7 6 2        Write this in the unit place.
  ---------
 ---------
```

Thus $2 + \boxed{7} = 9$ $7 + \boxed{2} = 9$

$6 + \boxed{3} = 9$ $8 + \boxed{1} = 9$

Ans: 1 2 3 7

3.7.2 Doing (-) by ULTA Method:

9 9 9 9
\- 8 7 6 3
\---------
1 2 2 9
\--------

Ans: 1 2 2 9

8 7 6 5
\- 7 9 8 7
\---------
0 7 7 8
\----------

Ans: 7 7 8

3.7.3 1 0 0 0 0 1
\- 8 7 6 4 2
\-------------
1 2 3 5 9
\-------------

By ‘SEEDHA’ method

Ans: 1 2 3 5 9

The same by ‘ULTA’ method:

1 0 0 0 0 1
\- 8 7 6 4 2
\-------------
1 2 3 5 9
\-------------

Ans: 1 2 3 5 9

- In “ULTA” method subtraction problem is converted into ADDITION problem. Many people find this easier.

Exercises - Chapter 3

Ex III.1 Read aloud (in English or at least in mother tongue)

1
21
331
4331
54331
654321
7654321
87654321
987654321

- Rewrite these, inserting commas (,):

E.g.: **Indian system:** 87654321 → 8,76,54,321
American system: 87654321 → 87,654,321

- Now read aloud (in English) in these 2 methods.

Ex III.2 Write these numbers:

Nine
Eighty nine
Eight hundred eighty nine
Nine thousand eight hundred and eighty nine
Ninety thousand nine hundred and eleven
Nine hundred thousand and one
Nine lakh ninety thousand and one
Six million five hundred thousand and four
Seventy lakhs sixty thousand and five
4 billion and one
40 crore and one

Ex III.3

a. One hundred thousand = ……………………

b. 1 million = …….. lakhs
c. 1 crore = …….. lakhs
d. 1 crore = …….. millions
e. Population of India = ………… millions
f. Population of India = ………… crore
g. World population = ………… million
h. World population = ………… crores

Ex III.4 Example:

675 – 5 =?

675
- 5 (Written as 675 – 5 =? **Ans: 670**)

670

Do:

a. 605 – 5
b. 675 – 75
c. 675 – 575
d. 675 – 675
e. 675 – 1
f. 675 – 11
g. 675 – 111
h. 675 – 605
i. 675 – 615
j. 675 – 575
k. 675 – 585
l. 675 – 595
m. 675 - 85
n. 675 - 86
o. 675 - 98
p. 675 – 596

Ex III.5

a. 3 2 4 5 6 7 9 8
\- 4 5 6 7 8 8

----

b. 3 2 4 5 6 7 9 8
\- 4 5 6 7 9 8

----

c. 3 2 4 5 6 7 9 8
\- 4 5 6 8 0 8

----

d. 3 2 4 5 6 7 9 8
\- 4 5 6 8 1 8

----

e. 3 2 4 5 6 7 9 8
\- 4 5 7 8 0 8

----

f. 3 2 4 5 6 7 9 8
\- 4 6 7 8 0 8

----

g. 3 2 4 5 6 7 9 8
\- 5 6 7 8 0 8

----

h. 3 2 4 5 6 7 9 8
\- 3 2 4 5 6 7 8 0 8

----

i. 3 2 4 5 6 7 9 8
\- 3 3 4 5 6 7 8 0 8

----

j. 3 2 4 5 6 7 9 8
-3 3 5 5 6 7 8 0 8

(First do it; write down the answers. Then check the results by using a calculator).

Ex III.6 Fill using $\boxed{+}$ or $\boxed{-}$ in boxes:

Example:

2 ☐ 6 ☐ 2 = 10 Ans: 2 + 6 + 2 =10

2 ☐ 1 ☐ 9 = 10 Ans: 2 - 1 + 9 =10

Ex III.7 Fill in using (+) or (-):

1		4		5	=	10
2		5		3	=	10
3		6		1	=	10
4		7		1	=	10
5		8		3	=	10
6		9		5	=	10
7		6		9	=	10
8		5		7	=	10
9		2		1	=	10
8		4		6	=	10
8		4		2	=	10
7		5		8	=	10
7		5		2	=	10
6		3		7	=	10
6		3		1	=	10
5		4		9	=	10
5		4		1	=	10
3		2		5	=	10
3		2		9	=	10

Ex III.8 Students make their own problems like the above using any numbers between 1 and 99.

==

Chapter 4 Multiplication

4. Multiplication **(X)**

See that multiplication is only many additions of the SAME number.

4.1 The elementary idea of 10+10+10+10 = 4 x 10 = 40
/ how many times / (or by adding) = 40.

The simple idea can be a game by making students think of real life situations.

E.g.: How many tickets? One more
How many coffees? Two more
Wage/day = □, how many day's work?

4.2 Students are you comfortable with 'multiplication table' (= **Maggi**?) If not, it is OK.

4.3 When in doubt, add. i.e. when Maggi gets stopped, or doubt arises go to this method.

If 5 X 5 = 25; 6 x 5 =? 25 + 5 = 30

For 7 x 5 =? If (6 x 5) = 30, (7 x 5) is one more five (∴ 35)

2 methods:

a. (6 X 5) + 5 once = 30 + 5 = 35 or

b. (5 X 5) + (5 twice) = 25 + 10 = 35

4.4 Try method of 4.3 to great advantage in new and high number situations

a. If 29 x 10 = 290, what is 29 x 11
Answer: 290 + 29 = 290 + 30 (minus 1) = 320 – 1= 319

b. If 29 x 10 = 290, what is 29 x 9

Answer: 29 x 9 = 290 – 29
= 290 – 30 (plus 1) = 260 Plus 1 = 261

c. If (123456) x 10 = 1234560 Give the number
What is 123456 x 11

Ans.: 1 2 3 4 5 6 0
+ 1 2 3 4 5 6

Add & give this as answer 1238016

In the above, what is (123456) x 9?
Ans.: 1 2 3 4 5 6 0
- 1 2 3 4 5 6
----
1 1 1 1 1 0 4
----

d. **Game:** Play this game with a calculator (A group of 5, Leader has the calculator). He gives out the question as above. Asks the question. All the rest do by hand. He does using calculator. They compare.

E.g.: (56789) x (12345) = Give this number

What is (56789) x (12346)?

Or What is (56790) x (12345)?

4.5 **Game:** split the class into 9 groups. Let each group make 1 to 10 Maggi by this addition method.

i.e. Group I will do for 1 & 10
Group II will do for 2 & 10 etc

2 x 1 =
2 x 2 =

2 x 10

4.6 Students should complete the Multiplication Grid given below:

X	1	2	3	4	5	5	7	8	9
1	1								9
2									
3									
4									
5									
6									
7									
8									
9	9								81

4.7 Make many copies of the completed multiplication grid. Play different games.

a:

X	1	2	3	4	5	5	7	8	9
1	1							8	■
2									■
3									■
4									■
5									■
6									■
7									■
8	8							64	■
9	■	■	■	■	■	■	■	■	■

Cut off or blank out as given. Produce it by simple addition.

E.g.: 8 x 8 = 64; 8 x 9 = 64 + 8 = 72

Now 8 x 9 = 9 x 8 = 72

Therefore 9 x 9 = 72 + 9 = 81

b: Now cut off at 8

X	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	1						7	█	█
2								█	█
3								█	█
4								█	█
5								█	█
6								█	█
7	7						49	█	█
8	█	█	█	█	█	█	█	█	█
9	█	█	█	█	█	█	█	█	█

Recreate the full grid. Go like 8, 16, 24 …

from left to right and top to bottom.

Then do as in 4.7 a above.

4.8 Make the student realize that Maggi up to 9 is good enough. 10 is simple & easy. Now show them how 10 x 10 is good enough for all the rest.

4.8a Use 10 x 10 grid for double digit to convert multiplication into single digit

8 x 14 = 8 x 10 + 8 x4
= 80 + 32 = (80 + 30) + 2 = 110 + 2 = 112

3 x 17 = 10 x 3 + 7 x 3 = 30 + 21 = 51

4.8b Double-digit number multiplication
14 x 14 =?
(14 x 10) + (14 x 4) = 140 + (14 x 4)
= 140 + 40 + 16 = 180+ 16 = 196

Seems to be long but with a little mental arithmetic, this is faster than going 14 x 1, 14 x 2 …. up to 10. Secondly you are sure your answer is right.

4.9 Students can play game of 4.8. Form 2 groups: One, which is sure of their Maggi up to 20 x 20. Another who wants to try the 10 x 10 grid. Have games for both – either oral or written or in quiz format.

For quiz format, go back to 3.6 and use the examples given by the students.

4.10a Not knowing Maggi (or aversion to Maggi) need not be the reason to fear mathematics. Maggi is only one of the tools. We can do without it if we have a calculator. But we cannot eliminate it, better is to be bold about it. 10 x 10 is good enough.

4.10b Now we will show that even 5 x 5 is OK

X	**1**	**2**	**3**	**4**	**5**
1					
2					
3					
4					
5					

This is 5 x 5 multiplication grid
All the students would be able
to complete this, written or oral.

4.11

4.12 Now use this 5 x 5 grid to make a 10 x 10 grid. This is easy as given in 4.7.

4.13 Use 5 x 5 only for all purposes.

4.12a Same as 4.8(a)

8 x 14 = (5 x 14) + (3 x14)
= 50 + (5 x 4) + 30 + (3 x4)
= 50 + 20 + 30 + 12
= 112

3 x 17 = (3 x 10) + (3 x 7)
= 30 + (3 x5) + (3 x 2)

= 30 + 15 + 6 = 51

4.12b Same as 4.8b

14 x 14 = (10 x 14) + (4 x 14)
= 140 + 4 x 14
= 140 + (4 x 10) + 4 x 4
= 140 + 40 + 16 = 196

4.14 In every example, we have assumed that multiplication by 10 is very easy and innate in every student. Test this before going any further.

Do 3 x 10 = 30
4 x 100 = 400
7 x 100000 = 700000

Exercises - Chapter 4

Ex IV.1 Worked example:

1 2 3 4	The same can be: 2 x (1234)
X 2	= 2 x (1000 + 200 + 30 + 4)
--------	= 2000 + 400 + 60 + 8
--------	= 2468

Do by both the methods:

a. (123) x 3
b. (812) x 3
c. (922) x 4
d. (202) x 5
e. (123123) x 3
f. (812312312) x 3
g. (92222121) x 4
h. (102102102) x 5

Ex IV.2 Example: (1234) x 5 =?

1 2 3 4	Also 5 x (1000 + 200 + 30 + 4)
X 5	= 5000
--------	+ 1000
6 1 7 0	+ 150
---------	+ 20

	6170

Do:

a. (12) x 8
b. (22) x 8
c. (35) x 9
d. (9876) x 5
e. (9876) x 2
f. (9143) x 2
g. (9143) x 3
h. (2345) x 4
i. (2228) x 6

Ex IV.3 Example: (1234) x (12)

1234	Also (1234) x (10 + 2)
x 12	= 12340
--------	+ 2468
2468	-----------
12340	14808
-----------	-----------
14808	

Do both ways:

a. (34) x (12)
d. (2123) x (33)
g. (1004) x (35)
j. (909090) x (35)
b. (23) x (33)
e. (1234) x (35)
h. (9006) x (35)
c. (2234) x (22)
f. (9876) x (35)
i. (102030) x (35)

Ex IV.4 Do:

a. 56789 X 11
b. 56789 x 111
c. 56789 x 1112

d. 56789 x 12   e. 56789 x 123   f. 56789 x 1234

Ex IV.5 Example: (789) x (12345) =?

```
Method A:                Method B:
    789                     12345
x 12345                      x  789
-----------                ----------
   3945                     111105
  31560                     987600
 236700                    8641500
1578000                    -----------
7890000                     9740205
-----------               -------------
9740205
-----------
```

Method B is shorter than method A.
Therefore smaller number is taken at second place.

Do:

a. (12) x (6789)   b. (123) x (6789)
c. (1230) x (67890)   d. (12300) x (6789)

Chapter 5 Division - 1

5.1 Students have understood that:

- Multiplication is many times addition.
- Multiplying (by a number bigger than 1) increases the value.
- This increase is always in equal steps.

5.1.1 As said above, take 3 x 4

- 3 x 4 = 12 This can be done as:

```
   3
  +3
  +3
  +3       4 times
 ------
 = 12
 ------
```

- 3 increased to 12 because of multiplication.
- 3 became 6, the 9 and finally 12. It increased in equal steps

5.1.2 Exercise for students

Worked example: Do 6 x 3 by addition method

```
6 x 3 =    6      once
         + 6      twice
         + 6      thrice
         ------
          18                 Answer:      ∴ 6 x 3 = 18
         ------
```

- **Do by addition method:**
  - a. 2 x 5
  - b. 5 x 2
  - c. 8 x 3
  - d. 3 x 8
  - e. 17 x 4
  - f. 123 x3
  - g. 123456 x 3
  - h. 98765 x 4
  - i. 29 x 9 (here try 10 – 1 = 9)

5.2 If multiplication can be understood as, **‘many times addition’**, division can be called as ‘**many times subtraction’**.

5.2.1 3 x 4 = 12. This is known

Now $\frac{12}{3} = 4$ or 12 ÷ 3 = 4

This can also be done as:

```
 12
- 3   once
----
  9
- 3   twice
----
  6
- 3  thrice
------
  3
 -3  four times
-----
 0
```

This means, if you take away 3 at a time and like this four times, nothing is left. i.e., $\frac{12}{3} = 4$

5.3 Students can now understand:

- Division is many times subtraction.
- Dividing a number by another number (bigger than 1 & smaller than the first number) decreases the first number.
- Division decreases the original number always in EQUAL STEPS.

5.4 See $\frac{12}{3} = 4$ $\frac{8}{2} = 4$ $\frac{6}{2} = 3$

Now try = $\frac{13}{3}$ **Ans.** = 4; but 1 remains

$\frac{9}{2}$ **Ans.** = 4; but 1 remains

$\frac{7}{2}$ **Ans.** = 3; but 1 remains

5.4.1 Let us learn some words.

$\frac{13}{3}$: Ans = 4. Left out = 1

Here 13 is an **integer**
3 is an **integer** (smaller than 13)
Answer 4 is an **integer**
Left out 1 is an **integer**

The Answer is called **QUOTIENT**.
What is left out is called **REMAINDER**. (Remainder = that which remains)
Like copier = that which copies; Joke: Should it not be 'remainer'?

13 is called **dividend** and 3 is called **divisor**. We can live without these two words. But we should know:
TOP NUMBER (here 13) is also called **NUMERATOR**
BOTTOM NUMBER (here 3) is also called **DENOMINATOR**

5.4.2 **Exercises:**

Example: Write A ÷ B in fraction form and write down the numerator & denominator.

A $\div$ B $= \frac{A}{B}$ Numerator = A, Denominator = B

Now Do:

a. $15 \div 5$ b. $9 \div 3$ c. $21 \div 7$ d. $16 \div 4$
e. $36 \div 9$ f. $24 \div 6$g. $12345 \div 25$ h. $9676 \div 8$

5.4.3 Exercises:

Worked examples:

Write down the quotient (= Answer) and the remainder (zero is no remainder).

$8 \div 2$ $\frac{8}{2} = 4$ **Ans** = 4, remainder = 0

$9 \div 2$ $\frac{9}{2} = 4 + 1$ remains; $\therefore$ **Ans** = 4, Remainder = 1

Now Do:

a. $15 \div 5$ b. $16 \div 5$ c. $18 \div 5$ d. $9 \div 3$ e. $8 \div 3$
f. $\frac{10}{3}$ g. $12345 \div 25$ h. $12375 \div 25$ i. $9676 \div 8$ j. $9672 \div 8$

5.5 Multiplication tables (= ‘maggi’) are useful for multiplying and dividing. That is why primary schools insist on every child learning it. Learning maggi and remembering is necessary. Every student should know multiplication tables from 2 to 9.

5.5.1 Let MT (= multiplication table).
MT of 1 is easy 1 x 1 =1, 1 x 2 = 2 etc.,

Mt of 10 is also easy 1 x 10, 2 x 10 = 20 ……. 9 x 10 = 90, 10 x 10 = 100
These need not be memorized.

5.5.2 MT of 2 should be known. But it is easy to memorize.
2 x 1 = 2, 2 x 2 = 4 …… 2x 6 = 12 etc.,

MT of 5 is also easy and does not trouble the students.
5 x 1 = 5, 5 x 2 = 10 …… 5 x 6 = 30 etc.,
Just see the answer has 0 or 5 as the last digit.

5.6 Let us learn how to use MT for dividing.
Start with 9 x1 = 9, 9 x 2 = 18 …… 9 x 7 = 63, 9 x 8 = 72, 9 x 10 = 90
Now take 9 x 10 = 90. This is multiplication

What is $\frac{90}{10} = ?$ Ans = 9

What is $\frac{90}{9} = ?$ Ans = 10

Similarly $\frac{72}{8} = ?$ MT of 8 helps to find the answer.

Similarly $\frac{72}{9} = ?$ MT of 9 is used.

Thus Division is reverse of multiplication.

5.7 (Number 1) (Number 2) = Product.

Now, $\frac{\text{Product}}{\text{number 1}} =$ (number 2)

Also $\frac{\text{Product}}{\text{number2}} = (\text{number } 1)$

Example: (1234) x (56789) = 70077626

What is $\frac{70077626}{1234} = ?$ **Ans**. = 56789

Given that (13) x (17) = 221; $\frac{221}{17} = ?$ **Ans**. = 13

5.8 Exercises

5.8.1 a. Write down MT of 13 up to 5, now seeing this, answer:

(a1) $\frac{65}{13} = ?$ (a2) $\frac{52}{4} = ?$ (a3) $\frac{26}{2} = ?$ (a4) $\frac{39}{13} = ?$

b. Write down any MT. Frame your own questions from this.

c. Given (137) x (17) = 2329, $\frac{2329}{17} = ?$

d. Given (13579) x (24) = 325896 (Given)

Find $\frac{325896}{13597} = ?$ $\frac{325896}{24} = ?$

5.8.2 a. Same method as above can be used; even when there is a remainder.

E.g.: 7 x 1 = 7, 7 x 2 = 14 ……. 7 x 4 = 28, 7 x 5 = 35

$\frac{28}{7} = ?$ **Ans.** = 4

Let us ask $\frac{30}{7} = ?$ now nearest smaller number divisible by 7 is 28.

i.e., 7 x 4 = 28

$\therefore \frac{30}{7} = \frac{28}{7} + \text{remainder}$, **Ans.** =4 with remainder 2

b. Given (137) x (17) = 2329, $\frac{2335}{17} = ?$

Given number which we know is 2329, 2335 – 2329 = 6

$\therefore \frac{2335}{17} = 137 + \text{remainder } 6$

5.8.3 a. Write down the table of 13 up to 5. Now, using this table find answer (i.e., after dividing) and remainder.

(a1) $\frac{67}{13} = ?$ (a2) $\frac{50}{4} = ?$ (a3) $\frac{25}{2} = ?$ (a4) $\frac{42}{13} = ?$

b. Use another MT; Frame your own questions.

c. Given that (13) x (17) = 221, $\frac{228}{13} = ?$

d. Given that (13) x (17) = 221, $\frac{228}{17} = ?$

e. Given that (1234) (56789) = 70077626, (Known)

(e1) $\frac{70077626}{56789} = ?$ (Clue – no remainder)

(e2) $\frac{70077628}{56789} = ?$ (Clue – with remainder)

(e3) $\frac{70087626}{56789} = ?$ (Clue – with remainder)

5.9 Division by single digit numbers (i.e. 1 to 9):

5.9.1 Division by 2 is very easy.

Eg: 122 ÷ 2 2 | 122 **Ans**. = 61
61

121 ÷ 2 2 | 121 **Ans.** = 60, Remainder = 1
60 – 1

- It is good for the students to learn some words and concepts, related to 2.
- Numbers like 2, 4, 6, 8, 10 etc are all divisible by 2. These are called **EVEN numbers**.
- All the numbers 1, 3, 5, 7, 9 and any number ending in any of these numbers are called **ODD** numbers.
- EVEN numbers are divisible by 2.
- ODD numbers are not divisible by 2 (i.e., there will be a remainder, 1).

5.9.2 We have seen that if you know 'maggi' (=MT, Multiplication Table), division becomes easy. Assuming the student knows MT of 1 to 10, we can say division by single digit number is easy. This is the reason multiplication is done as shown in 5.9.1, in case divisor (= denominator) is a single digit number.

5.9.3 Exercises

Example 1: 92345678 ÷ 3 =?

Ans: 3 | 92345676 **Ans.** = 30781892
30781892

Example 2: 9236 ÷ 3 =?

Ans: 3 | 9236 **Ans**. = 3078 + (Remainder 2)
3078 – 2

Now do:

a. 92345676 ÷ 2
b. 92345676 ÷ 4
c. 92345676 ÷ 5
d. 92345676 ÷ 6
e. 92345676 ÷ 7
f. 92345676 ÷ 8
g. 92345676 ÷ 9
h. 92345676 ÷ 10
i. 124 ÷ 6
j. 125 ÷ 5
k. 126 ÷ 7
l. 121 ÷ 11

5.9.4 When you write down MT of 2 to 10, you see some pattern. These patterns have given the clue to find whether a given large number could be divided by another (single digit) number without any remainder. These are given here as **divisibility rules**. [They are not rules; they are qualities of the behavior].

5.9.5a **1. Divisibility:**

- All numbers are divisible by 1 and by itself.
- Odd numbers, even numbers identification
- 2 – all even numbers.
- 3 – add all digits – result divisible by 3
- 4 – last 2 digits divisible by 4
- 5 – numbers ending in 0 or 5
- 6 – divisible both by 2 and 3
- 8 – divisible both by 2 and 4
- 8 - last 3 digits divisible by 8
- 9 – add all digits – divisible by 9
- 10 – last digit 0

2. Prime Numbers:

- Can be divided by 1 or itself
- Factorization not possible.
- Odd numbers.
- Some examples: 11, 13, 17, 19, 23 ……

3. **Special rule for 11:** (Sum of even digits)–(Sum of odd digits)= 0

5.9.5b **Examples:**

- 123 – sum = 6 div by 3 $\therefore$ number 123 is divisible by 3
- 12345 – sum = 15 div by 3 $\therefore$ number 12345 is divisible by 3
- 1218 & 1318 are not divisible by 4 because $\frac{18}{4} \neq$ integer
- But 1216 & 1308 are divisible by 4 because $\frac{16}{4}$ & $\frac{08}{4}$ = integers
- Any even number AND divisible by 3

  12346 - even number; div by 3, NO 16 = digits addition

  Try $\frac{12346}{6} = 2057\frac{4}{6}$

  123462 - even number; div by 3

  Try $\frac{123462}{6} = 20577$ OK

  - 12345 - yes div by 3; By 9?

    Try $\frac{12345}{9} = 1371\frac{6}{9}$; Not div by 9

    Try 22446 Sum = 18, $\therefore$ div by 9

    Check $\frac{22446}{9} = 2494$ OK

    Or Try 123453 Sum = 18, $\therefore$ div by 9

    Check $\frac{123453}{9} = 13717$ OK

  - 1221 – div by 11? 1+2 = 1+2; $\therefore$ yes

    Check $\frac{1221}{11} = 111$

  12321 - div by 11? 1 + 3 + 1 =5, 2+ 2 = 4; $\therefore$ No

  Check $\frac{12321}{11} = 1120\frac{1}{11}$

5.9.6 Divisibility Exercise:

A. Without actually dividing, state whether the following number is divisible by 92345676.

a. by 3	c. by 2	e. by 6	g. by 5
b. by 4	d. by 9	f. by 11	

B. Students can make their own questions.

5.9.7 Factorization

Divisibility rules help us in factorizing a given number.

Eg: 6 = 2 x3 2 and 3 are FACTORS of the number 6.
Eg: 21 = 3 x 7 3 & 7 are factors of 21
Eg: 42 = 2 x 3 x 7 2, 3 & 7 are factors of 42

Exercises:

i. Say if 2 is a factor or not: a. 1235 b. 1235 c. 5000 d. 5005

ii. Say if 3 is a factor or not: a. 27 b.31 c. 121 d. 123 e. 1234 f. 1232

iii. Say if both 5 & 3 are factors a. 275 b. 285 c. 1230 d. 1235

iv. Write all the factors: a. 362880 b. 3628800 c. 399168 d. 420
(Clue: Try shortcut method of ALL single digits one by one)

5.9.7.1 (Extra) Factories: 16, 21, 28, 144, 153

- 16 = 2 x 8 = 2 x 2 x 4 = 2 x 2 x 2 x2
  Now, this can be written as: 16 = 2 x 2 x 2 x2
  = 2 x 8 = 4 x 4

- 21 = 3 x 7 (& no more)

- 28 = 2 x 14 = 2 x 2 x 7
  ∴ 28 = 2 x 14 = 4 x 7

- 144 = 2 X 72 = 2 x 2 x 36 = 2 x 2 x 2 x 18
  = 2 x 2 x2 x 2 9 = 2 x 2 x2 x 2 x 3 x 3
  ∴ 144= 2^4 x 3^2
  This can be written in many ways

- 153 = 3 x 51 (& try 51 now)
  = 3 x 3 x 17 (17 happens to be a prime number ∴ no more)
  ∴ 153 = 3 x 51 = 9 x 17

Exercise: Factorise

a. 6	b. 62	c. 98	d. 256	e. 22	f.198
g. 106	h. 162	i. 298	j. 10256	k.15	l. 47
m. 141	n. 255	o. 189	p. 115	q.147	r. 241
s. 55	t. 199				

5.9.8 Approximation:

Helps in many calculations. In division problems, a rough idea of the magnitude of the numbers helps.

E.g.: We have 26 mangoes. This is to be shared by 3 persons. How many per person?

We know 26 cannot be shared equally but how much. One can start by doing:

to A, B, C $\boxed{1}$ $\boxed{1}$ $\boxed{1}$

$\boxed{1}$ $\boxed{1}$ $\boxed{1}$

Then + + + to A, B, C etc.,

Or Give $\boxed{2}$ $\boxed{2}$ $\boxed{2}$ to A, B, C

Then + +$\boxed{2}$ +$\boxed{2}$ to $\boxed{2}$ C etc.,

Instead try this way: If I had 30 mangoes. I could have given 10 each. But I have four less. Therefore each will get approximately 7 or 8. Now try dividing you will get.

Thus approximation helps in division. This helps when we forget our “maggi” (= multiplication tables).
E.g.: Divide 299 by 13

Actual method:

$$\begin{array}{r|l} & 23 \\ \cline{2-2} 13 & 299 \\ & 26 \\ \cline{2-2} & 39 \\ & 39 \\ \cline{2-2} & 0 \\ \cline{2-2} \end{array}$$

Approximation method: 13 x 20 = 260. Remaining is ~ 40. This gives ~ 3
Ans: 20 + 3 = 23

Exercise: Do both ways - Actual Division & Approximation Method

a. 6860 ÷ 7 b. 788 ÷ 8 c. 12423 ÷ 123
d. 1678 ÷ 17 e. 1768 ÷ 17 f. 525252 ÷ 51
g. 122232 ÷ 1202 h. 1222301 ÷ 1201 i. 998 ÷ 19 J. 1028 ÷ 19

Exercises - Chapter 5

Ex V.I Learn how a question can be asked.

Eg: Divide 25 by 5 or 25 ÷ 5 or $\frac{25}{5}$ or If 25 items are equally distributed (or divided or shared or …..) (Among/by) 5 (Persons/portions/…), what will be each person’s share?

All the above questions have the same answer= $\frac{25}{5}$ = 5

Divide:

a. Divide 27 by 3 b. 27 ÷ 9 c. 12 ÷ 4 d. $\frac{36}{4}$ e. $\frac{15}{5}$ f. $\frac{40}{8}$

g. $^{18}/_{9}$ (new computer notation)

h. If 4 persons share a basket of mangoes and the basket contains 12 dozen mangoes. How many mangoes each one will get.

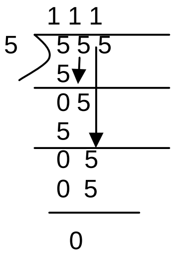
i. If the basket in (h) cost Rs. 600, how much each person should pay?

j. In (i) above, what will be approximate cost of each mango.

k. A hostel has 30 students, each, will eat 4 idles. How many idles will be needed? [Clue: This is a question on multiplication].

l. In (k), above only 92 idles are available. How many idles will each one get? [This is a question on division].

Ex V.2 Simple single digit division exercises.
Example: 555 ÷ 5



Start from left, one digit at a time. Write quotient above. Bring one digit down see arrow.

Ans: 111

[Do not write 5) 555 (111 This is not good]

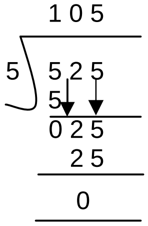
Do the following by the correct way of writing, as shown above.

a. 242 ÷2 b. 369÷3 c. 848 ÷4 d. 1055 ÷5 e. 660 ÷6

f. 707 ÷7 g. 888÷8 h. 909 ÷9 i. 4848484 ÷4 j.2424242 ÷2

k. 369369369963 ÷3 l. 84884848848 ÷8

Ex V.3 Eg: 525 ÷ 5



Bring down one digit at a time. If one is not enough, bring the next also. But put one 0 at the top.

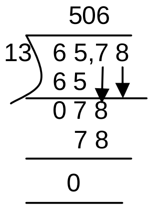
Ans: 105

Do:

a. 327 ÷3 b. 927 ÷9 c. 812 ÷4 d. 436 ÷4 e. 1015 ÷5 f. 840 ÷8

g. 918 ÷9 h. 949 ÷9 i. 981 ÷9 j. 749 ÷7

Ex V.4 Dividing by 2 digit number [knowing maggi up to 16 is helpful]
Eg: 6578 ÷13



, introduced by us.

Ans: 506

Do:

a. 121 ÷11 b. 5566 ÷11 c. 8472 ÷12 d. 3952 ÷13

e. 5642 ÷14 f. 9870 ÷14 g. 4590 ÷15 h. 4695 ÷15

i. 8032 ÷16 j. 9696 ÷16 k. 345167 ÷17 . 3672 ÷18

m. 9557 ÷19 n. 9728 ÷ 19

Chapter - 6 Division –2

6.1 How to write A $\div$ B means A is divided by B. Also written as A/B or $\frac{A}{B}$ (/ is the computer key). Some persons call $\frac{A}{B}$ as fraction and A $\div$ B as division. But both are the same. There is an advantage if one writes division as $\frac{A}{B}$.

$\frac{A}{B}$ = A ($\frac{1}{B}$). When ($\frac{1}{B}$) is a known quantity, $\frac{A}{B}$ (i.e., division) can be considered as A x ($\frac{1}{B}$). When ($\frac{1}{B}$) is a known quantity, $\frac{A}{B}$ (i.e., division) can be considered as A x ($\frac{1}{B}$) (i.e., multiplication).

Example: $\frac{12345}{3}$ is the same as 12345 $\div$ 3.

12345 is the dividend, 3 is the divisor.

In this form: $\frac{12345}{3}$ = (12345) is called numerator, 3 is called denominator.

6.1.1 For Convenience, we can say: $\frac{A}{B}$ is division when A > B

$\frac{A}{B}$ is a fraction when A < B. Thus $\frac{12}{3}$ is division (Ans: 4). $\frac{2}{3}$ is a fraction.

Do $\frac{13}{3}$ Ans = 4 (remainder= 1). This remainder is a fraction (=$\frac{1}{3}$).

6.1.2 Write A $\div$ B in $\frac{A}{B}$ form. Write down the numerator and denominator.

Example: 9876 $\div$76 Ans= $\frac{9876}{76}$; Numerator = 9876, Denominator = 76

Do:

a. 15$\div$ 3 b. 1501$\div$501 c. 999 $\div$ 9 d. 101 $\div$ 101
e. 1010 $\div$ 10

6.1.3 Exercises

Write $\frac{A}{B}$ in A $\div$ B form.

a. $\frac{15}{5}$ b. $\frac{10}{3}$ c. $\frac{123123}{123}$

6.1.4 Exercises

Write $\frac{A}{B}$ as (quotient) + fraction

Example: $\frac{7}{3}$ = 2 + $\frac{1}{3}$ or $2\frac{1}{3}$

Do:

a. $\frac{9}{2}$ b. $\frac{10}{3}$ c. $\frac{17}{4}$ d. $\frac{19}{5}$ e. $\frac{15}{6}$ f. $\frac{20}{7}$

g. $\frac{31}{8}$ h. $\frac{17}{9}$

6.2 Dividing by single digit number.

Example: $\frac{10}{3} = ?$ $\frac{10}{3} =$ *3 (remainder 1)* = $3+\frac{1}{3}$

Do:

a. $\frac{10}{2}$ b. $\frac{11}{3}$ c. $\frac{10}{4}$ d. $\frac{12}{4}$ e. $\frac{10}{5}$ f. $\frac{11}{5}$

g. $\frac{12}{6}$ h. $\frac{13}{6}$ i. $\frac{14}{7}$ j. $\frac{15}{7}$ k. $\frac{15}{8}$ l. $\frac{16}{8}$

m. $\frac{17}{9}$ n. $\frac{18}{9}$ o. $\frac{10}{10}$ p. $\frac{10}{9}$ q. $\frac{10}{8}$ r. $\frac{10}{7}$

s. $\frac{10}{6}$

6.3 We have seen:

0 x 5 = 0 and 5 x 0 = 0
0 x 1 = 0, 0 x 10 = 0 etc

(Anything) x 0 = 0 and 0 x (Anything) = 0

Similarly

$0 \div 1 = 0$ i.e. $\frac{0}{1} = 0$

$0 \div 5 = 0$ i.e. $\frac{0}{5} = 0$

$0 \div 10 = 0$ i.e. $\frac{0}{10} = 0$

$$\therefore \quad \frac{0}{\text{(anything)}} = 0$$

BUT $\frac{\text{anything}}{0}$ is NOT ZERO. It has a special meaning.

6.4 Dividing by 10, 20, etc

6.4.1 Dividing by 10

Compare with multiplying by 10
5 x 10 = 50
51 x 10 = 510
60 x 10 = 600
12345000 x 10 = 123450000

i.e. Add one zero for multiplying by 10. Similarly cut one zero for dividing by 10.

$$\therefore \frac{50}{10} = 5 \qquad \frac{510}{10} = 51 \qquad \frac{600}{10} = 60 \qquad \frac{1234500}{10} = 1234500$$

Exercises:

a. 123 x 10
b. 132 x 10
c. 1001 x 10
d. 10101 x 10
e. 9080701 x 10
f. $1230 \div 10$
g. $1231 \div 10$
h. $1320 \div 10$
i. $10010 \div 10$
j. $100105 \div 10$
k. $101010 \div 10$
l. $101026 \div 10$

m. $908070 \div 10$ n. $9080701 \div 10$

6.4.2 Dividing by 20

a. $\frac{120}{20} = \frac{12}{2} = 6$. Cut one zero for 10 first and then divide by 2.

b. $\frac{121}{10}$ To do this divide 121 by 10

i.e. $\frac{121}{10} = 12$ and (remainder 1)

$\therefore$ $\frac{121}{20} = \frac{12}{2} = 6$ and (remainder 1)

c. $\frac{129}{20}$ first $\frac{129}{10} = 12$ (+ remainder 9)

Then $\frac{12}{2}$ = 6. $\therefore$ 6 (+ remainder 9)

d. Exercises:

d1. $\frac{40}{20}$ d2. $80 \div 20$ d3. $\frac{100}{20}$ d4. $\frac{2000}{20}$

d5. $2020 \div 20$ d6. $12340 \div 20$ d7. $88880 \div 20$ d8. $99980 \div 20$

d9. $10101020 \div 20$

e. Exercises:

e1. $\frac{41}{20}$ e2. $\frac{81}{20}$ e3. $\frac{99}{20}$ e4. $\frac{2009}{20}$

e5. $2025 \div 20$ e6. $12348 \div 20$ e7. $88888 \div 20$

e8. $99989 \div 20$

6.4.3 Dividing by 20 (Contd) [This is difficult to understand $\therefore$ optional].

6.4.3a $\frac{130}{20} = \frac{130}{2 \times 10} = \frac{13}{2} = 6$ (+ remainder 1)

But this remainder 1 is really = 10.

$\therefore$ Ans. = 6 (+ remainder 10)

Now Do: i. $\frac{90}{20}$ ii. $\frac{30}{20}$ iii. $\frac{990}{20}$

6.4.3b $\frac{130}{20} = \frac{139}{2 \times 10}$

First $\frac{139}{10} = 13$ (+ remainder 9)

Next $\frac{13}{2}$ = 6 (+ remainder 1) But this last remainder is really = 10

$\therefore \frac{130}{20} = 6$ (+ remainder 10 + 9= 19)

Now Do: i. $\frac{98}{20}$ ii. $\frac{36}{20}$ iii. $\frac{988}{20}$

6.5 **Don't do:** Splitting the numbers as you like

6.5.1 $\frac{20}{7} = 2$ (+ remainder 6) This is Correct

Now try $\frac{20}{2 + 5} = (\frac{20}{2})$ first 2 then by 5

$= \frac{20}{2} = 10$ then $\frac{10}{5} = 2$ (no remainder) This is Wrong

6.5.2 Try $\frac{70}{7} = 10$ (OK)

Now $\frac{70}{2 + 5} = \frac{35}{5} = 7$ (wrong)

i.e. the bottom number (=denominator) **should not** be split into **addition** of 2 numbers.

6.6 Now you see **WHAT YOU CAN DO.**

6.6.1 $\frac{24}{4} = ?$ Ans = 6 (by 'maggi')

Can also do $\frac{20+4}{4} = \frac{20}{4} + \frac{4}{4}$
$= 5 + 1 = 6$

i.e., TOP NUMBER can be written as a SUM (i.e., addition). But division should be done for each one on the top.

Do (by converting numerator as addition of 2 number):

a. $\frac{205}{5}$ b. $\frac{2025}{5}$ c. $\frac{5555}{11}$ d. $\frac{2821}{7}$ e. $\frac{333378}{3}$

f. $\frac{17171734}{17}$ g. $\frac{34516817}{17}$

(For some of these, sum can be more than two numbers).

6.6.2 Splitting the numerator (as a sum). This is ALLOWED. Thus $\frac{24}{4}$ can be written as $\frac{20+4}{4}$.
This is possible because 20 is divisible by 4.
Similarly $\frac{225}{5}$ can be written as $\frac{200+25}{5} = \frac{200}{5} + \frac{25}{5} = 40 + 5 = 45$
This is because 200 can be divided by 5.
$\frac{3577}{7} = ?$ If you split as 3000 + 577 it does not help.
But you can split it as $\frac{3500+77}{7} = \frac{3500}{7} + \frac{77}{7} = 500 + 11 = 511$

Do by suitable splitting of numerator:

a. $30021 \div 3$ b. $\frac{1339}{13}$ c. $\frac{2200121}{11}$

6.6.3 Splitting need not be at 10 or 100. It can be anywhere.

Eg: $\frac{49}{7} = ?$ If we are not sure of our maggi and we know 7 x 5 = 35.

Do 49 = 35 +14 $\therefore \frac{49}{7} = \frac{35+14}{7} = 5 + 2 = 7$

Do as suggested by clues:

a. $\frac{156}{12}$ (We know 12 x 12=144) b. $\frac{143}{11}$ (Clue 11 x 10 = 110)

c. $\frac{169}{13}$ (We know 13 X 10 = 130) d. $\frac{209}{19}$ (Clue 19 X 10 = 190)

e. $\frac{544}{17}$ (Clue 17 X 3 = 51)

6.7 **Can Do**: $\frac{70}{14} =$

14 is a tough number
∴ 14 = 2 x 7

6.7.1 $\frac{70}{2 \times 7} = \frac{10}{2} = 5$ Answer

6.7.2 $\frac{48}{6} = 8$ (OK)

Try this wrong method: $\frac{48}{6} = \frac{48}{4 + 2}$

$= \frac{48}{4} = 12$ then $\frac{12}{2} = 6$ (Wrong)

Instead 6 = 2 x 3. $\therefore \frac{48}{6} = \frac{48}{2 \times 3}$

First $\frac{48}{2} = 24$ then $\frac{24}{3} = 8$ (OK)

6.7.3 Rule: DENOMINATOR can be split as product of 2 or more numbers (i.e., factorization is OK). Sum (i.e., +) in denominator is not correct.

6.7.4 Exercises:

a. $\frac{12345}{55}$ b. $\frac{12321}{33}$ c. $\frac{10094}{98}$ d. $\frac{16807}{49}$ e. $\frac{2401}{49}$ f. $\frac{16807}{343}$

6.8.1 Top number (= numerator) can be split into two parts. It can be addition or subtraction or multiplication.

6.8.2 Top numbers 25 = 20 + 5

a. $\frac{25}{5} = 5$ (OK)

$\frac{20 + 5}{5} = \frac{20}{5} + \frac{5}{5} = 4 + 1 = 5$ (OK)

b. Top number 25 = 5 x 5

$$\frac{25}{5} = \frac{5 \text{ x } 5}{5} \quad \text{(Here only once)}$$

$$= \frac{5}{5} \text{ x } 5 = 1 \text{ x } 5 = 5 \quad \text{(OK)}$$

c. $\frac{15}{5} = 3$ (OK). Now top number 15 = 20 – 5

$$\frac{15}{5} = \frac{20 - 5}{5}$$

$$= \frac{20}{5} - \frac{5}{5} = 4 - 1 = 3 \text{ (OK)}$$

d. $\frac{63}{7} = ?$ Top number 63 = 3 x 21

$$\frac{63}{7} = \frac{3 \text{ x } 21}{7} = 3 \text{ x } 3 = 9$$

e. $\frac{35}{7} = ?$ Top number 35 = $\frac{70}{2}$

$$\therefore \frac{35}{7} = \frac{70}{7} \div 2 = 10 \div 2 = 5 \text{ (OK)}$$

6.8.3 **Exercises: Do by at least 2 different methods for each problem.**

a. $\frac{28}{14}$ b. $\frac{650}{26}$ c. $\frac{154}{11}$ d. $\frac{12321}{33}$ e. $\frac{9595}{5}$ f. $\frac{95}{5}$

g. $\frac{9595}{95}$

Exercises - Chapter 6

Ex. VI.1 a. $\frac{456780}{4}$ b. $\frac{456780}{3}$ c. $\frac{456780}{5}$ d. $\frac{456780}{20}$ e. $\frac{456780}{12}$

f. $\frac{456780}{15}$ g. $\frac{456780}{60}$ h. $\frac{45678}{6}$

Ex. VI.2 Fill up using +, -, x, ÷ only

1. 10 □ 10 □ 10= 10
2. 10 □ 10 □ 10= 2
3. 10 □ 10 □ 10= 200
4. 10 □ 10 □ 10= 0
5. 10 □ 10 □ 10= 1000
6. 10 □ 10 □ 10= 90
7. 10 □ 10 □ 10= 30
8. 10 □ 10 □ 10= 110
9. 10 □ 10 □ 10= 11

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Chapter - 7 Basic Operations

7. **Four basic operations**

These are +, -, x, and ÷
When do we use them? Other than in the classroom? Yes, we use them almost daily, almost everywhere.

7.1 **Addition** of many numbers is a very common phenomenon.

a. Accounts book of any company
b. Daily accounts of any household
c. Daily sales of any small shop
d. Totaling of marks in an exam papers
e. Counting of total number of students in a school

Students can make this list and make it longer.

7.2 Real life **subtraction** also will be similar to 7.1.

7.3 Multiplication events:

a. Bus conductor
b. Any shopkeeper
c. Budget planner, accountant

Let the students make this list longer.

7.4 Division events:

a. Markets – per unit price.
b. Calculating property share
c. Calculating run rate – runs / over etc

7.5 Play a game

Make 8 groups of students.
Let there be a panel of judges consisting of students from each group.
Allot mark for each idea given and bonus marks for every good idea
Find a winning team etc.

Exercises - Chapter 7

Ex. VII.1 Example: Select the right one:
8 x (9876) =? a. 7200058 b. 790008 c. 79008 d. 79004

Ans: c

How did you arrive at this selection? Not by actual working, but by guessing. 9876 is almost 10000 (but less) 8 times 10000 = 80000. Answer should be less than 80000. So Ans ≠ (a) or (b). Out of (c) & (d) which one? Last digit is (8x 6) = 48. ∴ Last digit in the answer should be 8. ∴ (c) Is the answer?

Now do:

a. 19 x 99=?	a. 1881	b. 18081	c. 19991	d. 1991
b. 21 x 101=?	a. 2101	b. 2121	c. 21011	d. 2221
c. 41 x 19=?	a. 719	b. 819	c. 779	d. 829
d. 3 x 18=?	a. 21	b. 34	c. 74	d. 54

e. Let the student use a calculator and make his own questions.

Ex. VII.2 E.g.: If the product of two numbers is 21, which are these numbers?

Ans: 3 and 7 (21 and 1) also.

Now Do:

a. Product = 9 b. 15 c. 49 d. 55 e. 299

f. 8 g. Student can make own questions.

Ex. VII.3 E.g.: If the product of two numbers is 42. Which are the numbers?
Answers: 42 = 42 x 1(Omit this simple answer).
= 2 x 21 = 6 x 7 = 3 x 14
Any of the three above is correct.

Now Do: Let the product be = P

a. P = 18 b. 60 c. 98 d. 165 e. 1495 f. 16

g. Students own questions.

Ex. VII.4 Example: 7 x 49=? Do not start multiplying. Do mentally (or by shortcut with a paper). 49 is almost equal to 50. 7 x 50 = 350.
Less (7 x 1). ∴ 350 – 7 =343.

Do:

a. 4 x 19 b. 14 x 9 c. 99 x3d. 8 x 15 e. 8 x 151 f. 18 x 15

g to J – students can make his own questions and answer.

Ex. VII.5 E.g.: if 50 x 12 = 600, 49 x 12 =?
Ans.: Mentally 12 less than 600 = 588

Do:

a. If 41 x 19 = 779, 41 x 18 =?
b. If 3 x 18 = 54, 103 x 18 =?
c. Student can extend the same questions.

Ex. VII.6 Example: if 3 X 18 = 54, 54 ÷ 3 =? Ans.: 18

Do:

a. If 7 x 19 = 133, 133 ÷ 19 =?
b. If 7 x 19 = 133, 133 ÷ 7 =?
c. If 41 x 19 = 779, 779 ÷19 =?
d. If 41 x 19 = 779, 779 ÷ 41 =?
e. If 41 x 19 = 779, 7790 ÷ 41 =?
f. 41 x 19 = 779, 779000 ÷19 =?
g. If 103 x 18 = 1854, 1854 ÷ 18 =?
h. If 103 x 18 = 1854, 1854 ÷ 206 =?
i. If 103 x 18 = 1854, 1854 ÷ 618 =?

Ex. VI.7 Example: Let 2 x 3 x 4 x 5 x7 = 840. Then,

$$\frac{840}{2} =(3 \text{ x } 4 \text{ x}5 \text{ x } 7) =$$

$$\frac{840}{6} =(4 \text{ x } 5 \text{ x } 7) =$$

Do: Given 6 x 7 x 8 x 9= 3024

a. $\frac{3024}{9}$ b. $\frac{3024}{72}$ c. $\frac{3024}{504}$ d. $\frac{3024}{42}$ e. $\frac{3024}{56}$ f. $\frac{3024}{54}$

Ex. VII.8

Students should know how to convert written words into mathematical problems. E.g.: You are a conductor in a bus. There are 50 passengers, 20 are children. There are adults. Adult ticket is Rs. 5/- Half ticket is Rs. 3 /-. How much do you collect?

No. of children = 20, Ticket per child = Rs. 3 $\therefore$ Total = 20 x 3 = Rs. 60
No. of adults = 50 – 20 = 30, Ticket per adult = Rs. 5
$\therefore$ Total = 30 x 5 = Rs. 150
Total money collected = Rs. 60 + Rs. 150 = Rs. 210.

Questions:

a. In the example above 10 adults have pass. How much money is collected?

b. You are a sales person. One pencil costs Rs. 3 and sale price of a pen is Rs. 7. If a person buys 4 pens and 4 pencils and gives you a fifty rupee note, how much will you return?

c. You are treating your friends in a hotel. Everyone gets 2 vadas, 1 Gobi and 1 ice cream. Cost of vada is Rs. 6, Gobi is Rs. 10, ice-cream is Rs. 15. How much will be the bill? (Clue: Ask how many persons).

d. 1 kg of glucose biscuits packet was bought. We opened and counted. There were 100 biscuits. 20 students were present in the class. How many biscuits each student will get? If 5 more students come and join in, how many biscuits per person?

e. A company was run using the capital raised by 50 shareholders. All have equal number of shares. Profits are shared equally. Annual profit was Rs. 25,000. How much profit share will each shareholder get?

f. 1 dozen mangoes cost Rs. 72. What is the cost of each mango?
1 dozen pens cost Rs. 72. Each pen?
Now put notebook, sweet, etc., in place of mango (Clue: 1 dozen = 12 items).

g. A quintal of rice costs Rs. 3000 (as in May 2009). What is the cost of 1 kg Rice? (Clue: 1 quintal = 100 Kg).

h. Students can form their own questions, taking from day-to-day life.

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Chapter - 8 Rule of Three

8. **Rule of three.**
This is very common – is very important and is always bothering (giving trouble) the students.

8.1 If a box of 10 pencils costs Rs. 22. What is the cost of 3 pencils?
Go to price of one by division and get price of 3 by multiplication.
[This is also called UNITARY method].

8.2 A box of 3 dozen Alphonso mangoes costs Rs. 720.
What is the cost of 3 mangoes?

8.3 If a quintal of dhal costs Rs. 5000.
What is the lowest possible retail price of 1 kg of dhal?
[1 quintal = … kg should know].

8.4 Basava has 2 acres and 20 guntas of land. If land cost is settled at Rs. 5 lakhs per acre. How much will Basava get?
[Teacher can put in real life jokes of document cost, chai-pani cost, dalal's fees etc for fun and keeping the class of rural students alive].

8.5 Unitary method or **Rule Of Three** is very important. It has applications in business, commerce, economics, politics, budgeting and certainly in engineering. **METHOD of SOLVING** is important

8.6 Examples:

a. IF 1” = 2.5 cm, 1 foot = ? Cm

b. If 1 mile = 1.6 km, marathon runner of 26 miles runs how many kilometers?

c. If 1.6 km = 1 mile, what is the mph of a car running at 80 kmph.

d. Give a fast bowler’s ball speed in both the units
or
What is Nadal’s ace service speed?

e. Someone’s weight is 40 kg. How much in pounds (1 kg = …..)

f. If 1 US Dollar = Rs. 45, $ 100 = Rs ?.

g. If you have Rs. 10000, how may US dollars?

h. If a 300 gm biscuit packet of Parle costs Rs. 24 and a 90 gm packet of Britannia costs Rs. 8, which is cheaper? By how much per kilogram?

8.7 Rule of TWO or UNIT VALUE
E.g.: 10 items is worth 20.
Unit value (or 1 item) is worth how much?

$$10 \longrightarrow 20$$

$$1 \longrightarrow \frac{20}{10} = 2 \qquad \text{Ans} = 2$$

Do:

1. Pencils cost Rs. 10, each =?
2. 3 shirts for Rs.99, each=?
3. 40 students ate a total of 200 idles. How many each?
4. 30 days (one month) electricity use was 240 KWH (i.e., units). How many per day?
5. In (4) above what is annual consumption?
6. Old (ancestral) house was sold for 12 lakhs. 4 sons equally share. How many each one will get?
7. In (6) above if 2 daughters also should be given equal share, how much each son will get? How much each daughter will get?
8. In (7) above, 8 lakhs was used for starting a business. What was each Son’s contribution?
9. In (8) above, the capital was increased to 9 lakhs and daughters also join in, what is each person’s share?

8.8 **Rule of three;** Unitary method (or many to one and then one to many).
E.g.: 1 dozen bananas cost Rs. 24. What is the cost of of 4 bananas?

Ans.: (1 dozen) = 12 bananas $\longrightarrow$ Rs. 24

$$1 \text{ banana} \longrightarrow \frac{24}{12} = 2$$

$$4 \text{ bananas} \longrightarrow 2 \times 4 = 8$$

Do:

1. 5 kg of sugar Rs. 120. How many for 2 kg of sugar?
2. Annual salary of an IT professional is Rs. 10 lakhs. What is his monthly Salary? (Approximate also OK).
3. In (2) above, if he saves 3 months total salary, can he buy a scooter? Or a Car?
4. An office workers monthly income (total) is Rs. 5000. She gets 2 month bonus during Diwali. She could same only 1 month’s average income? Can she buy a car? Scooter?
5. A vegetable seller gets 100 kgs from wholesale market by paying Rs. 600. 10 kgs are wasted. He sells the remaining for Rs. 8 / kg. Is this OK?
6. In (5) above another 15 kg are not sold? Is this OK? What should he have done?

Chapter - 9 Average

9. **Average:** It is also called **mean**.
Please note that here only arithmetic mean, called as average, is our concern. No standard deviations or statistics.

9.1 Average of two numbers.

Let A be the average

$$A = \frac{\text{Sum of the 2 numbers}}{2}$$

9.1.1 Find A of (1,5); (2,6); (8,10) etc

Ans: Mean of (1,5) = $\frac{1+5}{2} = 3$

Mean of (2,6) = $\frac{2+6}{2} = 4$

Mean of (8, 10) = $\frac{8+10}{2} = 9$

9.1.1 Examples:

(a) Sometimes Mr. G eats 10 idlees for breakfast. Sometimes he eats only 6 idlees.
What is the average? (Or how many idlees shall I make and keep ready for him)? **Ans.:** 8

(b) Last week tomato was Rs. 12/kg. This week it is Rs. 8/kg.
What is the average price of tomato (in April in Mysore)? Ans: Rs. 10

(c) Some good motorbikes cost _____, Some other motorbikes cost _____ Approximately how much money is needed for a medium level motorbike? (Use suitable numbers here).

(d) Generate information from students asking for mean of 2 items.

Exercise:

1. Maximum temperature (some day in May 2009, at Mysore) was 38°c. Minimum was 26°c. What was the average on that day?
2. Yesterday's average temperature was 32°c. Day before yesterday it was 36°c. What do you expect today's average temperature?
3. A Scooter gives 50 km per liter of petrol on highway. A liter of petrol is Over if the same scooter runs for 30 km inside city. What is the average mileage given by this scooter? (Mileage = distance for 1 liter of fuel).

9.1.2 When only 2 numbers are involved mean can be looked at as the midpoint (or mid value).

a. A of 8 and 10 =? A = $\frac{8+10}{2} = \frac{18}{2} = 9(OK)$

b. A of 18 & 20 =? A = $\frac{18+20}{2} = \frac{38}{2} = 19$

c. A of 78 & 80 =? A = $\frac{78+80}{2} = \frac{158}{2} = 79$

Instead try: Mid value between 8 & 10 =19

Mid value between 18 & 20 = 19
Mid value between 78 & 80 = 79

d. Now try 123458 and 123498

9.1.3 In averaging two numbers (large or small) midpoint concept is ok. Since average is, in any case, an approximation it is ok to round off.

Thus in 9.1.3 (d) above 123458, 123498 A = 123478

But you can make these as 123460, 123500 $\therefore$ A = 123480

9.1.4 In the above try:

a. 123458, 123578. A =?

b. Round off to 123460, 123580
Keep 123 out 460, 580 Difference = 120
Half of difference = 60 $\therefore$ Value = 460 + 60 or 580 – 60 = 520
Now add 123: $\therefore$ A of 123460, 123580 = 123520

Doing such things without calculators increases self-confidence and removes the fear of simple arithmetic.

c. In the above example it is OK to make still more approximations:

123458, 123578 Can be 123500, 123600 A = 123550

d. Compare now a, b, c

a. Correct was 123518
b. First approximation was 123520
c. Rough value was 123550

9.1.5 Exercise

a. Find the mean by finding the midpoint.
   1. Distance of 3 and 5 km.
   2. 1 and 2
   3. Rupee equivalent of 1 US dollar: Rs. 42, Rs 48

b. Find mean by approximation & mid point.
   1. Population of a place 9 lakhs 9 thousand and 10 lakhs one thousand.
   2. 9900 and 11100
   3. 9912, 11162

9.2 Average of many numbers:

In real life situation average of many numbers are very much needed.

Rule: $A = \frac{\text{Sum of all the numbers}}{\text{Number of items}}$

E.g.: For 2,4,6,8,10. $A = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$

Later: Negative (-) number concept & hence assume mean value.
Keep common high numbers out and do only small numbers.

9.2.1 Marks obtained in an examination.

E.g.: 6 subjects 600 marks

Average: Add all marks; divide by 6

[If it is 625 marks, this small idea leads to percentage concept].

9.2.2 Average expenditure in a house (say on electricity) (over 3 or 10 months)

9.2.3 Average rainfall in a place (say over the last 10 years).

9.2.4 Average price of a famous company's share (say over last 4 weeks)

Extend this list using the students' ideas.

Students can take these values from a newspaper and make their own questions.

9.2.5 Exercises:

a. Total marks obtained by a student is 365 total subjects. What is the average?

b. If an average of 35 marks are needed for passing. What are the total marks required for a total of 6 subjects?

c. In (b) above what, for I class result?

d. In (c) above, which total marks will be called distinction?

e. In a class of 9 students, marks obtained by them in maths are as follows: 10,15, 15, 30, 35, 45, 50, 55, 85. What is the average in this class?

f. In (e) above one more comes and gets zero marks. What is the new average?

Exercises - Chapter 9

Ex. IX.1 Ages of children in 9th standard were: 14,15,13,14,15, 16, 13, 15, 14, and 13. Find the average age?

Ex. IX.2 In (1) above two more persons joined. Their ages were 18 and 19. What is the new average?

Ex. IX.3 In a KG class children's ages were: (in years and months; y, m):
3y, 4y, 2y 10m, 2y 11m, 3y 1m, 3y 6m, 4y 2m, 4y 6m, 3y 8m, 3y. Find the average age in years and months? [I year = 12 months]

Ex. IX.4 Marks obtained in a class are tabulated:

Marks	No. of Persons
0	2
5	3
12	15
13	10
14	5
15	2
20	2
25	1

Maximum Marks: 25
None was absent.

a. How many students (total) wrote the exam?
b. How many got zero marks?
c. How many got full marks?
d. How many first class marks?
e. What is the class average?

Ex. IX.5 Mean Values and a branch of mathematics called statistics are clearly related. Do you know some words related to statistics? If yes, write down

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Chapter - 10 Fractions - 1

10.1 If you have 2 items (one each for 2 of you) and 2 more join what will you do? Share by 4 so that each gets ½.

Write it down:

a. 2 items / 2 persons = $\frac{2}{2} = 1$

b. 2 items / 4 persons = $\frac{2}{4} = \frac{1}{2}$

Reverse 1 + 1 = 2 x 1 = 2

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 4 \text{ x } \frac{1}{2} = 2$$

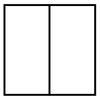
10.2 Do the same as 13.3.1, but now only with one item.

a. 1 item / 2 persons = $\frac{1}{2}$ (half)

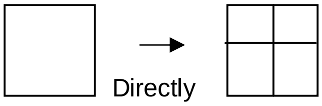
b. 1 item / 4 persons = $\frac{1}{4}$ (quarter)

Take a concrete example (piece of paper). First show (a) & (b) above

(a)



(b)



(c) Now take one half piece of (a) above and make it into 2 parts, i.e. $\frac{1}{4}$

Now say $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{4}$

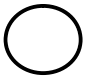
10.3 Handwork Exercise:
Take rectangular or square or circular piece of paper. Fold, cut and show:

a. $\frac{1}{2}$ b. $\frac{1}{3}$ c. $\frac{1}{4}$ d. $\frac{1}{6}$ e. $\frac{1}{24}$ f. $\frac{1}{12}$

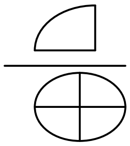
10.4 In (10.3) above, create your own questions.

Eg: $\frac{*}{***} = ?$ Ans= $\frac{1}{3}$ $\frac{□□}{□□□}$ =? Ans = $\frac{2}{3}$

Eg:



If this is 1,



=? Ans = $\frac{1}{4}$

10.5 Go to 3 items to be shared by 3. It gives one each $\frac{3}{3} = 1$

Now 3 more join. Then what happens? See 13.3.1 (a) & (b) & do similar here.

a. 3 items / 3 persons = $\frac{3}{3} = 1$

b. 3 items / 6 persons = $\frac{3}{6} = \frac{1}{2} =$ half

$$\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2} = (\frac{1}{2}+\frac{1}{2}) + (\frac{1}{2}+\frac{1}{2}) + (\frac{1}{2}+\frac{1}{2}) = 1+1+1= 3$$

Eg: $\frac{1}{2}$ added 6 times = $\frac{1}{2}$ x 6 = 3

Do:

a. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ b. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ c. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

d. $\frac{1}{5}+$........ (How Many) = 1 e. $\frac{1}{8}+$.......(How Many) = 1

f. In (e) how many = 2

10.6 Addition of simple fractions.

a. $\frac{1}{2} + \frac{1}{2} = 1$ (Use coins) i.e., 50 Paise + 50 Paise = Re 1

$[\frac{1}{2}\text{Re} + \frac{1}{2}\text{Re} = \text{Re } 1]$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = (\frac{1}{2} + \frac{1}{2}) + \frac{1}{2}$ = 1 plus $\frac{1}{2}$ Here 3 coins of 50 Paise

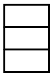
b. $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ (Show graphically)

c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = (\frac{1}{4} + \frac{1}{4}) + (\frac{1}{4} + \frac{1}{4}) = \frac{1}{2} + \frac{1}{2} = 1$

d. or $(\frac{1}{4} + \frac{1}{4}) + \frac{1}{2} = 1$ or $(\frac{1}{4} + \frac{1}{4}) + \frac{2}{4} = 1$

e. $\frac{1}{3} + \frac{1}{3} = 2 \text{ x } (\frac{1}{3}) = \frac{2}{3}$

$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} =$



All pieces put together = 1

10.7 Now give the rule:

a. $\frac{1}{2} + \frac{1}{2} = \frac{1 + 1}{2} = \frac{2}{2} = 1$

b. $\frac{1}{4} + \frac{1}{4} = \frac{1 + 1}{4} = \frac{2}{4} = \frac{1}{2}$

c. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{1 + 1 + 1}{4} = \frac{3}{4}$

d. $\frac{1}{4} + \frac{1}{4} + \frac{2}{4} = \frac{1 + 1 + 2}{4} = \frac{4}{4} = 1$

But we know that $\frac{2}{4} = \frac{1}{2}$

$$\therefore \frac{1}{4} + \frac{1}{4} + \frac{1}{2} = \frac{1 + 1 + 2}{4} = \frac{4}{4} = 1$$

10.8 Exercises (Addition of only 2 fractions). Denominators are the same.

E.g.1. $\frac{1}{5} + \frac{1}{5} = \frac{1 + 1}{5} = \frac{2}{5}$

E.g.2. $\frac{1}{4} + \frac{3}{4} = ?$ Ans: $\frac{1 + 3}{4} = \frac{4}{4} = 1$

Do:

a. $\frac{1}{6}+\frac{1}{6}$ b. $\frac{1}{6}+\frac{4}{5}$ c. $\frac{1}{2}+\frac{1}{2}$ d. $\frac{1}{2}+\frac{3}{2}$ e. $\frac{1}{7}+\frac{1}{7}$ f. $\frac{1}{7}+\frac{5}{7}$

g. $\frac{1}{7}+\frac{6}{7}$ h. $\frac{1}{11}+\frac{10}{11}$ i. $\frac{1}{99}+\frac{98}{99}$ j. $\frac{89}{99}+\frac{10}{99}$ k. $\frac{17}{34}+\frac{51}{34}$ l. $\frac{1}{17}+\frac{6}{17}$

m. $\frac{3}{17}+\frac{31}{17}$ n. $\frac{1}{986543}+\frac{1}{986543}$ o. $\frac{986500}{986543}+\frac{43}{986543}$

10.9 a. $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1 + 1 + 1}{3} = \frac{3}{3} = 1$ [This is not $\frac{3}{9}$]

b. $\frac{1}{3} + \frac{1}{3} = \frac{1 + 1}{3} = \frac{2}{3}$

c. $(\frac{1}{3} + \frac{1}{3}) + \frac{1}{3} = \frac{2}{3} + \frac{1}{3} = \frac{2 + 1}{3} = \frac{3}{3} = 1$

(a) is the same as (c)

d. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{2 + 2 + 2}{3} = \frac{6}{3} = 2$ [This is not $\frac{6}{9}$]

e. $\frac{2}{3} + \frac{2}{3} = \frac{4}{3} = 1$ Plus $\frac{1}{3}$

(Show this graphically or cut pieces).

Exercise:

All the above can be demonstrated by folding / cutting out of cardboard or on a graph paper. Students can do it by both the methods and show to teacher / one another.

10.10 Go to negative (subtraction).

E.g.: $1 - \frac{1}{2} = \frac{2 - 1}{2} = \frac{1}{2}$

(If this is tough, go to $\frac{1}{2} + \frac{1}{2} = 1$ take away from RHS one half). What is left will be $\frac{1}{2}$.

Exercise: Do by craft method and by graph paper

a. $1 - \frac{1}{3}$ b. $1 - \frac{2}{3}$ c. $1 - \frac{1}{6}$ d. $1 - \frac{5}{6}$ e. $1 - \frac{1}{4}$ f. $\frac{4}{3} - \frac{1}{3}$

g. $1 - \frac{1}{16}$ h. $1 - \frac{15}{16}$ i. $2 - \frac{3}{8}$ j. $1 - \frac{3}{8}$

==

Chapter - 11 Fractions - 2

11. Fractions: Extension of division.

Go back to division and revise.

11.1 Division and remainder:

a. Select any single digit number to be divided by any other single digit number.

Make a set A (1 to 5); another set B (6 to 10)

Make A ÷ B

E.g.: 1 ÷ 6 written as 1/6 division not possible.
3 ÷ 7 written as 3/7 division not possible.

Whenever you say “not possible” to divide it is called a FRACTION.

b. In (a) above do B ÷ A

E.g.: 6 ÷ 1 i.e. ${}^{6}\!/_{1} = 6$ + no remainder
7 ÷ 2 ${}^{7}\!/_{2} = 3$ + remainder 1
8 ÷ 4 ${}^{8}\!/_{4} = 2$ + no remainder
9 ÷ 4 ${}^{9}\!/_{4} = 2$ + remainder 1
9 ÷ 3 ${}^{9}\!/_{3} = 3$ + no remainder
10 ÷ 3 ${}^{10}\!/_{3} = 3$ + remainder 1
10 ÷ 5 ${}^{10}\!/_{5} = 2$ + no remainder

If there is no remainder the answer is an INTEGER (i.e. it is a whole number).

If there is remainder, the answer contains FRACTIONS.

Thus $\frac{7}{2} = 3 + \frac{1}{2}$ also written as $3\frac{1}{2}$

$\frac{9}{2} = 4 + \frac{1}{2}$ also written as $4\frac{1}{2}$

$\frac{10}{3} = 3 + \frac{1}{3}$ also written as $3\frac{1}{3}$

(Sometimes a number containing both integer and fraction is called **MIXED FRACTION**)

11.1.1 Exercises: Write down <1 or >1

E.g.: $\frac{1}{3}$ Ans: <1, $\frac{4}{3}$ Ans: >1, $\frac{3}{3}$ Ans: 1

a. $\frac{9}{8}$ b. $\frac{8}{8}$ c. $\frac{7}{8}$ d. $\frac{19}{20}$ e. $\frac{99}{100}$ f. $\frac{101}{100}$

g. $\frac{1234}{1235}$ h. $\frac{1238}{1235}$

11.2 Division – bigger numbers: Follow the same method as above but use the written format as shown in earlier Chapter.

11.2.1 Examples:

(i) 987 ÷ 8

$$\begin{array}{r l} & \overline{123} \\ 8\,\big) & 987 \\ & \underline{8} \\ & 18 \\ & \underline{16} \\ & 027 \\ & \underline{024} \\ & 003 \end{array}$$

Ans: 123 remainder = 3

$\therefore \frac{987}{8} = 123 \frac{3}{8}$

(ii) 987 ÷ 12

$$\begin{array}{r l} & \overline{81} \\ 12\,\big) & 987 \\ & \underline{96} \\ & 017 \\ & \underline{012} \\ & 005 \end{array}$$

Ans: 81 remainder = 5

$\therefore \frac{987}{12} = 81 \frac{5}{12}$

(iii) 987 ÷ 111

$$\begin{array}{r l} & \overline{8} \\ 111\,\big) & 987 \\ & \underline{888} \\ & 099 \end{array}$$

Ans: 8 remainder = 99

$\therefore \frac{987}{111} = 8 \frac{99}{111}$

11.2.2 Exercises: Following the method given above, do:

a. $\frac{8}{8}$ b. $\frac{9}{8}$ c. $\frac{88}{8}$ d. $\frac{889}{8}$ e. $\frac{98988}{8}$

f. $\frac{13}{12}$ g. $\frac{132}{12}$ h. $\frac{1335}{12}$ i. $\frac{12346}{12}$ j. $\frac{1000}{111}$

11.2.3 Example:

$$\frac{88}{24} = \frac{88}{8x3} = \frac{88}{8} \text{ x } \frac{1}{3} = 11 \text{ x } \frac{1}{3} = \frac{11}{3} = 3\frac{2}{3}$$

Do:

a. $\frac{889}{32}$ b. $\frac{98988}{48}$ c. $\frac{52}{24}$ d. $\frac{396}{36}$ e. $\frac{3000}{333}$ f. $\frac{3330}{999}$

11.3 Mixed fraction conversion

11.3.1

(a) $\frac{1}{2} + \frac{1}{2} = 1 \quad 1\frac{1}{2} = 1\frac{1}{2}$ Write this as

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} \quad \therefore \quad \frac{3}{2} = 1\frac{1}{2}$

Try $1\frac{1}{2} = \frac{2 \text{ X}1}{2} + \frac{1}{2} = \frac{3}{2}$

This way we are able to convert mixed fraction (i.e. integer + fraction) into one single fraction.

11.3.2 Convert the mixed fractions into simple fractions.

$2\frac{1}{2}, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2\frac{3}{4}, 3\frac{1}{2}, 4\frac{3}{4}, 5\frac{1}{2}, 9\frac{1}{9}$

Method:

$$2\frac{1}{2} = 2 + \frac{1}{2} = \frac{(2 \text{ X } 2) + 1}{2} = \frac{4 + 1}{2} = \frac{5}{2}$$

$$1\frac{1}{4} = 1 + \frac{1}{4} = \frac{(1 \text{ X } 4) + 1}{4} = \frac{4 + 1}{4} = \frac{5}{4}$$

$$4\frac{3}{4} = 4 + \frac{3}{4} = \frac{(4 \text{ X } 4) + 3}{4} = \frac{16 + 3}{4} = \frac{19}{4}$$

$$9\frac{1}{9} = \frac{(9 \text{ X } 9) + 1}{9} = \frac{81 + 1}{9} = \frac{82}{9}$$

$$11\frac{1}{9} = \frac{(11 \text{ X } 9) + 1}{9} = \frac{99 + 1}{9} = \frac{100}{9}$$

11.3.3 Convert (i) $123\frac{3}{8}$ (ii) $81\frac{5}{12}$ (iii) $8\frac{99}{111}$

Clue: Go to Para 14.2

$3\,\underline{\big|\,81}$

$\underline{\ 27\ }$ This means $\frac{81}{3} = 27$ Also 3 X 27 = 81

$3\,\underline{\big|\,83}$

$\underline{\ 27\ }$ (reminder 2) This means 83 - 2 = 81 $\frac{81}{3} = 27$ & 3 X 27 = 81

11.4 Equivalent fractions:

Fraction is the same as division into equal parts; otherwise called sharing or making equal heaps (Gudde).

11.5.1 If 2 chocolates are to be shared by 2 children how many will each child get?

Ans: $\frac{2}{2} = 1$

If there are 4 chocolates and 4 children? Ans: $\frac{4}{4} = 1$

If there are 52 items and 52 receivers Ans: $\frac{52}{52} = 1$

We get the same answer (viz 1) $\therefore \quad \frac{2}{2} = \frac{4}{4} = \frac{52}{52} = 1$

11.5.2 If I have Re.1 and give to 2 persons, each gets?

Ans: Total = Re.1 Persons = 2 Each = Re $\frac{1}{2} = \frac{1}{2}$ rupee or 50 paise

If I have Rs. 5 and give to 10 persons? Ans: Rs. $\frac{5}{10} =$ Re. $\frac{1}{2}$ or 50 paise

11.5.3 The above concept is sometimes called SIMPLIFYING a fraction. If 100 items are shared by 200 persons each = $\frac{100}{200}$. This can also be called $\frac{1}{2}$. Thus $\frac{100}{200}=\frac{1}{2}$ (simplified). This is done by dividing both the numerators and the denominator by the same number.

Example: Simplify $\frac{8}{24}$

Ans: Divide numerator by 8; we get $\frac{8}{8} = 1$

Divide denominator by 8; we get $\frac{24}{8} = 3$

$\therefore \quad \frac{8}{24} = \frac{1}{3}$

This can also be written as $\frac{8}{24} = \frac{8 \ X \ 1}{8 \ X \ 3} = \frac{1}{3}$

11.5.4 Exercises:

Do as shown above: Simplify

a. $\frac{2}{6}$ b. $\frac{2}{16}$ c. $\frac{4}{64}$ d. $\frac{42}{98}$ e. $\frac{999}{81}$ f. $\frac{72}{999}$

g. $\frac{18}{81}$ h. $\frac{18}{72}$ i. $\frac{72}{18}$

11.5.5 We saw how to simplify by DIVIDING BOTH TOP AND BOTTOM by the same number. Some times MULTIPLYING with help.

Example 1: Simplify $\frac{1234}{5}$. One can divide by long method. Instead, do as follows:

$$\frac{1234}{5} = \frac{1234 \ \text{x} \ 2}{5 \ \text{x} \ 2} = \frac{2468}{10}$$

$$= \frac{2460}{10} + \frac{8}{10} = 246 \ \frac{8}{10}$$

(Here it is easy for us to divide by 10)

Example 2: $\frac{12345}{125}$(125 x 8 = 1000 is known)

$$\therefore \frac{12345}{125} = \frac{12345x8}{125x8} = \frac{88760}{1000} = \frac{88000+760}{1000}$$

$= 88 + \frac{760}{1000}$ Now Simplify $\frac{76}{100}$ only

Do:

a. $\frac{421}{5}$ b. $\frac{421}{25}$ c. $\frac{210}{25}$ d. $\frac{48}{50}$ e. $\frac{5250}{125}$

11.6 Some (real-life) examples:

11.6.1 Imagine this real life situation. You are giving 1 pencil per person (student). You have 10 students and 10 pencils (i.e. $\frac{10}{10}$=1). Suddenly 5 more join. What will you do? 5 extra persons came; so you get 5 extra pencils.

$\frac{10}{10} = 1$ $\frac{?}{10+5} = 1$ Answer is 15.

Extra needed = 15 – 10 = 5

This looks like $\frac{10+5}{10+5} = \frac{10}{10}$ (i.e., adding to top & bottom is OK)

11.6.2 In 11.6.1 above, let us say we are giving 2 pieces (notebooks, laddus or ruppes) each person. We have 20 items and ten persons. Each gets 2 [i.e., $\frac{20}{10} = 2$]. Sunddenly 10 more join. What will you do? You know the asnwer: you will get 20 more items.

Now let us try : $\frac{20}{10}$ was the first.

Extra came in $\frac{?}{10+10}$

If you make (20 + 10) it becomes $\frac{20+10}{10+10} = \frac{30}{20}$. This is wrong.

It should be done like this: Each should get 2. Orginally 10 persons, then we needed 20. We had 20. Now, 10 extra persons came. Now we have (10+10) = total 20 persons. Each gets 2. So we need (20 x 2) = 40 items. We had with us 20. $\therefore$ we need 20 more .

11.6.3 In 11.6.2 above, only one item per person was being given i.e., $\frac{10}{10}$=1. Here both the top number (= numerator) and the bottom number (= denominator) were the same. So, just adding was OK.

In 11.6.2, we were giving more than one per person i.e., numerator was not equal to denomiantor (i.e., it was a true fraction). In such cases adding will not be OK.

11.6.4 Imagine you cooked 10 idlees for 10 persons. 5 did not come. How many will be left $\frac{10}{10} = 1$ (Planned) Now $\frac{?}{10-5}$. Answer 5 is OK.

Suppose you planned to serve 3 idles per person and expected 10 persons 5 did not come. How many will be wasted?

Answer is NOT 5. It will be 15.

$\frac{30}{10} = 3(OK)$ $\frac{30-5}{10-5} \neq 3$ (Not OK)

11.6.5 From the above 4 paragraphs (11.6.1 to 11.6.4), we learn that: ADDING OR SUBTRACTING to a fraction (numerator and denominator) gives wrong results.

11.7.1 Mathematically, the situations given above can be given as: $\frac{10}{10}$ means 10 items shared by 10 receivers. If receivers number increases by 5, $\frac{10}{10}$ becomes $\frac{?}{10+5}$ If it is = 15, adding 5 is OK.

Similary 11.6.4 can be written as 10 shared by 10. If 2 are less, receivers number becomes 8.

$\frac{10}{10}$ becomes $\frac{?}{10-2} = \frac{?}{8}$ If is = 8

(reducing or subtracting 2 is OK)

All this because $\frac{10}{10}$ i.e., the original fraction was unity (=1, one). It is a unique case (sometimes called TRIVIAL). For any other fraction, whether >1 or < 1, these methods do not work.

11.7.2 Caution: Adding the same number to both the numerator and the denominator is not OK. Doing that changes the value of the fraction. Similarly subtraction is also wrong.

11.7.3 To clarify still more:

We know $\frac{3}{1} = 3$ and $\frac{30}{10} = 3$

3 X 10 = 30 bottom also 1 X 10 = 10

But 3 + 27 = 30 bottom 1+ 27 = 28 $\frac{30}{28} \neq 3$

11.7.4 We can DIVIDE both denominator and numerator by the same number. It is OK.

$\frac{30}{10} = 3$ Divide 30 by 10 Ans.: 3
Divide 10 by 10 Ans.: 1 Now $\frac{3}{1} = 3$

$\frac{30}{10} = \frac{6}{2} = 3$ OK Divide 30 by 5 Ans.: 6
Divide 10 by 5 Ans.: 2

11.7.5 Rule:

In fractions, multiplying or dividing both the numerator and denominator is OK.

Chapter - 12 **Fractions – 3**

12. Work with fractions:
Even college students make mistakes in handling fractions. Students will benefit if they do all the problems given here.

12.1 Addition of fractions:

A. (1) $\frac{1}{3}+\frac{2}{3}$ (2) $\frac{2}{3}+\frac{2}{3}$ (3) $\frac{1}{3}+\frac{5}{3}$ (4) $\frac{2}{3}+\frac{7}{3}$

B. (1) $\frac{1}{6}+\frac{2}{6}$ (2) $\frac{2}{6}+\frac{4}{6}$ (3) $\frac{4}{6}+\frac{8}{6}$ (4) $\frac{4}{6}+\frac{14}{6}$

C. (1) $\frac{1}{7}+\frac{6}{7}$ (2) $\frac{3}{7}+\frac{4}{7}$ (3) $\frac{4}{7}+\frac{10}{7}$ (4) $\frac{9}{7}+\frac{12}{7}$

D. (1) $\frac{1}{17}+\frac{16}{17}$ (2) $\frac{10}{17}+\frac{7}{17}$ (3) $\frac{10}{17}+\frac{24}{17}$ (4) $\frac{24}{17}+\frac{27}{17}$

(Did you notice that the denominators are the same?)

12.2 Subtraction of fractions:

A. (1) $\frac{2}{3}-\frac{1}{3}$ (2) $\frac{2}{3}-\frac{2}{3}$ (3) $\frac{5}{3}-\frac{1}{3}$ (4) $\frac{7}{3}-\frac{2}{3}$

B. (1) $\frac{2}{6}-\frac{1}{6}$ (2) $\frac{7}{6}-\frac{1}{6}$ (3) $\frac{12}{6}-\frac{8}{6}$ (4) $\frac{18}{6}-\frac{14}{6}$

C. (1) $\frac{8}{7}-\frac{1}{7}$ (2) $\frac{4}{7}-\frac{3}{7}$ (3) $\frac{10}{7}-\frac{3}{7}$ (4) $\frac{21}{7}-\frac{14}{7}$

D. (1) $\frac{17}{17}-\frac{1}{17}$ (2) $\frac{10}{17}-\frac{7}{17}$ (3) $\frac{24}{17}-\frac{7}{17}$ (4) $\frac{51}{17}-\frac{34}{17}$

(This is wrong: $\frac{2}{3}-\frac{1}{3}=\frac{2-1}{3+3}=\frac{1}{6}$ Not Ok)
After 12.1 & 12.2 stress that the denominator simply sits.
Operations are on the upper side (nominator) only.

12.3 Addition:

Show that $1+\frac{1}{2}=1\frac{1}{2}=\frac{3}{2}$, $2+\frac{1}{2}=2\frac{1}{2}=\frac{5}{2}$

$5+\frac{1}{2}=5\frac{1}{2}=\frac{11}{2}$ etc

Now show $\frac{2}{2}+\frac{1}{2}=\frac{2+1}{2}=\frac{3}{2}$ Also $1+\frac{1}{2}=\frac{2}{2}+\frac{1}{2}=$ ……
Students can see and understand that,

$\therefore$ $1=\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}$ etc Similarly $2=\frac{2}{1}=\frac{4}{2}=\frac{6}{3}=\frac{8}{4}$ etc

$\therefore$ To do $2+\frac{1}{2}$ write 2 as $\frac{4}{2}$. Reason is we want the denominators equal.

$\therefore$ $2+\frac{1}{2}=\frac{4}{2}+\frac{1}{2}=\frac{4+1}{2}=\frac{5}{2}$

12.4 Additions, in 12.3 above, use other simple denominators.

12.4.1 $1 + \frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$

$$2 + \frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{6+1}{3} = \frac{7}{3}$$

$$11 + \frac{1}{3} = \frac{33}{3} + \frac{1}{3} = \frac{33+1}{3} = \frac{34}{3}$$

12.4.2 Short cut for mixed fractions.

a. $1\frac{1}{3}$ is really equal to $1 + \frac{1}{3}$ $\therefore = \frac{4}{3}$

$\therefore\ 1\frac{1}{3} = (3 \text{ X } 1 + 1) \div 3 = \frac{4}{3}$

b. Similarly $1\frac{2}{3} = (3 + 2) \div 3 = \frac{5}{3}$

12.4.3 Exercises:

1. Express as mixed fraction:

a. $\frac{12}{5}$
b. $\frac{12}{7}$
c. $\frac{12}{8}$
d. $\frac{12}{9}$
e. $\frac{13}{4}$
f. $\frac{13}{3}$

g. $\frac{13}{2}$
h. $\frac{121}{4}$
i. $\frac{121}{2}$
j. $\frac{121}{3}$
k. $\frac{121}{9}$

2. Express as a fraction: Eg - $2\frac{1}{5} = \frac{11}{5}$

a. $2\frac{2}{5}$
b. $1\frac{5}{7}$
c. $1\frac{1}{2}$
d. $1\frac{4}{8}$
e. $1\frac{1}{3}$
f. $1\frac{3}{9}$
g. $3\ \frac{1}{4}$
h. $4\frac{1}{3}$
i. $6\frac{1}{2}$
j. $30\frac{1}{4}$
k. $40\frac{1}{3}$
l. $13\frac{4}{9}$
m. $4000\frac{1}{3}$
n. $300\frac{1}{4}$

12.5 Subtraction: see that

$1 - \frac{1}{2} = \frac{1}{2}$, $2 - \frac{1}{2} = 1\frac{1}{2} = \frac{3}{2}$, $5 - \frac{1}{2} = 4\frac{1}{2} = \frac{9}{2}$ etc

Now show that $\frac{2}{2} - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

See that $2 = \frac{4}{2}$ and $5 = \frac{10}{2}$ $\therefore\ 2 - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{4-1}{2} = \frac{3}{2} = 1\frac{1}{2}$

Similarly $5 - \frac{1}{2} = \frac{10}{2} - \frac{1}{2} = \frac{10-1}{2} = \frac{9}{2} = 4\ \frac{1}{2}$

(See that we need the denominator to be the same)

12.6 In 12.5 use other denominators

$1 - \frac{1}{2} = \frac{3-1}{3} = \frac{2}{3}$, $3 - \frac{1}{3} = \frac{9}{3} - \frac{1}{3} = \frac{9-1}{3} = \frac{8}{3}$

(Here 3 can be $\frac{6}{2}$ or $\frac{9}{3}$ or $\frac{12}{4}$ etc. We take $\frac{9}{3}$ because after – sign ($\frac{1}{3}$) is there. It has denominator 3).

$3 - \frac{1}{5} = ?$, $3 - \frac{1}{7} = ?$ $3 - \frac{3}{8} = ?$

12.7 Mixed fractions (same denominator)

$5\frac{1}{2} - 3\frac{1}{2} = ?$ Directly 2 is OK

The same problem is done in 2 ways below:

(a) Integers 5 – 3 = 2

Fractions $\frac{1}{2} - \frac{1}{2} = 0$

Total 2 ∴ Ans = 2 OK

(b) $5\frac{1}{2} = \frac{10+1}{2} = \frac{11}{2}$, $3\frac{1}{2} = \frac{6+1}{2} = \frac{7}{2}$

$\therefore\ 5\frac{1}{2} - 3\frac{1}{2} = \frac{11}{2} - \frac{7}{2} = \frac{11-7}{2} = \frac{4}{2} = 2$ This is also OK.

12.7.1 Do in 2 methods as shown above:

a. $3\frac{2}{3} - 1\frac{1}{3}$ b. $18\frac{5}{7} - 16\frac{3}{7}$ c. $20\frac{7}{10} - 6\frac{3}{10}$ d. $1\frac{2}{3} + 2\frac{2}{3}$

e. $10\frac{3}{7} + 8\frac{2}{5}$ f. $3\frac{1}{10} + 3\frac{2}{10}$ g. $1\frac{2}{3} + 2\frac{1}{3} - 1\frac{1}{3}$

h. $10\frac{3}{7} + 8\frac{2}{5} - 16\frac{3}{7}$ i. $20\frac{7}{10} - 3\frac{1}{10} - 3\frac{2}{10}$

Exercises - Chapter 12

Ex. XII.1 Example: $3 + 2\frac{1}{3} = 5\frac{1}{3}$

Do: a. $8 + 2\frac{1}{4}$ b. $9 + 1\frac{1}{4}$ c. $6 + 4\frac{2}{3}$ d. $2\frac{1}{4} + 8$ e. $4\frac{2}{3} + 5$ f. $1\frac{1}{5} + 9$

Ex. XII.2 Example: $3 + \frac{7}{3} = 3 + 2\frac{1}{3} = 5\frac{1}{3}$ or

$3 + \frac{7}{3} = \frac{9}{3} + \frac{7}{3} = \frac{9+7}{3} = \frac{16}{3} = 5\frac{1}{3}$

Can do whichever is easier for the students. Many persons find the first method easier.

Do:

a. $8 + \frac{9}{4}$ b. $9 + \frac{5}{4}$ c. $6 + \frac{4}{3}$ d. $\frac{9}{4} + 7$ e. $\frac{14}{3} + 5$ f. $\frac{6}{5} + 9$

Ex. XII.3 Example: $3\frac{2}{3} + \frac{7}{3}$ =?

Ans: Method A: $3 + \frac{2}{3}+\frac{7}{3} = 3 + (\frac{7+2}{3}) = 3+ \frac{9}{3} = 3 + 3 = 6$

Method B: $3\frac{2}{3}+\frac{7}{3} = \frac{11}{3} + \frac{7}{3} = (\frac{11+7}{3}) = \frac{18}{3} = 6$

Even here, method A is better because smaller numbers come as numerator.

Do:

a. $8\frac{3}{4} + \frac{9}{4}$ b. $9\frac{3}{4} + \frac{5}{4}$ c. $6\frac{1}{3}+\frac{14}{3}$ d. $\frac{9}{4} + 7\frac{1}{4}$ e. $\frac{14}{3} + 5\frac{2}{3}$ f. $\frac{6}{5} + 9\frac{2}{5}$

Ex. XII.4 Example: $3 – 2\frac{1}{3}$ =?

Ans: $3 - 2\frac{2}{3} = 2 + 1 - 2\frac{1}{3} = 2 - 2 + 1 - \frac{1}{3}$

$= 1 - \frac{1}{3} = \frac{3-1}{3} = \frac{2}{3}$

Do:

a. $8 - 2\frac{1}{4}$ b. $9 - 1\frac{1}{4}$ c. $6 - 4\frac{2}{3}$ d. $2\frac{1}{4} - 1\frac{1}{4}$ e. $1\frac{1}{4} - \frac{3}{4}$ f. $4\frac{2}{3} - \frac{1}{3}$

Ex. XII.5 Example: $3 - \frac{7}{3}$ =?

Method A: $3 - \frac{7}{3} = 3 - 2\frac{1}{3} = 1 - \frac{1}{3} = \frac{2}{3}$

Method B: $3 - \frac{7}{3} = \frac{9}{3} - \frac{7}{3} = \frac{9-7}{3} = \frac{2}{3}$

Use whichever is easier:

Do:

a. $8 - \frac{9}{4}$ b. $9 - \frac{5}{4}$ c. $6 - \frac{14}{3}$ d. $\frac{9}{4} - 1\frac{1}{4}$ e. $\frac{5}{4} - \frac{3}{4}$ f. $\frac{14}{3} - 3\frac{1}{3}$

Ex. XII.6 Mixed addition and subtraction:

Example: $\frac{1}{3}+\frac{4}{3}-\frac{2}{3} = \frac{1+4-2}{3} = \frac{5-2}{3} = \frac{3}{3} = 1$

a. $\frac{3}{5}+\frac{4}{5}-\frac{2}{5}-\frac{1}{5} = ?$ b. $\frac{15}{7}+\frac{22}{7}-\frac{14}{7}-\frac{20}{7} = ?$ c. $\frac{9}{11}-\frac{8}{11}+\frac{10}{11}-\frac{5}{11} = ?$

==

Chapter - 13 Fractions - 4

13. Fractions (Contd.)

13.1 Try $\frac{1}{2} + \frac{1}{4}$ orally, say (half + quarter).
Students will answer correctly (In local language it works better).
The SECRET of doing right is: **"MAKE THE BOTTOM NUMBERS EQUAL".**

$\therefore \frac{1}{2} + \frac{1}{4}$ Let us write $\frac{1}{2}$ as $\frac{2}{4}$

$$\therefore \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{2 + 1}{4} = \frac{3}{4}$$

13.2 Caution: Before going any farther, student should learn HOW TO ADD FRACTIONS when the denominators are equal.

13.2.1 Take $\frac{1}{2} + \frac{1}{2}$ =?
Say it in words (local language also OK)
(Half) + (Half) equal one.
Now write this down:

$$\frac{1}{2} + \frac{1}{2} = 1$$

Now let us do by steps:

$$\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = \frac{2}{2} = 1$$

13.2.2 Now try $\frac{1}{4} + \frac{1}{4}$ in words and by mathematical steps

$$\frac{1}{4} + \frac{1}{4} = \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$$

13.3 Extra Caution: Primary and middle school children, very often write as below:

$\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$ (Why?)

(Why?) I guess they put the plus everywhere. Thus $\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2+2} = \frac{2}{4}$
If you explain the same sum in words they realize their mistake.

Rule: Never Add Denominators.

13.4 Let us clarify the caution given above. Let us try to understand.

13.4.1 Denominator (hereafter Dr.) is only a tag (=symbol, label etc). It just says how the big piece is. Numerator (Nr. Hereafter) says how many.

13.4.2 Statement made above can be said in other words also. Nr. Indicates how many are taken. Dr. says how small (or what kind) each piece is. Thus Nr. and Dr. together say, how many pieces of what type of item was taken.

13.4.3 If you are selling milk in packets you can understand what is stated above.
A person A takes 1 item of 1 liter milk packet (Item = bag, packet)
How much money will you collect? Say, Rs.15 (as on May, 2009)

Another person B takes 6 packets of the same.
You will collect; 6 x 15 = Rs. 90. Right?
3^{rd} person C takes 1 packet (or item) of ½ liter packet.
How much money will you collect? Rs. 7.50. Right?
4^{th} person D takes 2 packets (=items) of ½ liter milk packet.

Money collected = 2 x $7\frac{1}{2}$ = Rs. 15

A took 1 bag	Paid Rs. 15
D took 2 bags	Paid Rs. 15 (same)
C took 1 bag	Paid Rs. 7.50 (only)

C took ½ liter only. ∴ Less i.e., Rs. $\frac{15}{2}$ = 7.50
D took ½ liter + ½ liter = 1 liter ∴ Rs.15
A took 1 liter packet ∴ Rs. 15

This means ½ < 1 $\frac{1}{2}+\frac{1}{2}=1$

13.5 Exercise:

Some students have answered a few questions on fractions. You have to find right or wrong. Right - √ or Wrong – X

a. $\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2}=\frac{2}{1}$

b. $\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2+2}=\frac{2}{4}$

c. $\frac{1}{3}+\frac{5}{3}=\frac{5+1}{3+3}=\frac{6}{6}=1$

d. $\frac{2}{7}+\frac{5}{7}=1$

e. $\frac{2}{7}+\frac{5}{7}=\frac{7}{14}=\frac{1}{2}$

f. $\frac{1}{2}+\frac{1}{4}+\frac{3}{4}=\frac{5}{10}=\frac{1}{2}$

g. $\frac{1}{2}+\frac{1}{4}+\frac{3}{4}=\frac{1}{2}+\frac{4}{4}=\frac{5}{6}$

h. $\frac{1}{2}+\frac{1}{4}+\frac{3}{4}=\frac{2+1+3}{4}=\frac{6}{4}$

i. $\frac{27}{50}+\frac{33}{50}=1$

j. $\frac{27}{50}+\frac{33}{50}=1\frac{1}{5}$

k. $\frac{27}{50}+\frac{33}{50}=\frac{50}{100}$

l. Collect wrong answers from your friends, remove their names and explain how to do it right.

13.6 (a) $\frac{2}{3}+\frac{1}{6}$. Here $\frac{2}{3}$ should be written as $\frac{4}{6}$

$$\therefore \frac{2}{3}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{4+1}{6}=\frac{5}{6}$$

Standard method:

$\frac{2}{3}+\frac{1}{6}=\frac{?+1}{6}$

(?) is decided by the question. If 3 becomes 6, 2 becomes what? The answer is 2 x 2 = 4

$= \frac{4+1}{5}$

How to get $\frac{2}{3}=\frac{4}{6}$ Ask $\frac{2}{3}=\frac{?}{6}$ $? = \frac{6}{3} \text{ X } 2 = 2 \text{ X } 2 = 4$

(b) (i) $\frac{1}{4}+\frac{5}{8}=\frac{1 \text{X } 2}{4 \text{ X } 2}+\frac{5}{8} = \frac{2}{8}+\frac{5}{8} = \frac{7}{8}$

(ii) $\frac{1}{4} + \frac{5}{8} = \frac{1 \text{ X } (\frac{8}{4})}{8} + \frac{5}{8} \qquad = \frac{2}{8} + \frac{5}{8} = \frac{7}{8}$

(c) To make the above (i.e. 13.6.b (ii)) more useful.

$$\frac{2}{17} + \frac{39}{102} = \frac{2 \text{X}(\frac{102}{17})}{102} + \frac{39}{102} \qquad \text{To find } 102/17 \quad \begin{array}{r} 6 \\ 17\overline{)102} \\ \underline{102} \\ 0 \end{array}$$

$$= \frac{2 \text{ X } 6}{102} + \frac{39}{102} = \frac{12}{102} + \frac{39}{102} = \frac{51}{102}$$

13.7 The above (viz 13.6 c) is a very long leap in maths, some who can understand can go ahead. Others please follow in smaller steps.

13.8 Bigger & smaller

13.8.1 Given 2 fractions find, which is bigger.

E.g.: $\frac{2}{3}$ & $\frac{1}{3}$ Obvious $\frac{2}{3} > \frac{1}{3}$

$\frac{1}{2}, \frac{3}{4}$ Obvious $\frac{3}{4} > \frac{1}{2}$. But this could be done more logically as

$\frac{1}{2}, \frac{3}{4}$ is the same as $\frac{2}{4}, \frac{3}{4}$. Now $\frac{3}{4} > \frac{2}{4}$

In the above, if you take $(\frac{1}{4})$ as one piece or unit.

3 pieces = $\frac{3}{4}$ 2 pieces = $\frac{2}{4}$ 3 pieces > 2 pieces

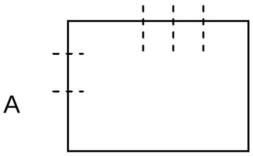
$\therefore \frac{3}{4} > \frac{2}{4}$

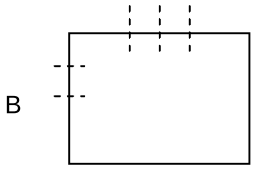
Making the measuring “unit” the same is the principle behind this. The same is the principle in making denominators identical.

13.8.2 Which is bigger $\frac{2}{3}$ or $\frac{3}{4}$?

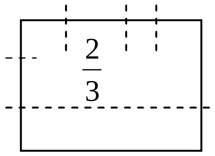
For this, one has to go to a little “paper work” i.e. take 2 identical rectangles of paper 3 units by 4 units, i.e. take 3 inch x 4 inch piece of paper.

Two such rectangles

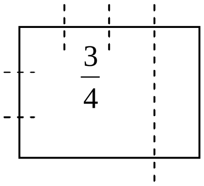




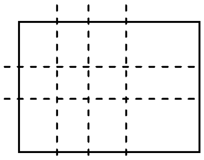
Fold $\frac{2}{3}$ from A



Fold $\frac{3}{4}$ from B



Now, you can SEE which is bigger?
How much bigger? Cut & give
Now take an identical piece of paper C, fold both ways & open



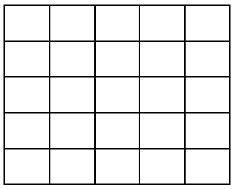
C

You get 12 equal pieces. Each = $\frac{1}{12}$.

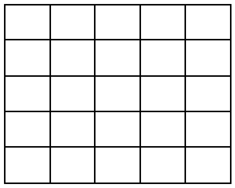
Comparing A, B & C, you get $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

13.8.3 Cutting squares (or rectangles) into smaller squares is an easy method of understanding. This is the reason graph papers contain big squares and small squares. Which is bigger, $\frac{2}{3}$ or $\frac{3}{4}$. This can be done "graphically" also. Take 1 big square each on 2 graph papers.

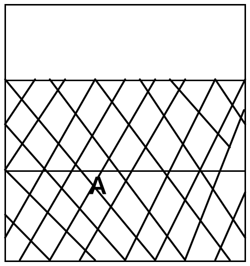
A

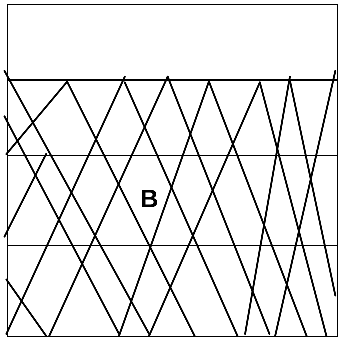


B



Mark $\frac{2}{3}$ on A Shade the area. Mark $\frac{3}{4}$ on B Shade the area.





Cut shaded A & B and place one over the other. You can find big / small. How much bigger also can be found by counting the smaller squares.

13.9 Do the same problem by standard method [Same problem as in 13.8.2. Viz which is bigger $\frac{2}{3}$ or $\frac{3}{4}$?

$\frac{2}{3}$ can be written as $\frac{?}{12} = \frac{8}{12}$

$\frac{3}{4}$ can be written as $\frac{?}{12} = \frac{9}{12}$

$\therefore \frac{3}{4} = \frac{9}{12}$ $\quad \frac{2}{3} = \frac{8}{12}$ $\quad \therefore \frac{3}{4} > \frac{2}{3}$ because $\frac{9}{12} > \frac{8}{12}$

Now how much bigger?

$\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{9 - 8}{12} = \frac{1}{12}$

Here, why did we write $\frac{2}{3}$ as $\frac{8}{12}$ instead of $\frac{4}{6}$ or $\frac{6}{9}$?

Why did we not write $\frac{3}{4}$ as $\frac{6}{8}$ or $\frac{15}{20}$?

Because we wanted a common denominator between 3 & 4. That is 12. ie 12 is such a number which can be achieved either by 3 or by 4 (by multiplying). Go to the paper and show the small squares are useful either to make 3 parts or 4 parts.

13.10 The above is called **Least Common Multiple** (LCM). LCM of two numbers is easy to find.

(3, 4) LCM = 12 (=3 X 4) (2, 3) LCM = 6 (=2 X 3) (2, 5) LCM = 10
(3, 5) LCM = 15 (4, 5) LCM = 20 (6, 7) LCM = 42
(7, 8) LCM = 56 etc (x, y) LCM = x X y

Exercise: Find LCM

a. (2,3) b. (2,5) c. (2,7) d. (2,9) e. (2,11) f. (3,4)
g. (3,5) h. (30,50) i. (30,5) j. (3,30) k. (5,50) l. (4,12)
m. (4,3) n. (6,4) o. (3,8)

13.11 Find which is bigger:

a. $(\frac{2}{3},\frac{4}{5})$ b. $(\frac{5}{7},\frac{4}{5})$ c. $(\frac{1}{2},\frac{4}{9})$ d. $(\frac{2}{3},\frac{6}{5})$ e. $(\frac{3}{4},\frac{11}{12})$ f. $(\frac{5}{6},\frac{4}{5})$

g. $(\frac{4}{5},\frac{23}{30})$

13.12 ASIDE on LCM

LCM is needed for addition & subtraction of fractions. In other places also it is needed. Concept of LCM comes from (integer) factors of given numbers.
i.e. 10 = 2 x 5

If you have 10 items it can be equally shared by 5 persons (i.e. 2 each) or by
2 persons (i.e. 5 each)

20 = 2 x 10 = 2 x 5 x 2 = 5 x 4. Numbers Involved are 2, 4, 5, 10.

Here 2, 4, 5, 10 these four numbers have LCM = 20
Even 4, 5 these two numbers have LCM = 20
Even 2, 4, 5 these three numbers have LCM = 20
Even 4, 5, 10 these three numbers have LCM = 20

In the above example, take only some pairs.

(2, 4) LCM = 8 (i.e. 2 X 4) OK but not necessary.
LCM = 4 is OK (This is because 4 is divisible by 2 and 4 is divisible by 4)

(2, 10) LCM = 2 x 10 = 20 OK. But not necessary
LCM = 10 is OK

(5, 10) LCM = 10 need not be 50
(4, 10) LCM = 20 need not be 40

13.13 Using LCM of the denominators, addition (or subtraction) can be done:

$$\frac{1}{3}+\frac{1}{2}=\frac{2\ \mathrm{X}\ 1+3\ \mathrm{X}\ 1}{6}=\frac{2+3}{6}=\frac{5}{6}$$

$$\frac{1}{4} + \frac{1}{2} = \frac{1 + (2 \text{ X}1)}{4} = \frac{1 + 2}{4} = \frac{3}{4}$$

$$\frac{1}{3} + \frac{1}{4} = \frac{(4 \text{ X } 1) + (3 \text{ X } 1)}{12} = \frac{7}{12}$$

$$\frac{1}{2} + \frac{1}{5} = \frac{5 + 2}{10} = \frac{7}{10}$$

All these can be shown by cutting paper, box full of balls, counting coins, materials etc.

Exercises: Use LCM Values given in 13.12 above. DO:

a. $\frac{1}{2}+\frac{1}{4}$ b. $\frac{1}{2}-\frac{1}{4}$ c. $\frac{1}{2}+\frac{3}{10}$ d. $\frac{1}{2}-\frac{3}{10}$ e. $\frac{4}{5}+\frac{7}{10}$ f. $\frac{4}{5}-\frac{7}{10}$

g. $\frac{3}{4}+\frac{7}{10}$ h. $\frac{3}{4}-\frac{7}{10}$

13.14 A separate session on how to find LCM can be given.
Many teachers think this knowledge is necessary for doing fractions.
Because of this problem, many students find fractions as a difficult subject. Therefore even without mentioning LCM one can go about handling fractions. Teachers, please do so, even if this leads to some large numbers and many steps.

Example:

Add: $\frac{1}{4}+\frac{3}{10}$

Method A: Regular Method:

Find LCM of the 2 denominators (4,10) LCM = 20. Convert the two fraction with this LCM as denominator.

Thus $\frac{1}{4}=\frac{5}{20}$

$\frac{1}{10}=\frac{2}{20}$ $\therefore \frac{3}{10}=\frac{3\text{X}2}{20}=\frac{6}{20}$

Now Add:

$$\frac{1}{4}+\frac{3}{10}=\frac{5}{20}+\frac{6}{20}=\frac{5+6}{20}=\frac{11}{20}$$

Method B: I' m afraid of LCM or GCM". Do not worry. Multiply both the denominators. Use this as the new denominator.
Thus, 4 x 10 = 40

$$\frac{1}{4}+\frac{3}{10}=\frac{10}{40}+3\text{X}\frac{1}{10}$$

$$= \frac{10}{40}+\frac{3\text{x}4}{40}=\frac{10}{40}+\frac{12}{40}$$

$= \frac{22}{40}$ (Divide both Nr. and Dr. by 2)

$$= \frac{11}{20}$$

Exercise: Do the following by both the methods:

a. $\frac{1}{2}+\frac{1}{4}$ b. $\frac{1}{2}-\frac{1}{4}$ c. $\frac{1}{2}+\frac{3}{10}$ d. $\frac{1}{2}-\frac{3}{10}$ e. $\frac{4}{5}+\frac{7}{10}$ f. $\frac{4}{5}-\frac{7}{10}$

g. $\frac{3}{4}+\frac{7}{10}$ h. $\frac{3}{4}-\frac{7}{10}$ i. $\frac{4}{9}+\frac{5}{12}$ j. $\frac{4}{9}-\frac{5}{12}$

Chapter - 14

Decimals - 1

14. Decimals:

14.1 Decimate as per dictionary means “**to destroy a great number or proportion**”. The earliest English sense of decimate is “**to select by lot and execute every tenth soldier of a unit”**).

Decimal = one tenth = $\frac{1}{10}$ Shown as .1

14.2 Our usual number system is also known as **decimal number system**. This is because the number (an integer) starting from one (=1) goes in steps of 10 as it is written.

i.e. 1
2
.
.
8
9 units
10 = 1 X 10 + 0
51 = 5 X 10 + 1
98 = 9 x 10 + 8
100 = 1 x 100 + 0 + 0
123 = 1 X 100 + 2 x 10 + 3 x 1

∴ If you write	a	b	c	d	e
	X	X	X	X	X
	10000	1000	100	10	1

14.2.1 Exercises:

Eg: 10010 = 1 x 10000 + 10 x 10
50403 = 5 X 10000 + 4 x 100 + 3 x 1

Write down in expanded form:
a. 12345 b. 10234 c. 10023 d. 10002 e. 908040

14.2.2 Exercises:

Which is bigger?

a. 98231, 100001 b. 8924, 9024 c. 88, 190

14.2.3 Exercises:

Write in ascending order (Take all the numbers given in 14.2.1, 14.2.2)

14.3 Decimals are fractions obtained by extending this system on the right side.

Thus .1 = 1 X $\frac{1}{10}$

$$.4 = 4 \text{ X } \frac{1}{10}$$

$$.8 = 8 \text{ X } \frac{1}{10}$$

Then $.01 = \frac{1}{100}$

$$.05 = \frac{5}{100}$$

$$.09 = \frac{9}{100}$$

Together $.12 = \frac{1}{10} + \frac{2}{100}$

$$.89 = \frac{8}{10} + \frac{9}{100}$$

3rd level $.001 = \frac{1}{1000}$

$$.123 = \frac{1}{10} + \frac{2}{100} + \frac{3}{1000}$$

If you write :

$$\begin{array}{ccccccccc} a & & b & & c & & d & & e \\ X & & X & & X & & X & & X \\ \frac{1}{10} & + & \frac{1}{100} & + & \frac{1}{1000} & + & \frac{1}{10000} & + & \frac{1}{100000} \end{array}$$

14.3.1 Note that .34 is the same as 0.34.
Also note that .34 is the same as .340
is the same as .3400
is the same as .340000
But not the shames as .341 or .3401 etc. .34 is the same as 0.34, 00.34 etc. But not the same as 10.34 etc.

14.3.2 Exercises:

E.g.: $.1001 = \frac{1}{10} + \frac{1}{10000}$

$$.50403 = \frac{5}{10} + \frac{4}{1000} + \frac{3}{100000}$$

[Note: $0.1001 \equiv .1001 \equiv 0.10010 \equiv .100100$]

Write down in expanded form:
a) .12345 b) .10234 c) .10023 d) .01002 e) .090804

14.3.3 Exercises:

Which is bigger?
a. (.982, .101) b. (.98231, .100001) c. (.89, .90)
d. (.893, .90) e. (.8, .91) f. (.9, .81)
g. (.88, .190) h. (.880, .199)

14.3.4 Exercises:

Write down in ascending order:

a. (.12345, .10234, .10023) b. (.01002, .090804, .10023)

c. Now take all the numbers in (a), (b).

d. Take (c) and also .1001, .50403

e. Collect all the numbers given in 14.3.3 and write in ascending order.

14.4 The **NUMBER SYSTEM**

1000000
100000
10000
1000
100
10
1
0 Here is zero
$\frac{1}{10}$
$\frac{1}{100}$
$\frac{1}{1000}$
$\frac{1}{10000}$
$\frac{1}{100000}$

Thus 123456789.123456 …… Each place can have 0 to 9.

14.5 Decimals everywhere:

Decimal system is the most convenient to write, understand, approximate etc.
It is also the best system to be accurate up to any desired level. So, it is used everywhere starting from science, engineering to daily life.

14.5.1 Unit of length is meter (m) $\frac{1}{10}$th of a meter is decimeter (dm). $\frac{1}{10}$th of a (dm) is centimeter (cm). $\frac{1}{10}$th of a (cm) is millimeter (mm). This whole set of lengths are in a decimal system.

Question:(millimeter and meter) are they in the decimal system? Ans: Yes.

Question: (1 crore and 1 lakh) are they in the decimal system? Ans : Yes.
(even though the names do not suggest)

14.5.2 Exercises: Are these in Decimal system?

a. Meter and kilometer
b. Millimeter and kilometer
c. Millimeter and liter
d. Kilogram and milligram
e. Cubic meter and cubic centimeter
f. Seconds and minutes
g. Minutes and hours
h. Days and week
i. Months and years
j. Volts and kilovolts
k. Ampere and milliampere

14.5.3 Exercises:

a. Do you know of any non-decimal systems of weights?

b. Can you write down different units for measuring areas and say which is decimal system?

14.6 Explanation: We saw that .89 is defined as $\frac{8}{10}+\frac{9}{100}$ (Defined meaning that we all agree to say so). Now let us try to see whether this definition is acceptable. We know how to ADD FRACTIONS.

Add $\frac{8}{9}+\frac{9}{100}$

This $= \frac{10\text{ x }80 + 9}{100} = \frac{80 + 9}{100} = \frac{89}{100}$ Now we should agree to put a (.) to denote (indicate, identify) decimal number.

Let us say $\frac{89}{100}$=.89

14.6.1 Exercises

Write in decimal form:

a. $\frac{1}{10}+\frac{2}{100}+\frac{3}{1000}$ b. $\frac{1}{10}+\frac{2}{10000}$ c. $\frac{5}{10}+\frac{4}{100}+\frac{3}{10000}$

d. $\frac{5}{10}+\frac{4}{1000}$ e. $\frac{5}{100}+\frac{3}{1000}$ f. $\frac{7}{1000}+\frac{8}{10000}$

g. $\frac{7}{100}+\frac{8}{10000}$ h. $\frac{7}{10}+\frac{8}{1000}$

==

Chapter - 15

Decimals - 2

15 Decimals (Contd.)

15.1 Fractions are natural way of expressing some division or some sharing or some fragmentation.

Decimals are fractions but always divided into 10 equal parts. One can say it is a specific or special type of fraction.

Both are usually less than 1. Decimals being special, they are written also in a special way (i.e. using a dot.). This is possible because the number system we generally use is also a decimal system. Otherwise called to **Base 10**. This will be appreciated by students when they come to know about powers and indices.

Also when measurements, calculations are involved decimals are convenient.

15.2 Activity:

a. Take a scale. See that it has both cm and inches. Usually they are on either side of the scale. Questions:

1. How many divisions (small) in 1 cm?
2. How many divisions (small) in 1 inch?
3. In your scale, are there different number of smaller divisions per inch? (Look at different parts of the scale).
4. Take a tape from a tailor and note down what you see.
5. Look at a thermometer and see how each degree is subdivided.

6. If a clinical thermometer is available, take your own temperature and state it accurately.

15.3 A fraction whose denominator is 10 can be called a decimal.

Thus $\frac{1}{10} = 0.1$ $\frac{2}{10} = 0.2$ $\frac{3}{10} = 0.3$

Then $\frac{1}{5} = \frac{2}{10}$ $\therefore \frac{1}{5} = 0.2$

Take this case if 5 become 10, 1 becomes 2. How ?

Denominator is multiplied by 2 or $\frac{10}{5}$

15.4 How to get a general rule for this? $\frac{1}{2} = ?$ (in decimal)

Let 2 become 10 by multiplying by 5.

Then $\frac{1}{2} = \frac{5}{10}$ $\therefore \frac{1}{2} = 0.5$

How did you get 5? $\frac{\text{Desired Denominator}}{\text{Actual Denominator}}$

In the case of decimal, desired denominator is 10.

$$\boxed{\therefore \textbf{ Factor needed} = \frac{\mathbf{10}}{\textbf{actual denominator}}}$$

15.5 Now compare 15.3 & 15.4

(a) $\frac{1}{5} = ?$ Factor needed = $\frac{10}{\text{actualdenominator}} = \frac{10}{5} = 2$

$$\therefore \frac{1}{5} = \frac{2}{10} = 0.2$$

(b) $\frac{1}{2} = ?$ Factor needed = $\frac{10}{\text{actualdenominato}} = \frac{10}{2} = 5$

$$\therefore \frac{1}{2} = \frac{5}{10} = 0.5$$

15.6 Exercises:

E.g. 1: Write $\frac{3}{5}$ as decimal: $\frac{3}{5} = \frac{?}{10} = \frac{6}{10}$

$$\therefore \frac{3}{5} = .6$$

E.g. 2: Write $\frac{13}{2}$ as decimal $\frac{13}{2} = 6 + \frac{1}{2}$

$$= 6 + \frac{?}{10}$$

$$= 6 + \frac{5}{10}$$

$= 6 + .5$
$= 6.5$

Write as a decimal number:

a. $\frac{1}{2}$ b. $\frac{2}{2}$ c. $\frac{3}{2}$ d. $\frac{7}{2}$ e. $\frac{55}{2}$ f. $\frac{1}{5}$

g. $\frac{2}{5}$ h. $\frac{3}{5}$ i. $\frac{4}{5}$ j. $\frac{5}{5}$ k. $\frac{55}{5}$ l. $\frac{56}{5}$

m. $\frac{59}{5}$

15.7 Problem cases: $\frac{1}{3},\frac{1}{7},\frac{1}{9},\frac{1}{11},\frac{1}{13}$ and many other denominators are real headaches. They are problem cases because; they cannot be converted to 10 easily. 2 x 5 will make 10 i.e., if you want to replace 2 by 10, just multiply by 5. Now, how will you make 3 into 10. The factor is not an integer. It is not only a complicated number, but number without any end.

Such fractions, when converted to decimals, become only APPROXIMATIONS.

E.g.: $\frac{1}{3} = .3333....$ (no end) $\frac{1}{7} = \frac{(^{10}\!/_{7})}{10} = .1428571......$ (no end)

15.8 Extend the concept of 15.5 to other denominators.

$\frac{1}{4} = \frac{?}{10}$ $\frac{10}{4} = 2\frac{2}{4} = 2\frac{1}{2}$ But we know $\frac{1}{2} = 0.5$

$$\therefore \quad \frac{1}{4} = \frac{2\,\frac{1}{2}}{10} = \frac{25}{100} = .25$$

$\frac{1}{8} = \frac{?}{10}$ $\frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}$ But we know $\frac{1}{4} = 0.25$

$$\therefore \quad \frac{1}{8} = \frac{1\,\frac{1}{4}}{10} = \frac{125}{1000} = .125$$

15.8.1 For an approximation, take

$\frac{1}{2} = .5;$ $\frac{1}{3} = .3\,;$ $\frac{1}{4} = .25\,;$ $\frac{1}{5} = .2$ $\frac{1}{6} = .6;$

$\frac{1}{7} = .14;$ $\frac{1}{8} = .125;$ $\frac{1}{9} = .11;$

15.8.2 Using the values given in 15.8.1 convert fractions into decimals:

a. $\frac{2}{3}$ b. $\frac{4}{3}$ c. $\frac{3}{4}$ d. $\frac{5}{4}$ e. $\frac{7}{4}$

f. $\frac{6}{5}$ g. $\frac{7}{5}$ h. $\frac{8}{5}$ i. $\frac{9}{5}$ j. $\frac{19}{5}$

k. $\frac{5}{6}$ l. $\frac{2}{7}$ m. $\frac{3}{7}$ n. $\frac{4}{7}$ o. $\frac{5}{7}$

p. $\frac{6}{7}$ q. $\frac{15}{6}$ r. $\frac{13}{7}$ s. $\frac{3}{8}$ t. $\frac{13}{8}$

u. $\frac{7}{9}$ v. $\frac{17}{9}$

15.9 Actual Conversion – Fraction to Decimal

From 15.8 it emerges that to get decimal from a fraction simply divide the numerator by denominator, supplying the decimal point where it is needed.

Thus $\frac{1}{4} =$

$$\begin{array}{r} 0.25 \\ 4\overline{)10} \\ 08 \\ \hline 020 \\ 020 \\ \hline 000 \end{array}$$

$\frac{1}{4} = 0.25$

$\frac{1}{2} = 0.5$

$$\begin{array}{r} 0.5 \\ 2\overline{)10} \\ 10 \\ \hline 00 \end{array}$$

$\frac{1}{8} = 0.125$

$$\begin{array}{r} 0.125 \\ 8\overline{)1000} \\ 08 \\ \hline 020 \\ 016 \\ \hline 0040 \\ 0040 \\ \hline 0000 \end{array}$$

15.9.1 Exercises:

Do all the conversions done earlier using actual division just now.

Thus $\frac{2}{10}, \frac{5}{10}, \frac{7}{10}, \frac{1}{5}$ (Given in 15.3) $\frac{1}{2}$ (Given in 15.4)

$\frac{3}{5}, \frac{13}{2}$ (Given in 15.6) $\frac{1}{8}$ (Given in 15.8)

15.10 Some more examples

a) $\frac{22}{5} = ?$ $\frac{22}{5} = 4\frac{2}{5}$ $\frac{2}{5} = .4$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ 0 \\ \hline 20 \\ 20 \\ \hline 00 \end{array}$$

This can be directly done also

$$\begin{array}{r} 4.4 \\ 5\overline{)22} \\ 20 \\ \hline 020 \\ 020 \\ \hline 000 \end{array} \qquad \frac{22}{5} = 4.4$$

b) $\frac{22}{8} = ?$

$$\begin{array}{r} 2.75 \\ 8\overline{)22} \\ 16 \\ \hline 060 \\ 056 \\ \hline 0040 \\ 0040 \\ \hline 0000 \end{array}$$

15.10.1 Exercises:

Eg: Write $\frac{56}{5}$ in decimal form.

Ans: $\frac{56}{5} = 11 + \frac{1}{5}$; $\frac{1}{5} = \frac{1.0}{5} =. 2$

$\therefore \frac{56}{5} = 11+. 2 = 11.2$

Method B:

$$\begin{array}{r} 11.2 \\ 5\overline{)56.0} \\ 55\downarrow \\ \hline 10 \\ 10 \\ \hline 0 \end{array} \qquad \therefore \text{Ans: } 11.2$$

Do by both the methods:

a. $\frac{7}{2}$ b. $\frac{20}{7}$ c. $\frac{59}{5}$ d. $\frac{27}{8}$ e. $\frac{25}{9}$ f. $\frac{2500}{9}$

15.11 Rule given in 15.8 applies to all numbers even if they are already decimals.

a) $\frac{.2}{2} = .1$ $\frac{.4}{2} = .2$ $\frac{.4}{4} = .1$

b) $\frac{4.2}{2} = 2.1$ $\frac{4.4}{2} = 2.2$ $\frac{4.4}{4} = 1.1$

c) $\frac{4.1}{2} = 2.05$ $\frac{4.5}{2} = 2.25$ $\frac{4.6}{4} = 1.15$

15.11.1 See examples given above. Do:

a. $\frac{2.2}{10}$ b. $\frac{2.33}{10}$ c. $\frac{2.404}{10}$ d. $\frac{5.15}{10}$

e. $\frac{5.001}{10}$ f. $\frac{1.1234}{2}$ g. $\frac{12.1234}{2}$ h. $\frac{3.330}{5}$

i. $\frac{6.6606}{10}$ j. $\frac{13.13}{2}$ k. $\frac{16.8}{8}$ l. $\frac{1.68}{8}$

m. $\frac{.168}{8}$ n. $\frac{8.0168}{8}$

15.12 Problem cases: we saw in 15.7 here we can convert by regular method.

Some divisions are endless. Then they have to be left at some point.

$\frac{10}{3} = ?$ $\frac{10}{3} = 3\frac{1}{3}$ This is OK

Division
$$\begin{array}{l} \quad 3.333 \\ 3\overline{)10.000} \\ \quad \underline{09} \\ \quad 010 \\ \quad \underline{009} \\ \quad 0010 \\ \quad \underline{0009} \\ \quad 0001 \end{array}$$
$\therefore \frac{10}{3} = 3.333...3$

15.12.1 Exercises:

a. There are many like this try $\frac{1}{6}, \frac{1}{7}, \frac{1}{9}$

b. Try $\frac{22}{7}$

15.13 SHORTCUT to where to put the dot.

[Note to teachers: $\frac{1}{2}$=0.5. 1 cannot be divided by 2. 1 can be taken as 1.0. This is because o has no value and only zero after **.** (Decimal point) is acceptable. But 10 can be divided by 2.

$\therefore \frac{10}{2} = 5$; $\frac{1.0}{2} = \frac{10}{2}$ (With decimal point to be put later)

= 5 (With decimal point $\frac{1}{2}$ =.5

Similarly $\frac{1}{8} = \frac{1.0}{8}$ (This is not enough)

But $\frac{1}{8} = \frac{1.000}{8}$ (This is OK because 1000 can be divided by 8)

$\frac{100}{8} = 125$; $\frac{1}{8}$=125 (with **.** somewhere)

=.125]

Chapter - 16 Percent

16. Percent:

Percent means per hundred.

16.1 Make the dividing number (i.e. denominator) as 100, the numerator you get is called percent.

Thus $\frac{1}{2} = \frac{?}{100}$ $\frac{100}{2} = 50$ $\therefore \frac{1}{2} = \frac{50}{100}$

Or $\frac{1}{2} = 50$ % (percent)

$\frac{1}{4} = \frac{25}{100}$ $\therefore$ = 25 %

$\frac{1}{5} = \frac{20}{100}$ $\therefore$ = 20 %

16.2 Exercises:

Convert the given fractions into percent:

a. $\frac{95}{100}$ b. $\frac{61}{100}$ c. $\frac{34}{100}$ d. $\frac{59}{100}$

b. In (a1) to (a4) above, assume they are marks obtained. Given $\geq 80\%$ distinction; $\geq 60\%$ is I class. < 35% fail. Write down who all got I class and who all failed.

c. Marks obtained by (a) to (d) out of maximum 10 are given below. Convert them into percent.

a. 3 b. 5 c. $7\frac{1}{2}$ d. 9

d. (a) to (d) are marks for total 25. What are the percentage values?

a. $7\frac{1}{2}$ b. $12\frac{1}{2}$ c. 23 d.18

16.3 In the case of decimals it is much simpler; this is because, for a decimal fraction, here is no denominator i.e. their denominator is 1.

Thus 0.5 = what % Ans: 0.5 X 100
= 50.00
= 50 %

0.25 = 0.25 x 100
= 25 %

0.2 = 0.2 x 100
= 20 %

16.4 Convert the given decimal numbers into percent.

Example: .125 = .125 x 100 = 12.5%

Do:

a) .55 b) .60 c) .6025 d) .01 e) .02
f) .05 g) .10 h) .99

16.5 Percent is very familiar to all the students. Their pass marks, mutual comparison of results are based on percent values.

Activity:
Students can bring their own mark sheets and verify the % marks calculation.

16.5.1 In a school, 80 students appeared for SSLC exam. 60 passed. What is the % result of the school?

16.6 Note to teachers:

Teachers, there are many real life examples of percent.

E.g.: Rebates, tax, commission, stamp duty, loan interest, economic growth rate. Use some of then to create interest in the subject.

16.7 Why percent?
Ans.: To remove the necessity of indicating any number.
E.g. 1: How Many persons voted? In different places, numbers will vary. These numbers will not be able to tell us whether many did not go to vote or not. Percent vote gives us an idea by which we can compare results of different places.
E.g. 2: How much allowance can we give to our employees? Any one answer (like Rs. 1000) will not be sufficient. Allowance may have to depend on the income (=salary) of each person. i.e., it should be appropriate So, instead of giving one number, we can say all will get 20% of their own salaries.
Thus percent is an easy way of comparing varying quantities; method of proportional allotment.

16.8 **Activity:** Go forward to the chapter on Graphs (in this book) and see how % can be shown in bar and pie chart.

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Chapter - 17 Profit and loss

17.1 A short note on what this chapter is: Students, do not think this is mathematics. It is not. It is commerce, business. Therefore easy? Yes, easy, if you go by steps. Necessary? Absolutely. Many activities depend on this subject. Many words like buying and selling, manufacturing and marketing, purchasing and retailing depend upon calculation of profit and loss. Many subjects like small-scale industry, large-scale industry, microeconomics, microeconomics national income. Global commerce etc refers to profit and loss. The subject is every big. So, some basic arithmetic will be shown here. Students and teachers may find their own examples and problems anywhere and everywhere.

17.2 Cost, Sale: Two terms buying price, selling price are very important.
[Price – not ‘prize’]
[‘Sale’ here does not mean cheap and below cost]
[Students from Mysore should be alert]
Let LHS = Cost
Let RHS = Sale Price (SP)
If you sell at cost, no gain, no loss
LHS = RHS (no gain)
If RHS > LHS, Profit
If LHS > RHS, Loss
Some persons write like this:
Sale price = cost price + profit
(Let SP be sale price, CP be cost price)
$\therefore$ Profit = SP – CP)

17.3 Student can do some simple exercises:
a. Bought a pen for Rs. 10, sold for Rs. 10, Profit/Loss?
b. Bought a shirt for Rs. 300 sold for Rs. 300, Profit/Loss?
c. Bought a cycle for Rs. 2000 sold for Rs. 2200, Profit/Loss?
d. Bought a saree for Rs. 500 sold Rs. 600, Profit/Loss?
e. Bought a scooter for Rs. 21000 sold Rs. 11,000, Profit/Loss?
f. Bought a site for Rs. 3 Lakhs sold Rs. 5 lakhs, Profit/Loss?

17.3.2 Instead of one item in the questions, we can buy many and sell one by one.

Eg: Buy 10 pens for Rs. 70 and sell each pen at Rs. 10. Students can make their own questions.

17.4 Bulk Buying and Selling: Every homemaker knows going to wholesale market and buying is cheaper. Many small scale vendors do this.

17.4.1 Exercises:

a. Bought a gross of pens for Rs. 1000. Sold each at Rs. 10. Profit? Percent? (Gross=12 Dozens. Dozen = 12 items).
b. Bought 1000 shirts for 1 lakh. Sold Rs. 150 per piece. Profit? Percent?
c. In (b) above 300 were small size 700 were adult sizes. Sold children's shirts at Rs.50. Sold adults shirts at Rs. 200. Profit or Loss? Percent?
d. Bought 100 sarees at Rs. 500 each. Selling Price was Rs. 600. But 20 Sarees were found defective. So only 80 should be sold. Profit or Loss? What can be done with 20 defective sarees, if there should be no loss?

17.5 Whole Sale and Retail: Retailing involves middleman, his profit or commission etc. In 17.4.1a above, 1 gross pens cost Rs. 1000. But sales man takes away Rs. 2 per pen. Sale price is the same Rs. 10 per pen. Now calculate profit percentage. Many such calculations are necessary students can easily manage them with the help of calculators.

17.6 Caution

17.6.1 The concept of cost price will change according to who you are. If you are a wholesaler your cost price can be small. If you buy from a wholesaler and want to sell, wholesaler's sale price is your cost price.

17.6.2 Calculation of loss or gain (especially) % calculations always refer to your cost price. Not sale price.

17.6.3 Discounts, concessions etc refer to sale price or the number written on the sales tag.

17.7 LHS = Cost RHS =Sale

17.7.1 (Cost price + all the expenditure made up to the point of selling) = LHS
RHS = Total money earned + any other benefit.
LHS includes transport cost, salaries of staff etc. This will be easily understood by ALL the students.

17.7.2 Now think of a bigger venture and associated heads of expenditure.
Cost price, transportation, insurance, godown and storage charges, cost of damaged articles. Now think of sale price required etc.

17.7.3 In addition to the items mentioned in 32.3, bigger ventures will have more heads of expenditure.
E.g.: Establishment charges, capital and its interest, advertisement expenditure, repacking, branding expenses, discounts and offers, if given.
So, the simple idea of profit or loss becomes very complicated.

17.7.4 Lastly take an example of a textile mill or car manufacture or any product of a factory and try to work out the final cost. It may be too much for high school level students. But it may be interesting to the students from a non-maths point of view. Teachers can discuss and help.

17.8 Problems are not given here. Any guidebook for competitive exams will have plenty of questions.

17.9 Students may realize many finance jargon (= technical terms) are related to profit and loss. Cost estimation, profit margin, price escalation, price reduction, profit share, returns on capital etc are all dependent on one another.

Activity: Collect some formulas from PUC (Commerce Students).

Chapter - 18 Borrowing and Lending

18. Borrowing and Lending:

Loan: Often means the money borrowed – rate of interest agreed upon and total sum to be returned after the duration of the loan.

18.1 If you borrow a pen or pencil, your are supposed to return it.

If you borrow a book, you are supposed to read (or use it) and return. You should return the book after a reasonable time, whether you had read it or not.

There are many such borrowings in real-life. They include small amounts of money taken (or given to help) in an emergency situation. In these cases there is no extra money involved.

All these can come under the term, **Interest- Free Loans**.

18.2 For calculation purposes, there has to be some Interest. This calculation depends on many items:

Principal – Amount of money taken by the borrower (not head of a college).

Duration – Time for which the borrower keeps the money with him.

Rate of Interest – The extra money to be paid by the borrower for having used the principal amount.

18.3 Exercises: (Activity)

a. Students can write down (or discuss) loans taken by their relatives and friends.

They can write:
- i. Amount of loan:
- ii. When taken:
- iii. From whom: Relative/Saukar/Bank/Sangha:
- iv. Purpose (Real):
- v. Rate of interest (clearly stated / no if clear; terms):
- vi. Still paying interest:
- vii. How much interest paid until now:

b. Students can be encouraged to discuss any moneylenders they know, then mode of paying, method and amount of collecting interest etc.

18.4 Rate of Interest in local small groups and social circles, is expressed in many different ways, sometimes per day, many occasions per week; mostly monthly. This is because in gambling and other places loan amount can range from Rs. 10 to Rs. 10000 and duration of loan can be as small as a day.

18.4.1 Activity: [Teachers, please do this activity. These may be less of arithmetic in this activity. But it will be enlightening socially]. Students could tell funny (or strange) stories of emergency loan taken and for what purposes.

18.5 10% rate of interest (R, hereafter) means, if the principal amount (P) is Rs. 100, duration of loan (T, time) is 1 year, interest is Rs.10.

Worked example:
What is the interest to be paid for a loan of Rs. 100 for 1 year at the rate of 10%?

Ans.: P=100 Rs., T= 1 year, R =10% (By definition of R, interest (I) = Rs. 10)

Exercises:

a. P = 100; what is I for 1 year? If R= 10%
b. P = 100; what is I for 1 year? If R= 20%

c. P = 100; what is I for R = 18%?

18.6 Interest to be paid increases if the loan amount increases. Simply stated Loan More, Interest More.

In mathematical language, they say, Interest is Directly Proportional to Loan.

Example1:
For a loan of Rs. 100 and 1 year duration, and rate of interest 10%? What is the amount of interest?

The same question can be written as, P = 100, T = 1 year, R = 10%, I =?
Ans: By definition of R, I = Rs. 10

Example2:
P = 200, T = 1year, R = 10%, I =?
Ans: IF P_1 =100, I_1 = 10
Now P_2 = 200, I_2 =?

$$I_2 = 10 \times \frac{200}{100} = \text{Rs. } 20$$

Exercises:

a. R = 10%, T = 1 year, I =? For P = Rs.500
b. R = 10%, T = 1 year, I =? For P = Rs.1500
c. R = 10%, T = 1 year, I =? For P = Rs.3000
d. R = 10%, T = 1 year, I =? For P = Rs.10000
e. R = 10%, T = 1 year, I =? For P = Rs.50
f. R = 10%, T = 1 year, I =? For P = Rs.10

18.7 Interest to be paid increases if the loan amount is kept with you for a longer time.
i.e., Time More, Interest More.
i.e., Interest is directly proportional to the duration of loan.

Example1:
P=100, R = 10%, T=1 year, I=? (Ans.: By definition, I = Rs. 10)

Example2:
If T = 2 years, I =? (Ans.: I = 2 x 10 = Rs. 20)

Exercises: All others are the same

a. T = 5 years, I =?
b. T = 10 years, I =?
c. T = 2 ½ years, I =?
d. T = ½ year, I =?
e. T = 6 months I =?
f. T = 1 month, I =?
g. T = 73 days , I =?
(Loan was from 01.01.09 to 18.03.09) I=?

18.7.1 In the examples given above, if the duration is in days (not full months). What can be done? Banks face this problem always. They calculate fraction of the year: i.e., (no of days)/365.

18.8 Interest Rate to be paid increases if the rate of interest agreed upon is more.
i.e., Rate More, Interest More.
i.e., The amount of Interest is directly proportional to the rate of interest, all others being the same.

Example:
P=100, T = 1 year, R =15% I=? (Ans.: By definition I = Rs.15)

18.8.1 Rate of interest is clearly state as % in banks and other neat transactions. In daily life all kinds of agreements are made. We should convert them into % per year. This is called Annual Rate of Interest.

Example: It was agreed that one rupee loan, will earn 1 paisa per day as interest. Minimum loan is Rs. 100. What is the interest in a month?

Ans:	1 Re 1 day	I= 1 paisa
	1 Re 30 days	I= 30 paisa
	100 Rs. 30 days	I= 30 x 100 Paise = Rs. 30

Exercise:

a. Interest of Re. 1 for loan of Rs. 10, every week.
b. Interest of Re. 2 for loan of Rs. 10, every month
c. Interest equal to loan amount, every 6 months

18.9 Put all things together (P, R, T, i.e. 18.6, 18.7, 18.8)

18.9.1 **Note for Teachers:**
Teachers allow the students to work out simple problems. Do not give out the formula at this time.

a. Loan Rs. 100 @ interest of 10%. Find the amount of interest to be paid after 1 year.

b. Let the interest rate remain at 10%. Increase the principal amount to 200, 300 ….1000. Ask the same question.

c. Let the loan amount be fixed at Rs. 100. Let the rate of interest go from 10% to 20, 30 …… 100%. Ask the same question**.** All Orally

18.9.2 From 18.9.1, try to find whether any student is able to make a formula. After some trials give out the formula.

18.9.3 **Total amount payable = Capital + Interest**
Try for various situations discussed earlier

I = $\frac{PTR}{100}$ Explain I = interest, P, T, R

If instead of T someone uses N it is also OK.

Here I = Amount of Interest
P = Principal (= Loan Amount)
T = Time (= duration of loan) in years
R = (annual) rate of interest (% per year)

Total amount = P + I = Loan + Interest

18.10.1 Worked Example:
What is the interest on a loan of Rs. 1000 for a period of 3 years at the rate of 20%?

Ans: I = $\frac{PTR}{100}$
P = Loan Amount
T = Period (=time) in years
R = Rate

Here: P = 1000 Rs.
T = 3 years
R = 20%

$$I = \frac{1000 \text{ x } 3 \text{ x } 20}{100} = \text{Rs. } 600$$

Example: In the above example, what is the total amount payable at the end of 3 years?

Ans: P=1000
I= 600
Total Amount = P + I
= 1000 + 600
= Rs. 1600

18.10.2 Exercises

a. Loan Rs. 1000, rate 20%, time 2 years, Interest=?

b. Loan Rs. 2000, rate 50%, time 1 year, Interest =?

c. A scooter on cash down payment costs Rs. 40000. They offer the same on monthly installment of Rs. 2000 for 3 years. Calculate the rate of interest charged [Approximate value is OK. Please do not go for complications].

d. If a bank gives loan for scooter at 12%. Which is better [compare with (c)].

18.11 More on Borrowing

18.11.1 **Note for Teachers:**

Teachers! Here's a chance to do some sociologically relevant mathematics

Ask the students to go and look for real life loan situations – their own kith & kin – some on daily, others on weekly or monthly rate of interest.

Calculate & show how much interest they are paying.
(As a manual writer, I cannot honestly say what all we discuss here is truly mathematics. But I will not mind if teachers digress far from maths to do some sociological education).

18.11.2 Now ask the same students to gather information on bank loans, mutual fund loans, money given by shree Shakti sanghas etc.

18.11.3 Calculate and show that these are less than the rates charged by friendly neighborhood moneylenders. Introduce the concept of banking. Let it first be customer point or view. What can you do with your money? Etc.

Let words like SB account, RD, FD be explained. If forms are brought and show also it is OK.

18.11.4 As per banking rules, calculations of interest follows some norms – teachers could try to find them and explain. (Alternatively, a visit to a friendly neighborhood branch of a banking institution could be arranged).

18.11.5 While on a visit to a bank, the student could learn about other available facilities.

18.11.6 Similarly a visit to a friendly electronics shop will reveal that they give (sell) goods on installment basis. One can collect information sheets on these items, their true prices on cash-down basis and compare.

18.11.7 If it is easy, teachers could even try to explain the term EMI. (If this involves compound interest or complicated calculations, one can wait or explain qualitatively. [EMI = Equal monthly installments].

18.12 Caution

18.12.1 Total amount to be paid,

A = P + I Where P = Principal Amount; I = Interest to be paid

And I = $\frac{\text{P x T x R}}{100}$ Where T = duration in years

R = rate of interest (% per year)

This is enough for us. This is called Simple Interest calculation.
Caution: Enough is enough. Learn no more.

18.12.2 Cautionary note for teachers:

Some persons make the pupils to memorize.

A = P $[1 + \frac{NR}{100}]$ NOT OK

Please DO NOT DO this. This kind of unwanted burden makes mathematics look like a monster.

Students! Remember that this kind of formula is neither necessary nor elegant. You do not have to remember this 2 formulas given in 18.12.1 given above together gives the above.

18.12.3 Example: If a person borrowed some money & returned a total sum of Rs. 120 after 1 year, what was the amount borrowed? Rate of Interest was 20%.

Ans: This can be easily (& elegantly as per some people's opinion) by the complicated formula A = P $[1+\frac{NR}{100}]$.

Let us do & see:

Method A: = P $[1+\frac{NR}{100}]$

Given: A = 120, P =? N = 1 year R = 20% / year

$\therefore$ 120 = P $[1+\frac{20}{100}]$ Change sides frist

P $[1+\frac{20}{100}]$ = 120

i.e., P x $\frac{(100+20)}{100} = 120$

i.e., P x $\frac{120}{100}$ = 120

i.e., P = 120 x $\cancel{\frac{100}{120}}$ = 100

Method B: I = $\frac{\text{P x T x R}}{100}$ T = 1, R = 20

= $\frac{\text{P x 1 x 20}}{100}$ = P x $\frac{20}{100}$

A = P + I

= P + P x $\frac{20}{100}$

= P $[1 + \frac{20}{100}]$ and so on.

Method B is OK; not longer; does not need any new formula to be memorized.

18.12.4 **Exercises: Do by both the methods seeing the book (open book)**

a. Total amount after 3 years at the rate of 15% was Rs. 690. What was the loan amount?

b. Total amount paid after 3 years on a loan of Rs. 200 was Rs. 750. What was the rate of interest?

18.13 Activity

Students write down (important points only).

a. How to calculate simple interest?
b. Sociological lessons you have learnt about borrowing and lending.

==

Chapter - 19 Compound Interest

19. Compound Interest:

Interest on a loan depends on the rate of interest agreed upon, duration of loan and the initial amount of loan. Such calculation shown earlier is called **SIMPLE INTEREST**.

19.1 What is the meaning of simple interest? If you have taken Rs. 1000 loan at 20% interest / year and keep it for 1 year, what is the amount of interest?

Rate of interest = 20% means,
Interest for Rs. 100 for 1 year = Rs. 20

$\therefore$ Interest for Rs. 1000 for 1 year = $\frac{1000}{100}$ X 20 = 200 Rupees

19.2 If a person A takes a loan of Rs. 1000 from B at 20% rate of interest and returns it after 4 years. What is the interest he should pay?

As per 19.1, interest for one year = 200
Interest for 4 years = 4 X 200 = 800
Now let us do it by formula

Interest = $\frac{PNR}{100}$ Here P = 1000, N = 4, R = 20

$\therefore$ I = $\frac{1000 \text{ X } 4 \text{ X } 20}{100}$ = 800 (OK)

19.3 Let us look at the problem once again. Annual rate of interest implies the debtor (person who took the loan) pays the interest at the end of the year (it is like a rent you pay for using something such as a house). If A had paid the interest of Rs. 200 at the end of the first year to B, B could have used the money. Isn't it so? He could have used it for 3 years. In fact B deserves to be paid more.

19.4 The argument given in 19.3 tells us that interest not paid in time is not correct. A calculation, which includes the interest on interest, is called **COMPOUND INTEREST**.

In other words, if interest is not paid in time, it should be considered as additional loan amount from that date.

19.5 Let us workout 19.2 on the basis of COMPOUND INTEREST.

Loan at zero time = Rs. 1000
Interest for 1 year = Rs. 200

Amount due at the end of I year = Rs. 1200

Now treat this as loan amount for the second year.

Loan at start of II year = 1200

Interest for I year = $\frac{1200 \text{ X } 1 \text{ X } 20}{100}$ = 240

Total amount due at the end of II year = 1200 + 240 = 1440

This money earns interest for I year = $\frac{1440 \text{ X } 1 \text{ X } 20}{100}$ = 288

$\therefore$ Amount due at the end of III year = 1440 + 288 = 1728

This money earns interest for I year = $\frac{1728 \text{ X } 1 \text{ X } 20}{100}$ = 345.60

$\therefore$ Total amount due at the end of IV year = 1728 + 345.60 = 2073.60

19.6 Compare the result of 19.5 and the result of 19.2.

Total amount payable at the end of 4 years:

a. On SIMPLE INTEREST = 1800

b. On COMPOUND INTEREST = 2073.60

It certainly makes a difference. Most of the real life situations are calculated on the basis of compound interest.

19.7 We can make the calculation of 19.5 in another way also.

A should have paid B, interest at the end of each year.
He did not do so.
He kept Rs. 200 for 3 years.

Interest on this = $\frac{200 \text{X } 3 \text{ X } 20}{100}$ = 120 (I)

Another 200 remained with him for 2 years.

Interest on that = $\frac{200 \text{X } 2 \text{ X } 20}{100}$ = 80 (II)

Another 200 was with A for one year

Interest on this last year = $\frac{200 \text{X } 1 \text{ X } 20}{100}$ = 40 (III)

Thus A has to pay all these amounts:
Principal amount = 1000
Interest on interest (I) = 120
Interest on interest (II) = 80
Interest on interest (III) = 40
Plus the basic interest = 800
(See 19.2) -------
Total = 2040

On the basis of this calculation, total interest payable is 2040.

19.8 Let us tabulate.
Loan = 100, Rate = 20%, Duration = 4 years
Total amount due:
SIMPLE INTEREST CALCULATION = 1800
INTEREST ON INTEREST = 2040
COMPOUND INTEREST = 2073.60
Correct method is the last one. Others are approximations; they favor the debtor.

19.9 Teachers! There is a formula for compound interest. It involves EXPONENTS. Not necessary for us. We can use the longer method shown in 19.5. If some students want to know it is here.

19.10 Compound Interest Calculation:

$$A = P\ (1 + \frac{r}{100})^{n}$$

Where A = Total amount to be paid
= (Principal + Interest)
P = Principal = Loan Obtained
r = Rate of interest (% per year)
n = number of years

[In the above example (section 19.2) P = Rs. 1000, r = 20%, n = 4

$$\therefore A = 1000\ [1 + \frac{20}{100}]^{4} \quad = 1000 \text{ x } [1.2]^{4}$$

19.11 Loan amount was Rs. 200. Rate of interest was 15%. Duration of loan was 3 years.

1. What is the total amount to be paid after 3 years on the basis of Simple Interest?
2. On the basis of Compound Interest? [Note: For simple interest calculations go to chapter 18; use formula given there. For compound interest calculation, use any formula given here].
3. A loan (say Rs. 1000) was offered to you with 2 options: option X: you can pay simple interest of 25% or option Y: you can pay compound interest at a lower rate of 20% loan is only for 2 years. Which option will you take?
4. In (3) above if the loan is for a period of 5 years, will your option be the same?
5. [This is for advanced level students]. In 3 and 4 above, does the option depend on the amount of loan? [Answers should have calculations and numbers to justify your answer].

Chapter - 20 Assessment – 1

After 19 chapters the students will have learnt some essentials of arithmetic. It is assumed in this manual that Functional Mathematics should enable a person to:

a. Do simple operations of multiplication, division, addition and subtraction.
b. Do some simple approximations (i.e. sensible guess work)
c. Boldly handle fractions with confidence.
d. Be at ease with percentages.
e. Handle with equal ease both fractions and decimals.

Some ideas on profit and loss and calculations of interest are given.
Now is a good time to assess the student's understanding.
Assorted (and Randomized) problems can be given at this stage.

Random Questions

1. List all odd numbers between 1 and 20.
List all even numbers between 1 and 20.

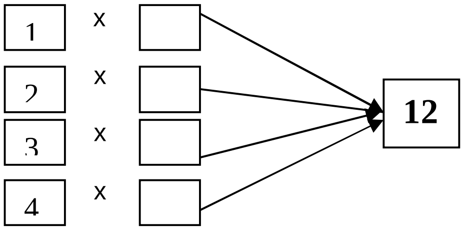
2. 83045 – Indicate space value of number 3.

3. If 4 + 6 = 10, then 10 – 4 =?

4. If A + B = C, then □ + B = C $\boxed{?}$ =

If A + B = C, then C – B = $\boxed{?}$ $\boxed{?}$ =

5. Fill up:



6. Fractions to decimals and percent:

a. $\frac{1}{2} = 0.........=$ ___% c. $\frac{3}{10} = 0.........=$ ___%

b. $\frac{3}{4} = 0.........=$ ___% d. $\frac{33}{50} = 0.........=$ ___%

e. $\frac{56}{70} = 0.........=$ ___%

7. Fill up: a. $\frac{2}{3} = \frac{4}{?} = \frac{10}{?}$ b. $\frac{4}{3} = \frac{8}{?} = \frac{20}{?}$ c. $\frac{?}{5} = \frac{9}{15} = \frac{?}{50}$

8. a. 12345 – 2340 =?
b. 10005 + 2340 =?

9. A water tank can be built by 2 people in 3 days. By 6 people in 6 days how many similar tanks can be made.

10. Convert the following:
a. 1 meter = ……. cm
b. 1 gram = …….. kg
c. 1 hour = ……… seconds
d. 1 liter = ……… gallons
e. 1 feet = ……… Inches

11. a. 1” = ….. cm
b. 1 minute = 60 seconds True / False
c. 1 hour = $\frac{1}{24}$ day True / False
d. 1mile = 1.6 km
e. 1 km = 1.6 miles $\Big\}$ which is correct d / e

12. a. 8 ÷ 2
b. 3 ÷ 3
c. 10; 20; 30; divided by ___, ___, ___ gives 1, 1, 1
d. $\frac{0}{5} = ?$
e. $\frac{9}{3} = 3$ or $\frac{9}{3} = 27$ Which is correct?

13. a. If 12345 x 10 = 123450
Find 12345 x 9

b. If 12345 x 10 = 123450
Find 12345 x 11

14. Calculate the percentage:
a. 20% of 350
b. 15% of 750

15. a. Write in ascending order $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}$

b. Write in ascending order $\frac{1}{2}, \frac{1}{22}, \frac{1}{25}, \frac{1}{250}, \frac{1}{260}$

c. Write in ascending order $\frac{1}{50}, \frac{2}{97}, \frac{3}{160}, \frac{4}{240}, \frac{5}{255}$

16. Convert the mixed fraction into simple fractions:
a. $2\frac{1}{2}$ b. $1\frac{1}{4}$ c. $1\frac{1}{2}$ d. $1\frac{3}{4}$ e. $4\frac{3}{4}$

17. Divide the following:
a. $\frac{22}{5}$ b. $\frac{22}{8}$ c. $\frac{10}{3}$ d. $\frac{22}{7}$ e. $\frac{1}{8}$

18. a. 17 x 12 = b. 17 x (10 +2) =

19. Use brackets in suitable places:
a. 5 x 2 + 4 = 14 b. 5 x 2 + 4 = 30 c. 5 x 2 + 4 –2 =12 d. 5 x 2 + 4 – 2 = 28

20. Subtraction of fractions:
a. $\frac{2}{3} - \frac{1}{3}$ b. $\frac{8}{7} - \frac{1}{7}$ c. $\frac{12}{6} - \frac{8}{6}$ d. $\frac{10}{7} - \frac{3}{7}$ e. $\frac{51}{17} - \frac{34}{17}$

21. Write ascending order: a. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ b. $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{6}, \frac{3}{4}$ c. $\frac{3}{5}, \frac{7}{12}, \frac{3}{4}, \frac{5}{6}$

22. a. $\frac{3}{4} - \frac{2}{3}$ b. $\frac{3}{4} - \frac{1}{4}$ c. $\frac{1}{2} - \frac{1}{4}$ d. $\frac{7}{12} - \frac{4}{12}$

[Help: Make the denominators equal, 4 is easy)

23. Which is greater: a. $\frac{1}{2}, \frac{2}{3}$ b. $\frac{5}{6}, \frac{2}{3}$ c. $\frac{5}{7}, \frac{4}{5}$

[Help: Make the denominators equal]

24. a. $\frac{1}{2} + \frac{1}{3}$ b. $\frac{1}{6} + \frac{1}{3}$ c. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

25. Find LCM:
a. 4, 8, 12 b. 3, 10, 6 c. 3, 10, 6, 5

26. Ramu had 100 sarees. The cost of saree sold for Rs. 300 each. But only 80 could be sold. 20 sarees were found damaged. Profit or loss? What can be done with those 20 sarees, if there should be no loss, no profit?

27. Cost price = Rs. 200. Percent profit wanted = 25%. Sale price =?

28. Onion price Rs. 200 per quintal sold by farmers. A buys and sells to wholesaler B. B sells to Mandy (APMC). D buys from APMC and sells. A, B, C, D all sell for 100% profit. What is the cost price for consumer per kg? (1 quintal = 100kg).

29. Find percent:

a. 5 subjects marks: $\frac{4}{10}, \frac{5}{10}, \frac{9}{10}, \frac{3}{10}, \frac{9}{10}$

b. Total marks 330 out of 600 find % marks.

c. I class is 60%. How many marks total put of 625?

30. Bought a gross (12 dozen) of pens for Rs. 1440 sold at Rs. 10 per piece. Profit or Loss? How much percent?

31. a. Bought a pen for 8, sold at 10. Profit / Loss? Percentage?
b. Bought a shirt for 100, sold at 150. Profit / Loss? Percentage?
c. Bought a cycle for 2000, sold at 1200. Profit / Loss? Percentage?
d. Bought a saree for 500, sold at 300. Profit / Loss? Percentage?
e. Bought a scooter for 16000, sold at 12000. Profit / Loss? Percentage?
f. Bought a house for 1 lakh, sold at 2 lakh. Profit / Loss? Percentage?

32. A Cyclist is moving with 10km constant speed. How much distance can be covered in 40 minutes? (1 Hr= 60 minutes)

33. A box of a 3 dozen-apple cost is 600. What is the cost of 3 apple?

34. If a quintal of rice costs Rs. 1000, what is the price of 5Kg rice?

35. A Bucket contains 10 liters juice. 1-liter juice can be given to 5 boys? Bucket juice can be given to how many boys?

36. A saree costs Rs. 2215. You have only Rs. 10,000. How many sarees can you buy?

37. A pen costs Rs. 23.00. You are given Rs. 200 only. How many pens you can purchase.

38. Bought wholesale 100 shirts for Rs. 10000. Some were small size (300). Others (700) were adult size. Sold the children’s shirts at Rs. 50 each. Sold the adult size at Rs. 200 each. What is the percent profit?

39. In 22 above 10% of junior size and 10% of adult size items were found damaged and useless. Now what is the percent profit / loss?

40.

17		1	8	15
	5		14	16
4		13		
10				3
	18			9

All add up to 65.
Fill up diagonals rows columns added = 65

41.

-	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8
2	-1	0	1						
3	-2	-1	0						
4				0					
5					0				
6						0			
7							0		
8								0	
9									0

Activity Based Problem:

A. Show by cutting shapes or rectangles or using graph paper.

a. $\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$ b. $\frac{1}{2}+\frac{1}{4}$ c. $\frac{1}{3}+\frac{1}{3}$ d. $\frac{1}{2}+\frac{1}{3}$

e. $\frac{1}{4}+\frac{1}{8}$ f. $\frac{1}{4}+\frac{1}{3}$ g. $\frac{1}{4}+\frac{1}{3}+\frac{1}{8}+\frac{1}{6}$

Activity Based Problem:

B. a. Which is bigger? $\frac{1}{2},\frac{1}{3}$

b. Which is bigger? $\frac{2}{2},\frac{3}{4}$

c. Which is bigger? $\frac{5}{6},\frac{7}{8}$

Show a, b, c above by 2 methods:

1. By cutting rectangles (or Circles).
2. By drawing on graph sheet.

Activity Based Problem:

C. Draw any small square. Show $\frac{1}{4}$ in 2 ways (at least).

2. Draw any rectangle. Show $\frac{1}{4},\frac{1}{6},\frac{1}{9}$ etc

3. Draw any circle. Show $\frac{1}{4},\frac{1}{8},\frac{1}{6},\frac{1}{3}$

Chapter - 21 Substitution

21. Substitution:
This is a very important operation in maths.

21.1 E.g.: Start with language: mother tongue, English any other.
What is his name? His name is X

Instead of X put the right name there.
What is her name? Her name is Y
Instead of Y put the right name there.

Exercise: Do the following:

My friend's name is

My another friend's name is

Students do the above and compare.

21.2 Activity

A. Class Activity: Needed – A dictionary
One student find the word 'substitute' reads out the meaning (local language also ok). Another writes on board. Class discusses this. Students can try to give some examples or stories.

B. Self Study Student's Activity:
[Students who read this book can and must do all the activities and exercises. Only some activities like games require a class (or teacher or many persons). Many activities can be done by the student]. Take a dictionary. Write down the meaning. Write a few occasions you know in which substitution has taken place.

C. After doing the activity given above, check whether the following items have been discussed:

a. Substitute player in games (Cricket, football etc).
b. Substitute teacher.
c. Substitute or temporary work.

21.3 Activity

A. Let the students bring any kind of forms available. Eg their own application form for the course or any other. Let them check one another's forms for correctness.

B. Teacher can try this:

My date of birth is
(Sample: 02.10.1869)
Let the students do this.

Now change the format
(Sample: 2nd October, 1869)
Let the students do as per this example.

21.4 Activity

A. Ganga is a good girl. Let all girl students substitute their own name, in place of Ganga. Let them read out.

B. Let boys have: Basava is a big boy (or anything else) Do substitution.

21.5 If 10 mangoes cost Rs. 60. 1 mango will cost Rs. 6. Now instead of mangoes, put any item. The answer is the same (10oranges, apples, pens, pencils….). Even unbelievable 10 cars, 10 scooters.
Students should do ALL these without getting bored (or thinking these are very easy).

10	Mangoes		Rs. 60
1	Mango	Costs	Rs. 6

Now put in place of mango [here we ignore singular plural of the nouns. Both are the same for our purpose]. Activity to Do:

a. $\boxed{\text{Apple}}$ b. $\boxed{\text{Banana}}$ c. $\boxed{\text{Pen}}$ d. $\boxed{\text{Pencil}}$

e. $\boxed{\text{Car}}$ f. $\boxed{\text{Scooter}}$

What do we observe here? The box may contain mango, apple or pencil or any other thing, the arithmetic remains the same. Now we understand substitution.

Exercises

21.5.1 In a “China Bazaar” (Mysore Local Slang). All items displayed have the same price. You see 1 plastic dabba, 2 pens in a bunch, 4 pencils in a box …. Take anything for Rs. 10.

a. What is the cost of a plastic dabba?
b. How much have you paid for 1 pen?
c. What is your cost of one pencil?
d. In the above china bazaar exercise you did not like the dabba you bought. You go and ask, “ Can I exchange this for something”. What was the shopkeeper’s reply?
e. Do you see “substitution”, “exchange” etc., have some condition? If yes, what is the condition?

21.5.2 Let us go back to (10.5) Mango example. Can we say m = mango, a = apple, b = banana, p = pen, c = car, s = scooter.
Cost of 10m = 60. $\therefore$ cost of 1m = 6.

Exercise:
Students don’t be lazy. Write down all the others here.

21.5.3 In the above exercise, did you write 10m, 10a, 10c etc…? If yes, Well Done! Mathematicians also do the same.

Just like them try now with letters of the English alphabet.

If 10a = 60; 1a = 6 (for 1a we can write simply a)
10 x = 60; x = 6
10p = 60; p = 6

21.6.1 We said: 10 of $\boxed{}$ cost $\bigcirc$ = 60

1 of $\boxed{}$ cost $\dfrac{\bigcirc}{10} = \dfrac{60}{10} = 6$

Now let us try:

10 $\boxed{\text{Kg}}$ of mangoes cost Rs.60

1 $\boxed{\text{Kg}}$ of mangoes cost $\frac{60}{10} = 6$

Exercise:
Students, please write down patiently: In place of $\boxed{\text{Kg}}$ write,

Baskets
Dozens
Heaps
bags

Do you see answer is the same? If yes, Well Done!

21.6.2 You can have double substitution also, with the same result!
Try

items
Kg
basket
bag

mango
apple
banana
pen

Any combination of two.

21.7 We learnt that substitution works when quantities are equal. To write this, symbols can be used. We used ▭ and ⬭. Some people use
★, ◉, ✳, ∆ etc.,

Note for DTP person. Any symbol is fine. Be careful to write in proper places same symbols.

21.7.1 A.

```
     5  ★ ◉ ✳ ∆
  +  ∆  ✳ ◉ ★ 5
     ----------------
     6  6 6  6 6
     ----------------
```

Find ★=? ◉=? ✳=? ∆**=?**

B.

```
     5  !   ? %  ◉
  -  ◉ %  ?  !   5
     -----------------
     4  1  9  7  6
     -----------------
```

Find !, ?, %, ◉

21.7.1 For using as symbols, letters of the language are very useful. Why not? What are letters (not a sheet of paper written to another); letters of the alphabet? They are Symbols telling us what sound to produce. “Letters are Phonetic Symbols”. Students, ask the meaning of this sentence from teachers (& elders).

21.8 Using letters (of the alphabet) as symbols, we can express some mathematical property. If we know, how to substitute, we can do many problems.

21.8.1 We know: 1 + 1 = 2 2 x 1 also = 2
7 + 7 =14 2 x 7 also = 14
(123) + (123) = 246 2 x (123) also = 246

If we write □ + □ = ○
2 x □ also = ○
i.e., □ + □ = 2 x □
This is the principle of “multiplication as addition” seen earlier.

21.8.2 Similarly
□ +□ + □ = 3 x □
□ +□ + □ + □ + □ = 5 x □

21.8.3 Let us use x instead of □
$x + x = 2 \text{ x } x = 2x$
$x + x + x = 3 \text{ x } x = 3x$
$x + x + x + x + x = 5 \text{ x } x = 5x$
[x symbol is not used; But 5 x means 5 x x]

21.9 Exercise

21.9.1 Students do this:
a. Take any one equation: $x + x + x + x = 4 \text{ x } x$
Put $x = 4$
$x = 6$
$x = 128$
x = (any number) Verify.

b. Now write the equation as:
$x + x + x + x = 4 \times x$
a + a +a +a = 4 x a
d + d + d + d = 4 x d

Put $x = 4$ or a = 4 or d =4. Verify & satisfy yourself; answers are the same.

21.9.2 $x + 2x = 3x$

Try $x = 1$ 1+ 2 x 1 = 3 x 1 i.e. 1 + 2 = 3
Try $x = 2$ 2 + 2 x 2 = 3 x 2 i.e. 2 + 4 = 6
Try $x = 33$ 33 + 2 x 33 = 3 x 33 i.e. 33 + 66 = 99

This shows that symbols help to generalize.

a. **Exercise:** Do it yourself
3y + 2y = 5y
b. 5y + 3Y = 8y
c. 5y – 3y = 2y
[Verify by substituting a number for y. This number can be any number]

21.10 Let us try some substitutions.

21.10.1 If a. $x = 2$ and $y = 5$ then $x + y$ =?
b. $x = 2$ and $y = 5$ then $y - x$ =?
c. $x = 2$ and $y = 5$ then $x - y$ =?
d. $x = 2$ and $y = 5$ then $xy + x^2 y$ =?

Ans: a. $x + y = 2 + 5 = 7$
b. $y - x = 5 – 2 = 3$
c. $x - y = 2 – 5 = -3$
d. $xy + x^2 y = x \times y + x \times x \times y$
$= 2 \times 5 + 2 \times 2 \times 5$
$= 2 \times 5 + 2 \times 2 \times 5$
$= 10 + 20$
$= 30$

21.10.2 Exercises:

A. Students can create many such simple examples. Try

B. Given $x = 100$, $y = 99$
Find $x + y$ =?
$x - y$ =?

C. Given $x = 10$, $y = 9$
Find 1. $xy + x$ =
2. $xy - x$ =
3. $xy - y$ =
4. $xy + y$ =
D. For fun, do also 1. $x(y+1)$=?
2. $x(y-1)$=?

21.11 Activity

21.11.1 Students can do and check with the teacher. Go to addition and subtraction of earlier sessions and do with symbols of earlier sessions.

21.11.2 Squares:
A number multiplied by itself is called the square of the number.

Examples:
If $x = 2$ $x^2 = ?$
$x = 5$ $x^2 = ?$

21.11.3 Square, square root, cube, cube root etc can be defined using symbols. This is called algebra. It helps to understand number system also. Any systematic study of science and engineering requires a basic understanding of algebra.
Where there is a formula, there will be an equation.
Where there is an equation there will be algebra.
If there is algebra or equation or formula (or any kind of symbolic representation).
Knowledge of Substitution is necessary

21.11.4 **Exercise:** Explain with examples at least 3 situations where substitution is necessary and used.

===

Chapter - 22 Concept of Negative Numbers

22. Concept of Negative Numbers:

[Teachers! Tell the students that positive numbers and integers can be physically shown, like fingers and objects. Zero is difficult to show. Negative is in the mind, but quite useful].

For self study students: This chapter helps you to understand. It is like reading a storybook. There are no exercise or exams, so please try to read. Some students never read any book, not even storybooks. Such persons neither read English nor their own mother tongue. Such persons will not even come to this para. Teachers (or other elders) may kindly read out and explain this chapter.

For Teachers:
Only very simple concepts are explained. This manual write has tried to make it as simple as possible (for him). Teachers may kindly read and explain to the students.

At the end of the chapter, some games and activities are suggested. Please do them & you can improve upon them too.

22.1 Concept of Zero.

22.1.1 Zero is a result of taking away what is there.

$10 - 10 = 0$	$5 - 5 = 0$	$12345 - 12345 = 0$
$\therefore\ 0 + 2 = 2$	$0 + 10 = 10$	$0 + 12345 = 12345$

This tells you that you can take away from a source, a little at a time or all together, until it gets empty. Now, can you take away, MORE THAN what is there? Usually not.

22.1.2 If your uncle is a business man and has a current account in a bank, he would have done what is not possible for you and me. i.e., taking out more than what is there.

Of course, he can put in later and bring the balance from negative to zero and may be positive.
[Bank officers will say that your uncle had a debit when he made an overdraft (drew more than the balance].

22.1.3 Now we can understand:

Adding (+ operation) any number to any number is possible. But subtraction is not so.

E.g.: $10 - 4 = 6$

4 – 6 =? It is not possible

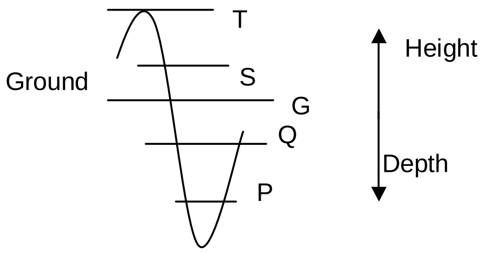
Therefore the subtraction grid (of Chapter 3) had some blank pages.
But one can view it this way
6 – 4 = 2. 4 – 6 = -2.

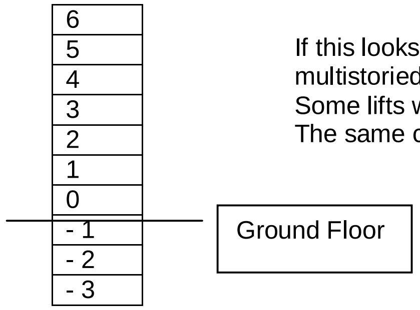
22.2 What follows is not maths. It is simple explanation. Students, please read through, you will understand algebra better. Teachers may kindly read and explain. There are no problems, exercises or exam, questions on this paragraph. Still explain the concepts to students.

22.2.1 **Concepts of depth & height**

P – down 4 feet
Q – down 2 feet
G – ground
S – up 2 feet
T – up 4 feet

Down is –
Up is +





If this looks like maths, ask the students to imagine that they are in a multistoried building & in a lift. Ground level is zero (= 0).
Some lifts will show B1, B2, B3 i.e. basements.
The same can be shown as –1, –2, –3, etc.

22.2.2 Concept of Number Line:
Many books give the concept of numbers as points on a continuous line. Where you start (or say you are standing) is zero. All to your right is positive and all to your left is negative. Such a concept is given as a game at the end of this chapter.

22.2.3 Concept of a long road:
Some others compare the system of natural numbers to a very long road, where milestones represent integers. Where you are is zero. What you left behind is negative, what is in front of you is positive.

A. Thus if you go back 2 km and come forward 2 km, you are at zero again. i.e., (-2) + (+2) = 0

B. If you go 2 km reverse, and go still more 3 km reverse, you will be 5 km back from the start. i.e., (-2) + (-3) = - 5

C. If you go 1 km forward, and again 4 km further, you will be where? i.e., (+1) + (+4) = +5.

22.2.4 The same concepts [i.e., A, B, C given above] can be explained by the “number line” also. Here left is –ve, right is +ve.

22.2.5 Concepts are described in this section. They are only analogies [i.e., examples, comparisons] given for the sake of understanding. Working rules come in the next section. They are easy.

22.3 Once we know +, - number system we can avoid writing (add) (sub) etc. Adding is just many positive numbers together. Subtracting is coming together of one positive number and another negative number.

To clarify this:

a. 1 + 2 = 3 Actually + 1 + 2 = +3 = 3

b. 2 – 1 = 1 Actually + 2 – 1 (together i.e. add +2 & -1) = + 1 = 1

1 – 2 =? Actually + 1 – 2 = $+\cancel{1}-\cancel{1}-1$ = -1 ? = - 1

22.3.1 Example: Write down the question differently including signs of the numbers and then simplify:

a. 18 + 12 = (+18) + (+12) = +30 =30
b. 1+ 2 + 3 + 4= (+1) + (+2) +(+3) + (+4) = +10 = 10

Exercises:

1. 101 + 99 =?
2. Add together 5, 41, 4, 10.
3. What is the sum of Rs. 100, 500, 1000?
4. You are a conductor. You issued 50 tickets of Rs. 5 and 20 tickets of Rs. 3. How much money is in your bag?

22.3.2 Example:
a. 18 – 12 = (+18) + (-12) = +

+ 18 items
- 12 items

+1
- 1

= 0

+
= + 6 = 6

b. 1 – 2 – 3 +4 = (+1) + (-2) + (-3) + (+4) = +(+1+4) +(-2-3)
= + (+5) + (-5) = 0

Exercises:

1. 101 – 99 =?
2. Add together 5, - 41, - 4, 10
3. What is the sum (=final balance) of bank deposits of Rs. 100, 500, 1000 on 3 days and a withdrawal (i.e., taking out money) Rs. 800 later? (Deposit = putting money into your bank account)
4. a. You are a conductor. After getting 10 tickets for Rs. 6 each a passenger gives you a hundred rupee note? How much will you return?
   b. You already had (before issuing the tickets) Rs. 240 in your bag. Including this hundred rupee note, how much will you have?
   c. You had already written on tickets (because you did not have change) change to be given latter. 4 persons have such tickets for Rs. 4, Rs.44, Rs. 94 and Rs. 14 with them. After giving them all how much money will you have?

   [a, b, c above look like long questions. But they are really simple. Do and see. You can even ask more questions about the conductor].

22.4 When Positives are together, net total is positive.
When Negatives are together, total is negative

a. 1 + 2 + 3 = 6 + 1 + 2 + 6 = + 9 = 9

b. –1 – 2 – 3 = - 6 - 1 – 2 – 6 = - 9 (writing – is a must)

i.e. Numbers carry with them (written before each number) their signs. Fortunately there are only two; positive (+), negative (-). Many times + is not written, it is assumed.

Exercise: Which one is a wrong answer?

1. 9 + 6 =? [a. 15 b. +15 c. +9+6 d. 3]
2. 9 – 6 =? [a. +9-6 b. –6+9 c. +3 d. –3]
3. – 6 – 9 =? [a. –15 b. – (15) c. – (+15) d. –3]

22.5 (+) and (-) symbols as operations. Explanation of addition process is given below. "Operation" word used here means 'method of doing some work'. Example: let us square 10 means 'squaring is an operation'. This means: Take a number. Multiply it by itself. The result is a square. Thus if we write (square 10) = 10^2 (small 2 sitting on top is a symbol for the operation squaring).

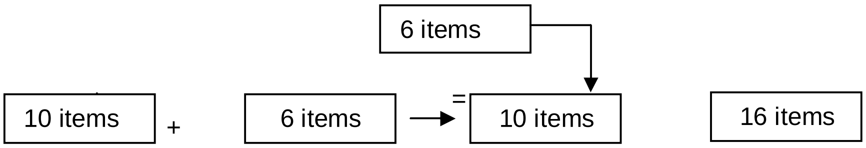
22.5.1 Repeat A from 22.5.

(+) and (–) symbols are also used for mathematical operations (i.e. working with numbers etc). When these symbols and the positive or negative signs come together, then we have to be careful.

If you put 10 items and 6 items together, total is 16 items. This can be written as 10, 6 = 16. But it is not done so. It is written as 10 + 6 = 16.

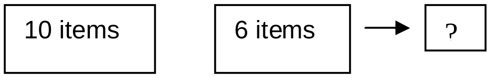
It can be considered as

Or,

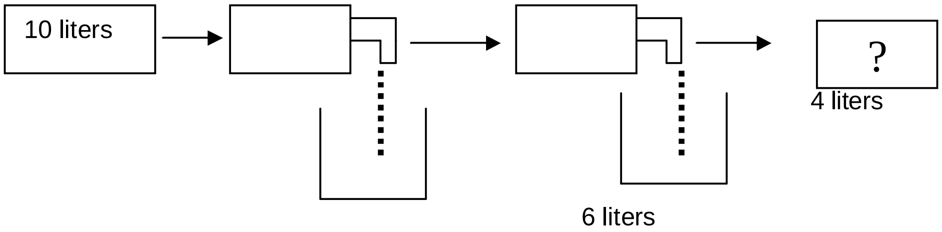


This means, in a basket containing 6 items. A more of the same are put in. now the basket has 16 items (all similar)

22.5.2 Now consider 6 items taken away from 10 items.



Or



This means, from a stock of 10 items 6 are taken away. Remaining = 4 items.

22.5.3 We have used numbers. They are all the same. So no problem. When we come to algebra, we will have different types (or kinds, categories) of quantities.

22.6 In the earlier section, we explained the operation of subtraction (-) as **Removing (=Taking Away)** of positive number. This works only when the first number is greater than the second.

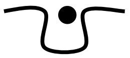
Exercise: (Students can do this). Give to primary level children (even 7th standard OK) & get the answers.

a. 3 – 2 =?
b. 3 – 3 =?
c. 2 – 3 =?
d. 10 – 6 =?
e. 6 – 10 =?
f. 100 – 98 =?
g. 99 – 99 =?
h. 99 – 100 =?
i. 1006 – 1000 =?
j. 1000 – 1006 =?

22.7 We saw earlier (Number – Line etc…) that if +ve numbers exist (on the right side) –ve numbers also exist (on the left side). If they exist, some of the problems in the earlier section could be explained. Give –ve numbers also a status. Call them by their status. Thus +6 or –6 you may say 6 = +6. But these can be a –6. So, let us give them a place.

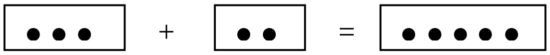
Exercise:

a. Point out which of the problems of 22.6 will have negative answer, Explain.

22.8 To make + & - operators under standable, let us assume:
+ve numbers as solid objects.
-ve numbers as holes (of the same size)
When a hole exists as a gap: it is = -1 ●
When an items exists as a solid: it is = +1 ʊ
If an item sits on a hole: it is = (+1) + (-1) = 0
Pictorially ● + ʊ =  = 0

22.8.1 Illustrate (= show by figures)

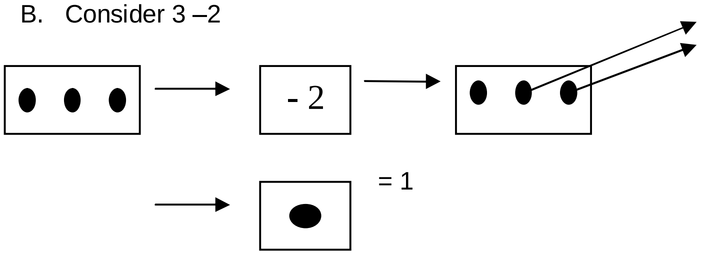
A. Consider 3 + 2



3 + 2 = 5

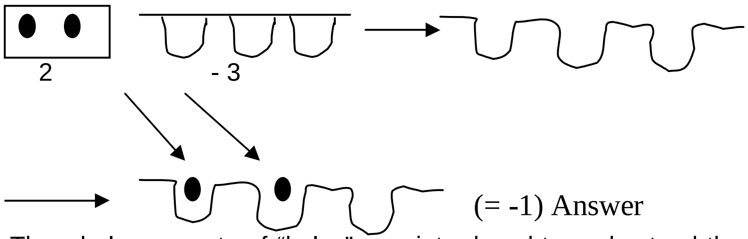
[Teachers! Please do and show. As seen, in the traditional game of 'aLa guLi maNe aata' **(ಅಳ ಗುಳಿ ಮಣೆ ಆಟ).** Using paper or black board does not convey the idea clearly].

B. Consider 3 –2



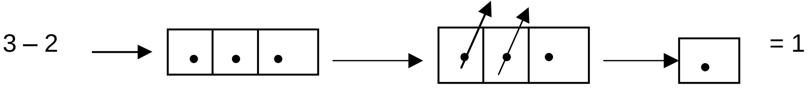
Arrow shows Removal (or Taking Out)

C. Now Consider 2 – 3

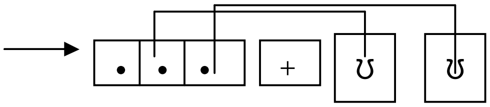


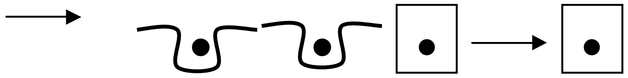
The whole concepts of "holes" was introduced to understand the above viz subtracting a bigger number from a smaller number. Students should go back and see why we wrote 2 – 3 as (+2) + (-3)
i.e., addition of (positive number) + (negative number)
i.e., ●● + ʊ ʊʊ

22.8.2 Further explanation: go back and see 22.8.1 (B)



Now see (c) above and write the same as 3 – 2 = (+3) + (-2)





= 1

Exercises: Pictorially do and show:

a. 5 – 3 b.5 – 4 c. 5 – 1 d. 3 – 5 e. 4 – 5 f. 1 - 5 g. 5 – 5
h. 100 – 100 [clue: just indicate]

22.9 Standard rules and shortcuts

22.9.1 Adding

+ 1 + 2 = 1 + 2 = 3

+ 2 + 1 = 2 + 1 = 3

+ 109 + 1 = 109 + 1 = 110

+ 1 + 109 = 1 + 109 = 110

(This can be easily extended to many numbers together 1+2+3+4 = 10).

22.9.2 Subtraction (only 2 numbers)

2 – 1 = 1 3 – 2 = 1
4 – 2 = 2 this is easy

Now consider

$2 - 3 = \boxed{+2}\ \boxed{-3}$

$\text{or} = \boxed{+\,+}\ \boxed{-\,-\,-}$

$= \boxed{\cancel{+\,-}}\ \boxed{\cancel{+\,-}}\ \boxed{-} = \boxed{-} = -1$

105 – 5 = 100

$\boxed{+ + + + +, \text{ 100 items}}\quad \boxed{- - - - -}$

i.e. $\boxed{+\,-}\ \boxed{+\,-}\ \boxed{+\,-}\ \boxed{+\,-}\ \boxed{+\,-}$ 100 items = 100 items

5 – 105 = - 100

i.e. $\boxed{- - - - -, \text{ 100 items}}\quad \boxed{+ + + + +}$

$= \boxed{+\,-}\ \boxed{+\,-}\ \boxed{+\,-}\ \boxed{+\,-}\ \boxed{+\,-}\ \boxed{- \ldots - \text{ (100 items)}} = -100$

22.10 Rule: When there are two numbers

+, +	Add the two numbers
+, –	If + is bigger, the result is +, after subtraction
–, +	If – is bigger, the result is – after subtracting the smaller number.
–, –	Careful, add both the numbers as if they were both + final result gets a sign (–)

22.11 Exercises

22.11.1 Teachers drill the students with many problems.

12.12(a) a. 2 + 5 b. 2 – 5 c. 4 – 3 d. + 2 + 5 e. + 2 – 5
f. – 4 – 3 g. 5 + 2 h. - 2 + 5 i. – 4 + 3
j. + 5 + 2 k. – 2 – 5 l. + 4 + 3

12.12 (b) a. 1 2 3 4 5 + 5 b. 5 + 1 2 3 4 5 c. 1 2 3 4 5 – 5
d. 5 – 1 2 3 4 5 e. – 1 2 3 4 5 + 5 f. – 5 + 1 2 3 4 5
g. – 1 2 3 4 5 + 5 h. – 5 – 1 2 3 4 5

(a), (b) above are just a few examples.

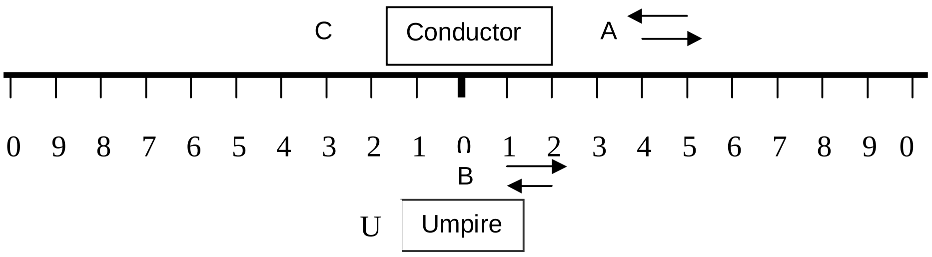
Go back to subtraction grid (Para 2.5). With the knowledge of negative numbers, the students could fill up

-	**1**	**2**	**3**	**4**	**5**	**5**	**7**	**8**	**9**
1	0	1	2	3	4	5	6	7	8
2	- 1	0	1						
3	- 2	- 1	0						
4				0					
5					0				
6						0			
7							0		
8								0	
9									0

22.12 Here is a game on +ve and –ve numbers. Open filled is better than the classroom.

Game on the number line. This should be played in pairs (2 persons on the field).

One instructor (=conductor)
One umpire (+ Scorer)
Draw a line on the ground (like Kho-Kho)



A Will be the runner B Will be the catcher

A will move only on the line and as per instructions of the conductor. B will be blindfolded and stationed at Zero. The distance can be approximately equal to A or B's one step distance. Distance between line A & line B will be less than Arm's length. How to play?

- A – open eyes – B eyes closed (tied) – facing each other. A on line A at zero point. B on line B at zero point.
- C (= conductor will give instructions for moving) Eg: He says 3 to the right; 2 to the right. A goes to #5 on the right. B now moves to catch (A) – like Kho Kho only one direction (i.e., as started) and goes not stopping – until he decides to stop. Then he stretches his hand and says "I got you".
- Umpire verifies and awards a point.
- C will give 3 commands. If B catches 2 points. The game goes up to 5 commands. Total of 10 points.

After the game, class moves to classroom. Then play the same instructions on the board.

(+3) + (+5) i,e., 3 to Right, 5 to Right
= + 8 (8 to Right)
(+5) + (+3) = +8 (same)

(+5) + (-3) = 5 to R, then 3 to left
= 2 to R
= +2

(+5) + (-5) = 5 to R, then 5 to left
= Starting point
= 0

(+5) + (-8) = 5 to R, then 8 to R
= 3 left of zero
= -3

Can also be
(-5) + (-3) = 5 to left, then 3 to left
= total 8 to left
= -8

(-5) + (+3) = 5 to left, then 3 to R
= 2 to left
= - 2

Chapter - 23 Basic Algebra

23. Basics of Algebra:

23.1 Instead of numbers, algebra uses symbols. The symbols used are usually the letters of the English alphabet.

Thus a, b, c, d ….. p, q …….. x, y z
Usually the small letters (lower case) are used.

Go back to the chapter on substitution where the same ideas are introduced. Come back and read more.

23.2 Symbols are very useful for generalizations.

E.g. What is your name?
My name is Lata.

This is quite OK. But when all the girls in a class repeat this, it generates laughter. When all the boys also say the same, it becomes a joke.

23.2.1 If the teacher insists that all the students should write the above answer, it becomes a ridiculous joke on the education system. What will you do?

What is your name? My name is *
* Write your name here.

What is your mother tongue? My mother tongue is * *
** Write here Kannada / Hindi / Tamil etc.

(Teachers, use such examples to show that blank spaces can be filled by symbols and those symbols could be explained).

23.2.2 What is your name? My name is x

What is your father's name? My father's name is y

(x, y – Fill up with suitable names. Here x, y are also symbols).

These are examples of filling up the blanks as in English or History exams or in forms for jobs, ration cards etc., Algebra is not very different.

23.3 Instead of word or data substitution as seen above, there can be substitution of x, y etc by numbers. It will then look like maths. Now consider:

A friend gives you US $ 100. How much money do you have?

Ans: I don't know (is OK).
A clever fellow asks, " How much is a dollar in rupees"? "
"I don't know is also OK.
The best answer is - Money in my hand = 100 X x rupees.
(Where x = value of one dollar in rupees on that date)
(Students can make examples like this one)

In the above example use of x, y or any letter helps. To use this effectively, students should be good at substitution.

23.4 The advantage of algebra (i.e., using x or y etc) is that the value of x or y need not be the same. It can change as per the situation in question.

23.4.1 In the above dollar example, let us assume that:
Exchange rate of US dollar (on one date) = Rs. 45

Then US dollars 100 = 100 X 45
= Rs 4500

If the exchange rate today is Rs. 50
Today's worth of US $ 100 = 100 X 50 = 5000 rupees

Both came from the equation, value of 100 US dollars = 100 x x
Where x = rupee value of 1$ (on that day)

Without doing anything you gained Rs. 500

23.5 Go back to substitution chapter & do the same again. Here are some more.

1. $x = 10$, $10x = ?$ Ans: 10 x 10 = 100
2. $x = 10$, $10 + x = ?$ Ans: 10 + 10 = 20
3. $x = 10$, $10 - x = ?$ Ans: 10 – 10 = 0
4. $x = 10$, $x - 9 = ?$ Ans: 10 – 9 = 1
5. $x = 10$, $\frac{10}{x} = ?$ Ans: $\frac{10}{10} = 1$
6. $x = 10$, $\frac{x}{10} = ?$ Ans: $\frac{10}{10} = 1$

Exercises:

a. 10a =? , If a = 8.5 b. 10 + a =? , If a = 90 c. 10 – b =? , If b = 8

d. c – 99 =? , If c = 100 e. $\frac{77}{d} = ?$, If d = 7 f. $\frac{x}{15} = ?$, If x =30

[(g) to (o) to be generated by students. One per student OK].

23.6 **More problems**

1. $x = 5$, $\frac{x+5}{5} = ?$ Ans: $\frac{5+5}{5} = \frac{10}{5} = 2$
2. $x = 5$, $\frac{x-5}{5} = ?$ Ans: $\frac{5-5}{5} = \frac{0}{5} = 0$
3. $x = 5$, $\frac{10}{x+5} = ?$ Ans: $\frac{10}{5+5} = \frac{10}{10} = 1$
4. $x = 5$, $\frac{10}{x-3} = ?$ Ans: $\frac{10}{5-3} = \frac{10}{2} = 5$
3. $x = 5$, $\frac{2}{x-3} = ?$ Ans: $\frac{2}{5-3} = \frac{2}{2} = 1$
4. $x = 5$, $\frac{2}{7-x} = ?$ Ans: $\frac{2}{7-5} = \frac{2}{2} = 1$

(7) to (10) to be generated by students.

Exercises: Given a = 11, find

a. $\frac{a+9}{5}$ b. $\frac{a-9}{2}$ c. $\frac{30}{a+4}$ d. $\frac{28}{a-4}$ e. $\frac{3}{a-8}$ f. $\frac{3}{14-a}$

g. $\frac{a-8}{14-a}$ h. $\frac{8-a}{a-14}$

23.7 Substitution of 2 variables

If $x=1, \ y=2$

a) $x+y=?$ Ans: 1 + 2 = 3

b) $y-x=?$ Ans: 2 – 1 = 1

Then ask c) $x-y=?$ (Explain negative numbers) Ans: 1 – 2 = -1

d) $\frac{x}{y} = ?$ Ans: $\frac{1}{2}=\frac{1}{2}$

e) $\frac{y}{x} = ?$ Ans: $\frac{2}{1}=2$

23.7.1 Exercise: Given that x = 20 y = 10 find:

a. $x + y$ b. $x - y$ c. $y - x$ d. $\frac{x}{y}$ e. $\frac{y}{x}$ f. $\frac{x+y}{x-y}$

23.7.2 *If* $x=1, \ y=2$

a) $2x+y=?$ Ans: (2 x 1) + 2 = 2 + 2 = 4

b) $2x-y=?$ Ans: (2 x 1) – 2 = 2 –2 = 0

c) $2y-4x=?$ Ans: (2 x 2) – (4 x 1) = 4 – 4 = 0

d) $\frac{2x}{y}=?$ Ans: $\frac{2 \text{ x } 1}{2}=\frac{2}{2}=1$

e) $\frac{y}{4x}=?$ Ans: $\frac{2}{4 \text{ x } 1}=\frac{2}{4}=\frac{1}{2}$

(Ask students to give x & y different valves and do the above problems in groups. Show the results).

23.7.2 Exercise: Given that x =20, y =10 find:

a. $2x + y$ b. $2x - y$ c. $2y - 4x$ d. $\frac{2x}{y}$ e. $\frac{y}{4x}$

f. $\frac{2x+y}{2x-y}$ g. $\frac{2x+y}{x+2y}$

23.8 Teachers, drill this.
You can use number bricks and letter bricks for this.
Let a group of 3 to 5 persons play. One is leader with calculators. Others work out by hand.
They check the results.

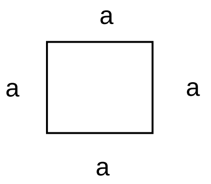
23.9 Substitution principle used in some formulas.

23.9.1 Single Variable:

$$\left.\begin{array}{l}\text{a. A = C + 273 where A} = {}^{o}\text{K} \\ \qquad\qquad\qquad\qquad \text{C} = {}^{o}\text{C}\end{array}\right\}\text{Temperatures}$$

What is the absolute temperature in ^{o}K, for 0^{o} C?
Ans: A = 0 + 273
= 273^{o}K
What is the room temperature in absolute scale, if the thermometer reads 27^{o} C.
Ans: A = 27 + 273 = 300^{o}K

b. The sides of a square are a units. Then Area, A = a^2 square units



What is the area of a plot of 12 meter by 12 meter square?
Area, A = $(12)^2$ = 12 x 12 = 144 square meters = 144 Sq. m

Exercise:

c. Akka is 5 years elder to Thangi. Write a small equation for this. If Akka is 15 years old now, what is Thangi's age? What will be Akka's age when Thangi's age becomes 60?

23.9.2 Two Variables:

A. Distance traveled = Speed x time
Or d = v x t [v for velocity ≈ speed]

A bus going at an average speed of 50 km/hr. takes 3 hrs to go from Mysore to Bangalore. What is the distance between these two cities?

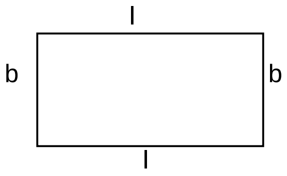
Ans: d= v x t v= 50 t = 3
= 50 x 3 = 150 km

Exercise:

A1. A cyclist is going at 15 km/hr. How long will he take?

A2. If a taxi took only 2 hours, how fast was the taxi?

B. If l is the length and b is the breadth (=width) of a rectangle, its Area
A = l x b. What is the area of a plot of 10 m x 14 m?
A = l x b l =14 m b = 10m
= 14 x 10
= 140 Sq. m



C. What is the least count (LC) of a Vernier? LC = (MSD) – (VSD) when
MSD = 1 mm, VSD = 0.9 mm.

Chapter - 24 Basic Operations in Algebra - A

24. Basic Operations in Algebra:
+, -, X, ÷ are the basic operations. We have learnt how to do these on real numbers. Now we will do the same with symbolic numbers (i.e. algebra).

24.1 Addition:
Same symbols will add just like numbers
1 + 1 + 1+ 1 = 4
a + a + a + a = 4a

$x + x + x + x = 4x$
u + u + u+ ………. 30 times = 30u

Since many times addition = multiplication
4a, 4x, 30u above also mean: 4 X a, 4 X x, 30 X u
i.e. multiplying symbol (X) need not be written.

Problem with numbers: The same cannot be done with numbers.

4 = 41 (this will become forty one i.e. 40 + 1)

Therefore we should write 4 = 4 x 1. This can be written as 4 = 4(1)

Exercises: Say True or False or Yes/No Desirable/Not Desirable

1. b + b + b = 3b
2. c + c + c = c3
3. d + d + d = ddd
4. If a = 1; b = 2, ab = 12
5. If a = 1; b = 2, ab = 2
6. ab = a + b
7. ab = a x b
8. ab = a .b
9. a.b = a x b
10. x^5 is not written; it is written as 5x
11. x^5 means x x x x x x x x x

24.2 Now try 3 + 3+ 3 = 9 also 3(3)
If we write a + a + a = 3 (a) = 3a
If we write 3a + 3a + 3a = 9(a) = 9a
3 + 2 + 1 = 6
3a + 2a + 1a = 6a
(1a is usually written as a)
Therefore 3a + 2a + a = 6a

Instead of 'a' anything can be written. This is the reason this is called algebra.

Thus 3b + 2b + b = 6b
$3x + 2x + x = 6x$
3p + 2p + P = 6p

Exercise:
Students can now go back to exercises of 24.1. They can check whether they have done right or not.

24.3 Mixing of Numbers and Letters:
Numbers all belong to one group. So they can all add up.

Thus 1 + 2 + 3 = ◯ some value = A

11 + 12 + 13 = ◯ some value = B

(any big number) + (any other number) = $\boxed{\text{Some value}}$ = C

Similarly ALL items belonging to ONE GROUP (i.e. algebra) can add up.

i.e. $x + 2x + 3x =$ $\boxed{\text{Some value}}$

$11x + 12x + 13x =$ $\boxed{\text{Some value}}$ = Bx

(Any big no.) x + (Any other no.) x = $\boxed{\text{Some value}}$ = Cx

24.4 Note for Teachers [Self – learning students can skip this section]

In writing numbers, we are using what is called PLACE VALUE and use digital system (base 10). Thus (4, 42, 421) here value of 4 is different depending upon where it occurs. (Teachers! If you feel like you can digress upon this system. Go if you like to the greatness of zero – how the zero after 1 to 9 is used back again to give 10. Use an abacus if you like. This is fully optional because such a discussion can only be suggested by a manual maker).

24.5 In number system 1111 means 1000 + 100 + 10 + 1
Similarly 5555 = 5000 + 500 + 50 + 5
But aaaa $\neq$ 1000a + 100a + 10a +a
LHS above means a X a X a X a = a^4
RHS above means (1000 + 100 + 10 + 1) a = 1111a
Similarly 5a5a5a5a does not exist. Do not write this way.
Instead 5555a is OK. But a5555 is not written.
This means 5000a + 500a + 50a + 5a

Exercise: Answer right or wrong (√ or X)
a. 1234 = 1000 + 200 + 30 + 4
b. 1234a = 1000 + 200 + 30 + 4 + a
c. 1234a = 1000 + 200 + 30 + 4a
d. 1234a = 1230 + 4a
e. 1234a = 1000a + 200a + 30a + 4a
f. 9876b = b x 9876
g. 9876c = 9876 x c
h. 4567d = d(4567)
i. 999e = 999(e)

24.6 Brackets:

24.6.1 Teachers, now is an opportunity to explain, expansion of brackets.

3(10 + 5 + 2 + 1) = 3 X 10 + 3 X 5 + 3 X 2 + 3 X 1
= 30 + 15 + 6 + 3
= 54
Or 3 X 18 = 54

Similarly a (10 + 5 + 2 + 1) = a X 10 + a X 5 + a X 2 + a X 1
= 10a + 5a + 2a + a
= 18a
LHS = a (18) = 18a

Exercises:
a. Expand: 5 (a + b + c)
b. Expand: a (5 + 3 + 2)
c. Expand: 5 (a + 3)
d. Expand: a (5 + b)
e. Expand: a (a + b + c)
f. Expand: 5 (1 + 2 + 3 + 4+5)
g. Expand: a (1 + 2 + 3 + 4 + 5)
h. Expand: 2 ($x + y + z$)
i. Expand: a ($x + y + z$)

24.6.2 Brackets for division and fractions:
Similarly dividing also

$\frac{10}{3} + \frac{5}{3} + \frac{2}{3} + \frac{1}{3}$ is the same as $\frac{1}{3}$ (10 + 5 + 2 +1)

Or = $\frac{10 + 5 + 2 + 1}{3} = \frac{18}{3}$ = 6

Much more clear way will be to write the above as $\frac{(10 + 5 + 2 + 1)}{3}$

i.e. with brackets.
Brackets are used when extra items are there in the numerator.

In algebra $\frac{10}{a} + \frac{5}{a} + \frac{2}{a} + \frac{1}{a}$

$$= \frac{1}{a}\ (10 + 5 + 2 + 1) = \frac{18}{a}$$

$$= \frac{10\ +\ 5\ +\ 2\ +\ 1}{a}$$

$$= \frac{18}{a}$$

Exercises:

a. $\frac{a}{5} + \frac{b}{5} = ?$ For a = 4, b = 1

b. Given that a + b = 10 Find $\frac{a}{5} + \frac{b}{5}$

c. Given that a= 4 find $\frac{5}{a} + \frac{3}{a}$

d. First Simplify $[\frac{4}{a} + \frac{3}{a} + \frac{3}{b} + \frac{5}{b}]$

Then find the value for a =7 and b = 8

24.7 Caution $a \neq b \neq c$
In number system

2 + 3 + 4 = 9 can add up, because they all belong to **ONE FAMILY**.

But 2a + 3b + 4c + a + b + c
Can be written, by grouping together same family members.

= 2a + a + 3b + b + 4c + c
= 3a + 4b + 5c

In the above example, if a, b, c, are substituted by numbers, then they can join.
Eg: Let a = 1, b = 2, c = 3
LHS = 3a + 4b + 5c

= 3 x 1 + 4 x 2 + 5 x 3
= 3 + 8 + 15
= 26

a. After substitution a, b, c etc do not exist any more. They have been converted to numbers.

24.8 Caution $a \neq a^2$
In number system, a number and its square can add up.

E.g.: $3 + 3^2 = 3 + 9$
$= 12$

Or $5 + 2\ (5)^2 = 5 + 2 \times (25)$
$= 5 + 50$
$= 55$

In algebra a, a^2 will remain independent

Thus $a + a^2$ = same only
$x + 2x^2$ = same only

In the above example, if a, x etc are substituted by numbers, then they can be added.
Thus If a = 3, $a + a^2 = 3 + 3^2 = 12$
Thus If $x = 5$, $x + 2x^2 = 5 + 2(5)^2 = 55$

Repeat (a) from sec 24.7

24.8.1 Exercises: Simply

a. a + b + 3a + 4b
b. a – b + 3a – 4b
c. –a + b –3a + 4b
d. a(a+1) + b(b+1) + c(c+1)
e. a(1 + 2 + 3) + b(2 + 3 + 4) + c(3 + 4 + 5)
(f) to (j) If (A) to (E) above, find the values if a = 3, b = 5, c= 7
k. $3^2 + 5^2 + 7^2 = ?$
l. 3 + 5 + 7 =?
m. $(3 + 5 + 7)^2$
n. Which is bigger m or k?

24.9 Caution: No mixing
The idea given in 35.7 is usually explained by resourceful teachers as “apples and oranges” do not mix. i.e. If a is for apple and b is for orange or banana.

“a” are kept in one basket
“b” are kept in another basket

Thus if there are 10 apples in one basket and 6 bananas in another basket.
10a in one and 6b in another.

Now if you empty both of them into a big basket.

(10a + 6b) is there in a big basket.

This is how algebra differs from numbers.

24.10 Students can skip this section (=omit, need not read)

(This Para is for teachers only).

Some times apples and bananas may be grouped into fruits.
In that case a, b both will belong to a big family of fruits.
Then a, b will be called subsets of a set of fruits (call ‘f’ if you like).
This is called **SET THEORY** and practical engineering does not need it.
If discussion comes to a ‘basketful of fruits’, then you can say,

a = f, b (also) = f

$\therefore$ 10a + 6b = 10f + 6f = 16f (This is the basketful of fruits).
In such a case a, b will be called subsets.

24.11 Subtraction or Handling –ve quantities

24.11.1 Subtraction can also be done in the same way as addition.

Remember	5 - 4 = 1	means 1 of (+) remains
	5 – 5 = 0	means nothing remains
	5 – 6 = -1	means 1 of (–) remains

A. This means subtraction is only a method of combining, including the minus sign.
B. In traditional subtraction we handle only 2 items at a time. Now let us see how we can handle many items.

24.11.2 Many –ve numbers

The idea of 24.11.1 helps in handling many numbers at a time.
Thus 100 – 40 – 30 – 10 – 5 – 4 =?
By the traditional concept of subtraction, you have to do one pair at a time.

Thus 100 – 40 = 60
60 – 30 = 30
30 – 10 = 20
20 – 5 = 15
15 – 4 = 11

Forget subtraction; call the problem as a combination of both (+) & (-) numbers.
In that case: (+) is 100

(-) is 40 + 30 + 10 + 5 + 4 = 89
Thus LHS = +100 – 89 = 11
This can also be written as 100 - (40 + 30 + 10 + 5 + 4)
i.e. 100 – 89 = 11

24.12.1 Same as 24.11.1

$5x - 4x = 1x = x$
$5x - 5x = 0x = 0$
$5x - 6x = -1x = -x$

Repeat (A) from 24.11.1
Repeat (B) from 24.11.1

Exercises:
Example: 1234a – 1033a =?
Ans: LHS = a (1234 – 1033)
= a x 201 = 201a

Do:
a. $5243x - 5243x$ =?
b. $243y - 242y$ =?
c. $5243x - 5244x$ =?

24.12.2 This is the same as 24.11.2, but with y added. Now we call it Algebraic quantity.

$100y - 40y - 30y - 10y - 5y - 4y$ =?
i.e. $100y - y(40 + 30 + 10 + 5 + 4)$
i.e. $100y - 89y$
$= 11y$

Exercises:
Example: 1234a – 1000a – 30a – 3a =?
Ans: LHS = 1234a – a (1000 + 30 + 3)
= 1234a – a (1033)
= a (1234 – 1033)
= a (201)
= 201a

This looks long. But doing this way makes a student good at doing algebra and no mistakes. The confidence gained helps. Do the following by steps:

A. $5243x – 5000x - 200x - 40x - 2x - x$
B. $243y - 200y - 39y - y - 2y$
C. 5243a – 5200a – 40a – 4a
D. 8a – 7a + 3a – 2a + 4a – 5a
E. 128a – 127a + 123a – 122a + 124a – 125a

24.13 Activity

24.13.1 Addition with numbers only. Earlier we have prepared bricks (some people call it 'Tiles' – In scrabble game also they call the letters as 'Tiles' – so, we start calling them Tiles). Those tiles were sets of 0 to 9 – i.e., one one-digit number on each tile. Now put –ve numbers also.

Teachers, go back to number bricks you had prepared earlier. Make equal number of negative numbers. Let students play in groups.

a. One from positive set; one from negative set. Put them together; pick the result.
b. 2 from positive set; two from negative set.
c. Some from positive set; some from negative set.

24.13.2 Addition with x, y (Algebra)

Prepare similar sets of x numbers and y numbers.

a. Play the game of (a) (b) (c) above with x set only.
b. Play the game of (a) (b) (c) above with y set only.
c. Play the game of (a) (b) (c) above with x, y, (+), (-) mixed.
d. Now mix number sets also.

For a self – study student, this looks very complicated. Such a student can get the help of teachers / elders / other senior students.

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Chapter - 25 Basic Operations in Algebra - B

25. Basic operations in algebra (Contd.)

We saw addition and subtraction. We also reduced both to joining together of positive and negative numbers or symbols (algebraic quantities). Here we see multiplication & division.

25.1 Observe carefully (better still write down):
10 X 2 = 20
10 X a = 10a
10 X 2a = 20a
10a X 2 = 20a
10 X 2 X a = 20a
20 X a = 20a
20 X 1 = 20 x 1 = 20
20 X 21 = (20 X 21) = 420
20 X 21a = (20 X 21) a = 420a

What is you observe?
These were not given for the sake of finding answers (in students language "doing sums" "Lekkaa Maaduvudhu"). These are given for you to observe the rule.

Rule: Numbers join up. Separately worked out.
Letters (=algebraic quantities) follow.

Convention: 10 x a, 20 x a, 420 x a are written as 10a, 20a, 420a.

Caution: a10, a20, a420 do not exist. They are not written like that. If you want algebraic quantities to be written first, you can write as:
a x10, a x 20, a x 420

25.1.1 Exercises: Say Right / Wrong

a. 5 x b = 5b
b. 5 x b = b5
c. 5 x 20 x b = b100
d. 5 x 20b =5b20

e. 5 x 20b = 100b
f. 25 x 2b = 50b
g. 25 x 2b = 50 x b
h. 25 x 2b = b50
i. 25 x 2b = b x 50

25.1.2 Exercises: Say Right / Wrong. They are the same as (a) to (i) above. Let b = 3, then

a. 5 x b = 5b = 53
b. 5 x b = b5 =35
c. 5 x 20 x b = b100 = 3100
d. 5 x 20 x b = 5b20 = 5320
e. 5 x 20 x b = 100b = 300
f. 25 x 2b = 50b = 150
g. 25 x 2b = 50 x b = 150
h. 25 x 2b = b50 = 350
i. 25 x 2b = b x 50 = 150

25.1.3 Activity

Let each student pick bricks from the number sets, add X and play this game (At least one problem per student). (Brick = Tile in number set).

25.2 Division

Just like you can divide a number by a number, in the case of algebraic quantities, you can (say x):

a. Divide x by any number
b. Divide any number by x
c. Divide x by x itself
d. Divide x by any other y

Let us do step by step (i), (ii) first.

25.2.1 $\frac{10}{2} = 5$ (Same as 10 ÷ 2 = 5)

$$\frac{10\ a}{2} = 5\ a$$

25.2.2 Activity

A. Play a game of division using only integers (go back to the chapter on fractions and learn how to do).

Make 'Tiles':	10	11	……	20	2 digit numbers.
Make 'Tiles':	1	2		9	1 digit numbers.

One set of students will pick only 2 digit numbers. They are numerators. (= Top numbers).
Second set of students will pick only 1 digit number. They are denominators. (=Bottom numbers).

Answers will be written down and checked by the teacher (or other students).

B. Now, play the same game with x. To play the game let one group have 2 digit numbers – the other group 1 to 9. Add x to the 2 digit numbers and let it be the numerator. Divide by the one digit number. Display the results. Let there be as many results as there are students. Teachers may go and check the results, one by one. (You may need x bricks, as many as the number of students.

C. Now let everyone be the numerator. Denominator will be a sweeping (mobile). 10 will come and stand as denominator. While leaving, it will leave a print (=point) on the numerator. Thus producing decimal numbers.

Later add X to the numerator and play the same game. Generate decimals.

How to play:
Starting: 5, 9, 14 standing
Decimal denominator comes around.
During game $\frac{5}{10}$, $\frac{9}{10}$, $\frac{14}{10}$
10 leaves.
End of game .5, .9, .14
When X is added to the numerator.
End of game .5X, .9X, .14 X

D. Vary (c) above to include 100, 1000 as denominators. First play with numbers only. Later add X to the numerator.
You will get .05X, .005X etc

25.3 Multiplication: Many Numbers

a. Drill multiplication of 2 numbers
Eg: 2 x 3 = 6 2a x 3 = 6a
2 x 3a = 6a
2 x 3 x a = 6a

b. Go to 3 numbers
2 x 3 x 4 = 12
2a x 3 x 4 = 12a
2 x 3a x 4 =12a
2 x 3 x 4a = 12a
2 x 3 x 4 x a = 12a

c. Exercises:
1. X x 15 x 2 =? 2. 15 x X x 2 =? 3. 2 x 15 x X =?
4. 1 X 15 x 2X =?5. 2 x 15X x 3 =? 6. 4X x 15 =?
7. 15X x 4 =? 8. 15 (X +4) =? 9. X + (15 x 4) =?

25.4 Division: Many Numbers

25.4.1 a. Drill division of 2 numbers

$$\frac{6}{2} = 3 \qquad \frac{6a}{2} = 3a$$

b. Make 2 numbers in numerator & one number in denominator.

$$\frac{6 \text{ x } 5}{2} = 3 \text{ x } 5 = 15$$

$$\frac{6 \text{ x } 5 \text{ x } a}{2} = 15a$$

$$\frac{6a \text{ x } 5}{2} = 15a$$

Exercises:

1. $\frac{x \text{ x } 15 \text{ x } 2}{15} = ?$ 2. $\frac{2 \text{ x } 15\ x}{15} = ?$ 3. $\frac{2 \text{ x } 15\ x}{2} = ?$

4. $\frac{15\ (x + 4)}{15} = ?$ 5. $\frac{15\ (x + 4)}{5} = ?$ 6. $\frac{x\,(15 + 4)}{19} = ?$

7. $\frac{x + (15 + 4)}{x + 19} = ?$

25.4.2 Many items in both numerator and denominator.

a. $\frac{1 \text{ x } 2 \text{ x } 3 \text{ x } 4 \text{ x } 5}{2 \text{ x } 3} = 1 \text{ x } 4 \text{ x } 5 = 20$

b. $\frac{11 \text{ x } 13 \text{ x } 15 \text{ x } 17}{11 \text{ x } 17 \text{ x } 5} = \frac{13 \text{ x } 15}{5} = \frac{13 \text{ x } 3 \text{ x } 5}{5} = 13 \text{ x } 3 = 39$

c. $\frac{22 \text{ x } 26 \text{ x } 30 \text{ x } 37}{11 \text{ x } 13 \text{ x } 15} = 2 \text{ x } 2 \text{ x } 2 \text{ x } 37 = 8 \text{ x } 37 = 296$

d. $\frac{22 \text{ x } 26 \text{ x } 30 \text{ x } 37}{44 \text{ x } 13 \text{ x } 60 \text{ x } 74} = \frac{1}{2} \text{ x } 2 \text{ x } \frac{1}{2} \text{ x } \frac{1}{2} = \frac{1}{2 \text{ x } 2} = \frac{1}{4}$

e. Now add X (or any algebraic quantity anywhere) (only one at a time)

1. $\frac{1 \text{ x } 2 \text{ x } 3 \text{ x } 4x}{2 \text{x } 3} = 4x$ 2. $\frac{11 \text{ x } 13 \text{ x } 15 \text{ x } 17}{13 \text{ x } 11 \text{ x } 17 \text{ x } x} = \frac{15}{x}$

3. In (2) above add y to numerator. Ans: Will be 296y.

If (c) above, add y to denominator. Ans: will be $\frac{296}{y}$

Exercises:

1. Students, you can make your own problems. [Help: use numbers only in the numerator. Use numbers and letters in the denominator. Do vice versa (=ulta)]

2. Simplify:

a. $\frac{9+8+7+6}{15x}$ b. $\frac{(9+8+7+6)x}{15}$ c. $\frac{(9+8+7+6)x}{15x}$

d. $\frac{9x+8+7+6x}{15x+15}$ e. $\frac{9+8+7+6}{15x}$ f. $\frac{(5+8)+10}{100x}$

g. $\frac{(5+8)x+10x}{50x}$

25.5 Division as fractions.
We have seen addition of fractions. Let us revise by doing some exercises.

25.5.1 Exercises: Say True / False

a. $\frac{1}{4}+\frac{2}{4}=\frac{3}{8}$ b. $\frac{1}{4}+\frac{2}{4}=\frac{2}{4}=\frac{1}{2}$ c. $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$

d. $\frac{2}{11}+\frac{3}{11}+\frac{5}{11}+\frac{1}{11}=\frac{11}{44}=\frac{1}{4}$ e. $\frac{2}{11}+\frac{3}{11}+\frac{5}{11}+\frac{1}{11}=\frac{11}{11}=1$

25.5.2 Recall (= remember, think about it again) the rule. "When the denominator is the same, numerators can be added".
Now apply this to the problems above and see how you fared (= did well or not).

25.5.3 Apply the above rule when the fraction contains letters instead of numbers.

a. $\frac{1}{n}+\frac{2}{n}=?$ Ans: LHS = $\frac{1+2}{n}=\frac{3}{n}$

b. $\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\frac{5}{n}=?$ Ans: LHS = $\frac{1+2+3+5}{n}=\frac{11}{n}$

Exercises:

a. $\frac{2}{x}+\frac{3}{x}=?$ b. If x =5, same =?

c. $\frac{1}{n}+\frac{2}{n}+\frac{3}{n}=?$ d. If n = 6, same =?

e. $\frac{1}{d}+\frac{2}{d}+\frac{3}{d}+\frac{4}{d}+\frac{5}{d}+\frac{6}{d}+\frac{7}{d}+\frac{8}{d}+\frac{9}{d}=?$ f. If d = 15, same =?

g. If d = 5, same =? h. If d = 3, same =? i. If d = 45, same =?

j. $\frac{a}{5}+\frac{b}{5}+\frac{c}{5}+\frac{d}{5}+\frac{e}{5}=?$ k. If a = b = c= d = e = 2, same =?

l. Instead of 5, substitute 'n', same=?

25.6 Teachers! Tell the students that mixed denominator is not allowed for simplifying.

E.g.: a. $\frac{10}{5+a}$ remains as such.

b. $\frac{10}{1+4}$ Becomes $\frac{10}{5}=2$ OK

c. $\frac{10a}{1+4}$ = $\frac{10a}{5}$ = 2a OK

d. $\frac{10+5}{a}$ Becomes $\frac{15}{a}$ OK

e. $\frac{10+5a}{5+a}$ ≠ $\frac{10}{5}+\frac{10}{a}$ This is wrong.

f. $\frac{10}{5+a}$ ≠ $\frac{10}{5}+\frac{5a}{a}$ Wrong.

(5), (6) above cannot be simplified. If we know a = same number, then all of it will join to become numbers.

25.7 Note to teachers: in the following examples, we assume some knowledge, i.e., squares, cubes etc., i.e., Simple Indices. Later, there will be a chapter on square and square roots. A few points here is worth learning before we handle mixed quantities (i.e., mixture of numbers and algebraic quantities).

25.7.1 Please note: Multiplication tables can be generated by addition.
i.e., 4 x 5 = 20 means
4 + 4 + 4 + 4+ 4 …… (5 times) = 20
or 5 + 5 + 5 + 5 + 5 (4 times) = 20
This is the reason we stated earlier, repeated addition is multiplication.

25.7.2 Now consider (a) 3 + 3+ 3 and (b) 3 x 3 x 3
(a) = 9 = 3 times 3 = 9
(b) = (3 x 3) x 3 = 9 x 3 = 27
(b) is different from (a)
(b) is repeated multiplication.

$4 + 4 + 4 \neq 4 \times 4 \times 4$

LHS = 12 = 3 x 4
RHS = 64

Symbol of repeated multiplication is called Index or Power*. It is written as a small size number on the right hand side top of the number.
[* strict maths calls this "Exponent"]

Thus, $2 \times 2 \times 2 = (2)^3$ or 2^3
$3 \times 3 \times 3 \times 3 = (3)^4$ or 3^4
$4 \times 4 = (4)^2$ or 4^2

25.7.3 How to read (Indices)
When x, y, a, b, alone occurs many times then use "Power".
y x y =y^2 (read as 'y square')
y x y x y = y^3 (read as 'y cube')
y x y x y x y = y^4 (read as 'y to the power of 4')
[4 onwards 'to the power of' is used while reading out]

25.7.4 How to write (Indices)
a. b x b x b …..8 times is written as $(b)^8$ or b^8 (8 is small).
b x b x b …..n times in $(b)^n$ or b^n.

b. If (x+1) is multiplied by itself, (x + 1) x (x + 1). This can also be written as (x + 1).(x + 1) [x or .].
Both mostly written as (x + 1) (x +1).
Since the same thing occurs 2 times (x + 1) (x + 1) is the same as (x + 1).
Similarly $(a + b + c)^4$ or $(a + 5)^n$ etc.,

25.8 Multiplication of mixed items.
A. Worked examples:
a. 2 x 3 =6
b. 2a x 3 = 6a
c. a x a = a2
d. 2a x 3a = 6a2
f. 2a x 3a x 4a x 5a = (2 x 3 x 4 x 5) (a x a x a x a) = $120.a^4$

B. Exercises: x y
1. 2x x 3 =? 2. 2x x 9x =? 3. 1.5x x 4 =?
4. .9x x 10 =? 5. 2x x 2x x 2x =? 6. y x y x y …(10 times) =?
7. d + d+ d + d + …. (10 times) =? 8. b x b x b …. (n times) =?
9. x x x x (m times) =?

25.9 Division: Mixed quantities.

A. Worked Examples:

a. $\frac{2 \text{ x } 3}{3} = 2$ b. $\frac{2 \text{ x } 3 \text{ x } 4 \text{ x } 5}{2 \text{ x } 3} = 4 \text{ x } 5 = 20$

c. $\frac{2 \text{ x } 3 \text{ x } 4 \text{ x } 5 \text{ x } a}{2 \text{ x } 3} = 4 \text{ x } 5 \text{ x } a = 20a$ d. $\frac{2 \text{ x } 3 \text{ x } 4 \text{ x } 5 \text{ x } a}{2 \text{ x } 3 \text{ x } a} = 20$

e. $\frac{2 \text{ x } 3 \text{ x } a3}{3a} = 2 \text{ x } 3 \text{ x } a \text{ x } a \text{ x } a = 2 \text{ x } a \text{ x } a = 2a^{2}$

B. Exercises:

a. $\frac{a \text{ x } 2 \text{ x } 3a \text{ x } 4 \text{ x } 5a}{2 \text{ x } 3} = ?$ b. $\frac{11 \text{ x } 13 \text{ x } 15a \text{ x } 17a}{11 \text{ x } 17 \text{ x } 5}$ c. $\frac{22a \text{ x } 26a \text{ x } 30a \text{ x } 37a}{11a \text{ x } 13a \text{ x } 15a} = ?$

25.10 Concept of reciprocal.

Definition: Reciprocal of a number = $\frac{1}{\text{the number}}$

This is simply to be accepted as a word and its meaning.

25.10.1 This, reciprocal of 10 = $\frac{1}{10} = .1$

Reciprocal of 100 = $\frac{1}{100} = .01$

Reciprocal of 3 = $\frac{1}{3} = 0.33$

Reciprocal of $\frac{1}{3}$ = $\frac{1}{1/3} = 3$

Reciprocal of .2 = $\frac{1}{.2} = 5$

25.10.1 Exercises: Write down the reciprocal of

a. 5 b. 50 c. 500 d. 0.1 e. 0.01 f. 0.001

g. 8 h. $\frac{1}{8}$ i. $\frac{8}{3}$ j. $\frac{3}{8}$ k. $\frac{8}{100}$ l. $\frac{100}{125}$

Why the idea of reciprocal?
It helps to remove the fear of 'DIVISION' more importantly, if you know there is a reciprocal, you need not have any division any more. Only multiplication will do. (Will do = is enough, is sufficient).

Example:

a) $\frac{8}{3} = 8 \text{ x } \frac{1}{3} = 8 \text{ x } (\frac{1}{3})$

b) $\frac{3}{4x7} = 3 \text{ x } (\frac{1}{4}) \text{ x } (\frac{1}{7})$ i.e., 3 multiplications

Exercises:

a. Given $\frac{1}{2} = .5$, find $\frac{x \text{ X } y}{2}$ where $x = 10$ $y = 6$

b. Find $\frac{axb}{125}$ where a = 12; b = 5

Given: $\frac{1}{125}$ = $.008$

c. Given $\frac{1}{333}$ = $.003$, find $\frac{66 \text{ x } 99}{333}$

(Clue: Do approximately; do not use calculator)

We know a Addition & Subtraction are Not two different actions. It is only one action i.e., subtraction (with a sign -) is a special case of addition of negative numbers. This we explained with a ball & hole concept.
Now we know that Multiplication and Division are not two different actions. It is only one action i.e., of multiplying with a reciprocal is a special case, also called division. In this system where do +ve & -ve numbers come in?

25.12 Let us clarify to the students one of the problems in algebra i.e., + & -

<u>Addition Rules:</u>

Rule 1. + (a) + (b) Ans: + ve (add both)

Rule 2. + (a) – (b) Ans: + ve if a> b
Do (a - b)
It is – ve if b > a
Do (b – a)

(Rule 2 is often stated as: subtract smaller from bigger and the final sign will be that of the bigger)

Rule 3. – (a) – (b) Ans: – (a + b)
- ve (add both)

[Rule 3 often stated as: Add the numbers & final sign is – ve]

Multiplication Rules:

Rule 1. $(+a)$ x $(+b)$ = +ab. +ve

Rule 2. $(+a)$ x $(-b)$ = -ve (a x b)
$(-a)$ x $(+b)$ = -ve (a x b)

Rule 3. $(-a)$ x $(-b)$ = +ve (a x b)

In students language

(+) x (+) = + (-) x (-) = +

(+) x (-) = -

This rule works. Follow it.

[For advanced – level teachers, there is an appendix on it]

Chapter - 26 Equations

26. Equations:

Equations are very useful in algebra. They are useful in all branches of science. Engineering is full of equations. Economics (selling, buying, estimating) transactions use equations. So equations are everywhere.

26.1 How to convert simple statements onto equations?

"I bought a bicycle" for 2000 rupees.

i.e. my bicycle cost me Rs. 2000
i.e. cost of my bicycle is Rs. 2000
i.e. cost of my bicycle = Rs. 2000

Let c be the cost of my bicycle c = Rs. 2000

Suppose I bought a new Atlas cycle. Then I can even say c = Rs. 2000 where c = cost of an Atlas cycle.

26.2 Simple statements:

Grandfather "Bachcha, I am three times older than you".

i.e. Grandfather's age is 3 times that of Bachcha's age.
i.e. Grandfather's age is 3 times that of Bachcha's age.
i.e. Grandfather's age = 3 x Bachcha's age
i.e. $y = 3x$

Where

$y = \dots\dots\dots$
$x = \dots\dots\dots$ (Let students fill this up)

Teachers should draw the attention of the students. The brevity and clarity of the equation given in 26.1 & 26.2. Try some of the statements.

a. My house is only 10 minutes walk from your house.
b. New road can take from Mysore to Bangalore in 2 hours.
c. I am taller than you by 6 inches.

[Note for teachers: very few maths books include problems of this type. CBSE books are helpful. Formulas found in economics, commerce, business organization high level books (eg: B.Com, BBM, MBA etc) are similar to what is written here). But we do not need all those for teaching this chapter. More knowledge helps]

26.3 Try some puzzles

a. Father is twice as old as the son. If son's age is 25 what is the father's?
b. In (a) above; the daughter is 3 years younger than the son. The daughter is 3 years older than me. My age is 15.What is the father's age
c. Father is four times as old as his son. In twenty years, son will be half his father's age. What are their ages?
d. Pens and notebooks were bought. 10 pens and 2 notebooks cost Rs. 100. What are their prices?
e. In (d) above for the same money I could have bought 5 pens and 6 notebooks. What are their prices?

[Help: Earlier sections had only equations No maths. The above can be converted into equation, and solved also.
In (d) above many answers are possible.

In (e) the same is restricted to one value.

26.4 Activity

Every shopkeeper, vegetable vendor, storekeeper, bus conductor, bank manager uses mental algebra and equations. Let the students try some examples.

Students can play games using this idea. One set can use actual money, another only equations**.**

26.5 Exercise:

If you are drawing Rs.1000 from a bank, how many different ways could the cashier pay you?

Assume a = One thousand rupee note
b = Five hundred rupee note
c = One hundred rupee note
d = Fifty rupee note
e = Ten rupee note
f = Five rupee note
g = One rupee note / coin

Many answers could be generated. Let the whole class try it come out on equation form.

Eg: 1000 = 1000g
1000 = a
1000 = b + 5c

26.6. Students, recall (= remember, think again) that we started with the process of substitution. In a blank space (as in an application form) you write your name, others write other names. Here you can write x. (or any letter).Then it looks like Maths. It is called algebra. It is simple. When something is not known, you call it x. This is how it started. Then the habit spread. Now any letter can be used to represent ‘ the unknown ’.

26.6.1 Equation form of normal statements.

a. There are some fruits in the basket.
You can take half of them.

Let fruits in the baskets = x

Your share = $\frac{x}{2}$

b. Question: If there were 8 fruits, how many will you take?

Ans: x = 8, your share = $\frac{x}{2}$

$= \frac{8}{2}$

= 4

c. Question: In (a) above, your share was 6 fruits. How many were in the basket?

Ans: Your share $= \frac{x}{2}$

$\frac{x}{2} = 6$

$x = 12$

Exercises A: Do As Per (a) above.

1. I have some money in my account.
2. I owe you some money

3. He commutes (= travels for work) up and down & covers some Km per day.
4. Do you eat only this many idlies? I eat 3 times that many.
5. You take this much time to do this work. Nirmala can finish in half the time.
6. With the money you pay for one person in Hotel Paradise, you can give lunch to 4 persons in Darshini.
7. In “SALE” time, we give 10% discount on all items of purchase.

Exercises B: Do as per 26.6.1 b and 26.6.1 c (See exercises A (1) to (7) above)

1. Minimum balance required is Rs. 5000. I have Rs. 500 in the bank. How much money (maximum) I could draw (= take out)?
2. I owe you Rs. 100.I’ll return it after 1 year with 10 % interest. How much will I pay you next year?
3. If he travels 200 km/day. What is the distance between his houses office?
4. I think 8 idlies will be needed for both of us. How many idlies each one eats?
5. Nirmala was given 50%(= half) the work. She finished her work in 1 day. How long will you take for your work? What was the total time taken for the work?
6. Darshini gives lunch for Rs.15 What is the bill for one person in Hotel Paradise?
7. MRP written on some items are as given below.

   Salt 1kg packet Rs. 6
   Dal 1kg packet Rs. 60
   Sugar 5kg packet Rs. 120
   Electronics items Rs5000.

   You bought 1 of all these items. How much (total) money did you save?

Exercises C: Combination of exercises A (5) & B (5)

1. K is a slow worker. N can do the work in half the time (i.e., compared to K). 300 pages of some work is to be done. If the work is shared between K & N and if you would like the work to be finished simultaneously (= at the same time), how many pages will you give to N, and how many to K?
2. If the work can be finished in 10 days, how will the work be shared on daily basis?

Chapter - 27 Working with Equations - A

27. Working with Equations:
Simple symbols (algebra) and equations go together. They make many activities simpler, easier and more accurate. Many guessing games could be avoided by using proper equations.

27.1 Some Symbols How to write:

$x = y$	5 = 5	10 = 10	12345 = 12345	$x > y$	5 > 4
10 > 9	12345 > 12344	$x < y$	5 < 6	10 < 11	12345 < 12350
$x \neq y$	$5 \neq 6$	$5 \neq 4$	$5 \neq 22000$		

These are very basic symbols. The most important is the first one (=).
There are some others also $\geq$, $\leq$, $\ngtr$, $\nless$ etc.

Some others are A > x < B
They are not very important.

27.1.1 How to read:

= ‘is equal to’ OR
Equal to OR
Equal to OR
Equals.

$x > y$	x	(is) greater than y (is) optional.
$x < y$	x	less than y
$x \neq y$	x	not equal to y
$x \geq y$	x	greater than or equal to y
$x \leq y$	x	less than or equal to y
$x \not> y$	x	not greater than y
$x \not< y$	x	not less than y
$x \approx y$	x	approximately equal to y

27.2 Simplification: How to write:
Writing (=) sign.

2(3 + 2) + 5 (4 - 3) + 25 =?
2 x (5) + 5(4 – 3) + 25 =?
0 + 5 (1) + 25 =?
10 + 5 + 25 =?
15 + 25 =?
$\therefore$? = 40

This is a poor way of writing; even with so many steps. Instead write:

2(3 + 2) + 5 (4 - 3) + 25
= 2 x (5) + 5(4 – 3) + 25
= 10 + 5 (1) + 25
= 10 + 5 + 25
= 15 + 25
= 40

Therefore put (=) sign one below another and each step follows the previous step.

27.2.1 EXERCISE

Follow the step-by-step principle and the Practice of writing = sign one below the earlier. Simplify

a. Product = 1 x 2 x 3 x 4 x 5 x 6
[Clue: 5 steps are needed]
b. Product = 2 x 5 x x x 3 x y x 4
c. Result = 2 (5+ x) + 3(y +4)
d. Result = 2 (5+ x) + 3(x +4)
e. Result = 3 (x +4) - 2(5+ x)
f. Result = 2 (5+ x) - 3(x +4)

27.3 Cost of one dozen bananas = Rs. 24

Cost of one banana = $\frac{24}{12}$

Cost of one banana = $\frac{2X\cancel{12}}{2X\cancel{6}}$

Cost of one banana = 2

What is wrong here? Waste of energy etc.

Therefore write cost of one banana = $\frac{24}{12}$

$$= \frac{2 \text{ X } \cancel{12}}{2 \text{ X } \cancel{6}}$$

= 2 (= sign one below the previous)

27.3.1 Students! From the example given above what did you learn?
Ans: you learnt how to do the problem (lekkaa maaduvudhu) and also how to write it. Until you learn how to neatly write & logically arrange your thoughts, do not go for shortcuts. [Ask for forgiveness from Parisara Devathe for wasting Paper-Use slates if take long time to learn]

27.3.2 Exercises:

Do step by systematically:

a. Cost of 1 kg rice is Rs.38 (as on May, 2009 Mysore)
   i) What is the cost of 5 kg?
   ii) What is the cost of 300 grams?

b. 5 liters of branded oil costs Rs 420, Loose oil ½ liter bought cost was Rs. 40. Which is cheaper?

27.4. LHS = RHS is the Rule.

An equation has LHS & RHS.
LHS must always be equal to RHS.
We know that **10 + 5 = 15.**
The equation is OK if we ADD the SAME thing to both LHS & RHS of the equation.

Thus **10 + 5** + 20 **= 15** + 20
Or 20 + **10 + 5 =** 20 + **15**

Similarly **10 + 5** + 123456 **= 15** + 123456
123456 + **10 + 5 =** 123456 + **15**

Similarly **10 + 5** + 1 + 2 + 3 + 4 + 5 **= 15** + 1 + 2 + 3 + 4 + 5
(i.e. add any number of items).

Similarly **10 + 5** + x **= 15** + x
10 + 5 + ($x^2 + x + y^2 + y + 25$) **= 15** + ($x^2 + x + y^2 + y + 25$)

27.4.1 Exercises:

You are given $\boxed{x = y}$
State which of the following is True.

a. $x + 2 = y + 2$
b. $x + 2 = y + 20$
c. $x + 2 = 2y + 1$
d. $x + 2 = y + 5 – 3$
e. $2x + 4 = 2y + 4$
f. $2x + 4 = y + 8$
g. x x x = y x y
h. $x + a = y + a$
j. $x + a + b = y + a + b$.

27.4.2 Subtraction is the same as adding a negative item. So the above rule applies to (-) minus also.

∴ Given that A = B

$$A + \underbrace{\ (\text{anything}) = B +\ }\ (\text{anything})$$

$$A - \underbrace{\ (\text{something}) = B -\ }\ (\text{something})$$

Exercises: You are given $\boxed{x = y}$ State which of the following is True.

a. $x - 2 = y - 2$

b. $x - 2 = y - 20$

c. $x - 2 = 2y - 1$

d. $x - 2 = y - 5 – 3$

e. $2x - 4 = 2y - 4$

f. $2x - 4 = y - 8$

g. $x \text{ x } x = y \text{ x } y$

h. $x - a = y - a$

j. $x - a - b = y - a - b$.

27.5 FOR TEACHERS.

This section is for teachers. Students may try to read. If it is difficult, skip and go to 27.6.

27.5.1 Given that A = B
We know A + 10 = B + 10
Is it correct A +10 = B + 8 + 2 It is OK because 10 = 2 + 8

Rules1: Adding the same to LHS & RHS is ok

Rules2: Adding equal items to LHS & RHS is ok

27.5.2 Same as above with (- ve).
Given that A = B

A – (an item) = B – (an item)

Also A – (item) = B – ($\frac{1}{2}$ item + $\frac{1}{2}$ item)

√ A – (10) = B – (10)
√ A – 10 = B – (5 + 5)
√ A – 10 = B – (2 + 8)
√ A – 10 = B – (5 + 2 + 3)
X A – 10 = B – 5 + 2 + 3 Wrong

27.5.3 General rule:

$$\boxed{\begin{array}{l} A + x = B + y \\ A - x = B - y \end{array}} \quad A = B \text{ and } x = y$$

(Teachers, give large number of examples for this and See if the written matter is OK).

27.5.4 Some more examples (for 27.8).

a. 100 = 50 x 2

8 = 4 x 2

100 + 8 = 108 = (50 x 2) + (4 x 2) √ OK
100 – 8 = 92 = (50 x 2) – (4 x 2) √ OK
Now try 100 + (50 x 2) = 8 + (4 x 2) NOT OK

b. Why? If A = B and $x = y$

A + 1234= B + $\boxed{?}$ only 1234 or its equals.

Similarly A + x = B + $\boxed{?}$ x itself

= B + x or items which are equal to x

= B + y because $y = x$

In the above example, even it $x = y$

If you do A + B = $\boxed{1}$ + $\boxed{2}$

$\boxed{1}$ Can only be something equal to A

$\boxed{2}$ Can only be something equal to B

A + B = $x + y$ is wrong NOT OK
Because A $\neq x$; B $\neq y$

27.6 For STUDENTS & TEACHERS
This rule is very simple. It says, you can add anything to LHS of an equation, equality will be true if you add the same thing to RHS also.
Same here applies to equals also.

Thus if

$$\boxed{\begin{array}{l} A = B \\ A + x = B + x \\ A + y = B + y \\ A - x = B - x \\ A - y = B - y \end{array}}$$

If $x = y$, then A+ x = B + y .
A - x = B - y .

If $x \neq y$ then this rule does not apply.

27.6.1 Exercises: State true/ false showing steps to prove your answer.

i. x + a – b = x – (b – a) ii. x + 100 = x + (50 x 2)
iii. $x + y$ +2z = x +2(y +z) iv. $x + c = x + \frac{c}{2} + \frac{c}{2}$
vi. x -8 – 8 b= x - 8(b+1) v. 4(x + a) = 4 x +4 a

27.7 For teachers only:
Self – learning students can skip this and go to 27.8.

27.7.1. Now try 100 = 50 x 2; 10 = 5 x 2

$$\frac{100}{10} = \frac{50 \text{ X } 2}{5 \text{ X } 2} = 10 \quad \text{OK}$$

Also $\frac{100}{50 \text{ X } 2} = \frac{10}{5 \text{ X } 2} = 1$ √ OK

How? It is because all are = 1 **Special Case**

Some more examples (27.9 c)

a. Let 100 = 50 x 2; 8 = 4 x 2

Now try 100 x 8 = (50 x 2) x (4 x 2)

800 = 800 √ OK

b. Again try (100) x (50 x 2) = 8 x (4 x 2) NOT OK

c. ∴ If A = B; $x = y$
A x x = B x y OK

A x B $\neq$ x x y

d. In (c) above

If A = B, A x B = $(A)^2 = (B)^2$ OK √

If $x = y$, x x $y = (x^2) = (y^2)$ OK √

And Ax = By OK √

Also A x y = B X x OK √

Also $\frac{A}{x} = \frac{B}{y}$ OK √

Also $\frac{A}{y} = \frac{B}{x}$ OK √

e. Thus $\frac{A}{x} = \frac{B}{y} = \frac{A}{y} = \frac{B}{y}$ all OK

A X x = B X y = A X y = B X y all are OK

Not A X B = x X y Not OK

27.8 For both Students & Teachers.

<u>Rule:</u>

If A = B

A x y = B x y

$\frac{x}{A} = \frac{x}{B}$, $\frac{y}{A} = \frac{y}{B}$

A x x **=** B x

$\frac{A}{x} = \frac{B}{x}$, $\frac{A}{y} = \frac{B}{y}$

Condition = $x \neq 0$, $y \neq 0$

<u>Sub rule</u> (Corollary)

If A = B and $x = y$

A x x = B x x = A x y = B x y

$$\frac{A}{x} = \frac{B}{x} = \frac{A}{y} = \frac{B}{y}$$

<u>Caution</u>: Even if $x = y$ & A = B

A x B $\neq$ x x y

<u>Funny:</u> But $\frac{A}{B} = \frac{x}{y}$, why?

(If A = B, $\frac{A}{B} = 1$ Similarly $\frac{x}{y} = 1$

These are called trivial result)

27.8.1 Activity.

a. Use addition rule to find x in the equations

1) $x - 5 = 10$ x =? 2) $5 - x = 0$

b. Use subtraction rule.

1) $x + 5 = 10 \quad x = ?$ 2) $10 - 2x = 0$

27.9. Working with equations:
All the four basic arithmetical operations (i.e., -, +, x, ÷) could be done with equations.

27.9.1 Note that 4 = (3 + 1)
Now 40 = 4 x 10 i.e. (3 + 1) X 10 = 30 +10 = 40

∴ If A = B

A x 10 = B x 10 and A X (….) = B X (….)

What is not allowed: (….) cannot be zero.

27.9.2 Similarly division

40 = (3 + 1) X 10

$$\frac{40}{10} = 4 \quad \frac{(3+1)}{10} = 4$$

∴ If A = B

$$\frac{A}{x} = \frac{B}{x} \qquad \frac{1}{x} = (A) = \qquad \frac{1}{x} = (B)$$

27.10 Exercises :

Worked example: Solve $x - 20 = 10$
[Solve means, find the value of x]

$x - 20 = 10$ Add 20 to each side
i.e. $x - 20 + 20 = 10 + 20$
i.e. $x + 0 = 30$
i.e. $x = 30$. Ans

SHORTCUT Method
(a -ve quantity can go to other side but becomes +ve. ie -LHS → + RHS.
ie.+ RHS → - LHS.)

This way $x - 20 = 10$
i.e. $x = 10 + 20$
= 30 Ans

Do (i.e. Solve the equations :)

i. $x + 4\frac{1}{2} = 10$

ii. I bought something and gave Rs.10, I got back Rs.4.50 what was the cost of the item bought?

iii. I bought $\frac{1}{2}$ kg of onion, and gave Rs. 10, I got change of Rs.2.50. what was the cost of onion per kg? (Mysore, May09)

iv. A party went to GRS (Fun Park). 5 adults and 10 children were there in the group. Children allowed for $\frac{1}{2}$ the money. A total of Rs.2000 was paid for the entry tickets. What was the cost of adult ticket? Child's ticket?

v. $30x + 50\left(\frac{x}{2}\right) = 110 \quad x = ?$

[Clue: (v) helps to do (iv)
(i) helps to do (ii)

vi. A bus was arranged for a marriage party. It was only for 55 persons. 70 persons came in. Nobody wants to be left. How many autos will you arrange? [clue for foreigners: auto will take only 3 persons/less]

vii 1 bus and 5 autos arrived. A total of 50 persons came in. What was the capacity of the bus?
[Clue: (vi) & (vii) are similar. For some student, one may be easier than the other, do that first]

27.11 Shortcuts

27.11.1 Worked examples:

1. Given that $2x = 220 \quad x = ?$

Divide $\frac{2x}{2} = \frac{220}{2}$ i.e. $x = 110$

Here we divided both LHS & RHS by 2.

Shortcut: if $ax = b$ (a of LHS becomes $\frac{1}{a}$ on RHS)

$$x = \frac{b}{a}$$

$$2x = 220$$

$$x = \frac{220}{2}$$

$= 110$ Ans

2. Suppose the question was $25x = 225$, $x = ?$
Let us do as in the example above

Method 1: $25x = 225$

$$\frac{25x}{25} = \frac{225}{25}$$

$$x = \frac{225}{25} = 9$$

Looking at the number 25 on LHS we can do by another method also, i.e. multiplying.

Method 2: $25x = 225$

$$4 \times 25x = 4 \times 225$$

$$100x = 4 \times 225$$

$$= 900$$

$$\frac{100x}{100} = \frac{900}{100}$$

$$x = 9$$

Method 2 appears to be longer, but if you really do the divisions, second method is easier. These methods help us when we have large number or fractions.

27.11.1 Summary:

In method 1, dividing LHS & RHS by the same number is OK.
In method 2, multiplying LHS & RHS by the same number is OK.

27.11.2 Rule:

The methods given above are sometimes shown as transferring to the other side. The shortcuts are multiplying factor on LHS goes as dividing factor on the RHS & vice versa.

$\therefore$ If A X x = B $\quad x = \frac{B}{A}$

If $\frac{x}{A}$ = B, $\quad x$ = B X A

27.11.3 Exercises: Using shortcuts given above, solve:

a. 10 x = 100 x =? $\qquad$ b. $\frac{x}{10}$ =100.

c. 125 x = 900 + 100 $\qquad$ d. 125 x = 250.

e. 100 x = 250 - 25 x $\qquad$ f. 200 x = 1000 + 75 x .

g. $\frac{1}{3}x + \frac{1}{2}x = 25$ $\qquad$ h. $\frac{1}{3}x - \frac{1}{2}x = 25$

i. a= 2b, b = 2c ; c = 5 a = ?

j. Taruna is twice as old as Beti. Beti is twice as old as Pappu. If Pappu's age is 5 years, how old is Tarun? [Clue: If you can solve (i), you can do (j)

k. If (j) above, Naani is 3 times as old as Taruna, What is the age difference between Naani and Pappu?

27.12 Rules for solving equations:

All the rules put together

If x + A = B, $\quad x$ = B – A
If x - A = B, $\quad x$ = B + A
If x X A = B, $\quad x = \frac{B}{A}$
If x X $\frac{1}{A}$ = B, $\quad x$ = B X A

27.13 For Teachers:

Teachers! Simple questions using the above rules could be generated by the students themselves & given to other students. Let them work in groups.

Many maths, puzzles, funs, Sunday magazine sections use these techniques. It will be very easy to find problems by yourselves; or if you just look around.

27.14 Exercises:

a. Ramu is 5 years elder to Radha. If Ramu is your age, What is Radha's age?

b. I gave away 5 rupees each to all the students in the class. The class had 40 students. I was left with 50 rupees. How much money did I have?

c. A player in the casino put some money in the first game. He won & got double the amount. He played like this 4 times. Each time he got double the amount. He added 40 rupees and paid a bill of Rs.200. How much money did he start with?

d. A businessman got a quintal of rice for 1500 rupees and sold in retail at the rate of 21 rupees per kilo. What was his profit? (=Total money earned)? What is his profit per kg? What is his percent profit?
[Help: 1 quintal = 100 kg]

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Chapter - 28 Squares & Square roots

28. Squares & Square roots:
A number multiplied by itself is its square. The reverse of it is the square root.

28.1 How to write:

$a \times a = a^2$ (small 2 on the right top)
$5 \times 5 = 5^2$ (small 2 on the right top)
a^2 and $(a)^2$ are the same
6^2 and $(6)^2$ are the same

Brackets must be used when more than one item is squared (= multiplied by itself)
Thus $(5a) \times (5a) = (5a)^2$
Writing this as $5a^2$ is wrong.
Similarly $(ab) \times (ab) = (ab)^2$ writing this as a^2b, ab^2 are wrong.
Similarly $(x + 5)^2$,$(x + y + 2)^2$ ok.
Similarly $(a + b)^2$,$(a – b + 5)^2$ ok.

28.2 How to read:

5^2 is read as “ FIVE SQUARE ”
[Note: some read as ‘ Squared ’ this assumes a passive voice idea of English grammar, meaning somebody came & squared it. This is Not Important
You can read either way.]
When brackets are there read upto the last item & say “Whole Square ”. [Some say, “ All square (D)”. It is also ok].

28.3 Activity

28.3.1 a. Teachers let the students write the squares of single digit numbers.

Thus 1^2 = …………
2^2 = …………
3^2 = …………
4^2 = …………
5^2 = ………..
6^2 = ………..
7^2 = ………..
8^2 = ………..
9^2 = ………..
10^2 = …………
Let them do it without using the calculator. Practice how to read also.

b. Teachers, let the students read out the following correctly. Allow the students to check right /wrong among themselves. You can act as a referee.

E.g. $(1 + 2)^2 = 3^2$ = …………The right way of reading this is, “One – plus – two whole – square” “ equals “ three square ”equal to “(nine)”[“one – plus – two” should be spoken in one single breath. ‘equal’ ‘equal to ’ ‘equals’, ‘is equal to’, anything is fine. The word ‘ whole ’ in ‘ Whole Square ’ is important]

28.3.2 Caution: Many Mysore children may not know (no!) ‘ Hole ’ as different from ‘ whole ’. In this book, both these terms occur with important significance. Teachers! Take this opportunity to explain.

28.4 Area and Square:

Finding the squares of numbers is very important. This is because area of a square or a circle needs squares.
A closed figure on a surface has Area.
A Chessboard has 64 squares.
A graph sheet has many big squares and hundreds of small squares.
A Square is a special geometric shape (on a plane surface).Circle, Triangle, Pentagon are also similar geometric shapes and they also have area.[In fact any closed figure on a surface contains area).
Whatever be the shape of the figure, the area contained is expressed in
Square Units
[Sq is used here to mean ‘ square’]
Sq. cm, Sq. m, Sq. feet, Sq. foot, Sq. miles, sq. kms are some of the
common terms used in daily life.

28.4.1 Activity
[Teachers! elicit info. from students experiences, the concept of area & its units]
a. Carpenter – for a table/plank.
b. (House) painter – for painting a wall
c. (House) seller – site area
d. From a book) – forest cover in the world
e. (From a book) – area of Karnataka.
f. Many more.

28.5 Calculation (Approximation)
Finding squares of large numbers is not easy.
Actual method is either by using calculator or long multiplication method. But one should always be able to approximate or guess.
Here are some tips (= clues, suggestions) on how to do approximation.(= finding near value)

28.5.1 Worked examples
a. Create your own square grid

$1^2 = 1$
$2^2 = 4$
.
$10^2 = 100$

Using these, how to approximate?

b. 11^2 =?
11^2 = 11 x 11 this is > 11 x 10
Therefore 11^2 >110. Actually 121

c. 20^2 =? 2^2 = 4 10^2 = 100
$(20)^2$ = (2 X 10) = 4 X 100 = 400

d. What is 19^2
19^2 < 19 x 20
i.e. < 380 Actually 361

One can guess, how much less also
192 = 19 x 19
19 x 20 = 380
19 x 19 = 380 – (1 x 19) = 361

e. What is 32^2
32^2 = 32 x 32

$= 4^2 \times 8^2 = 16 \times 64$

Or $(32)^2 > (30)^2$
$> (30)^2 + (2)^2$
> 904 Actually it is 1024

28.5.2 Activity (Game)
One group has calculator. The other goes for mental maths. Use only 2 digit numbers for squaring. [Even upto 20% difference is ok].

28.6 Decimal

28.6.1 Finding squares of decimals.
Teachers, let the students do some simple sums by both the methods i.e.
By multiplying 2 fractions
Or by multiplying 2 decimals (fraction)

a. $(\frac{3}{2})^2$ or $(.5)^2$

$(\frac{3}{2})^2 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4} = 2.25$

$(1.5)^2 = 1.5 \times 1.5 = 2.25$

b. $(\frac{1}{4})^2$ or $(0.25)^2$

$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = 0.0625$

$(.25)^2 = .25 \times .25$

$= \frac{25X25}{10000} = \frac{625}{10000} = 0.0625$

c. Let students select their own, do and show.

28.6.2 Teachers should show that squares of numbers > 1tend to be big.

Square of numbers < 1 (i.e., fractions) tend to be small (less than the original fraction).

Thus 2 x 2 = 4 but 3 x 3 jumps to 9

Similarly $\frac{1}{2} \times \frac{1}{2} << \frac{1}{2}$ (much less than)

$\frac{1}{4} \times \frac{1}{4} << \frac{1}{4}$ (much less than)

28.6.3 Exercises:
Do by all 3 methods:
a. Approximation
b. Actual (manual) rigorous calculation
c. Using calculating machine.

a. 15^2 b. $(1.5)^2$ c. $(0.15)^2$ d. $(15.5)^2$ e. $(21)^2$

f. $(2.1)^2$ g. $(4.2)^2$ h. $(\frac{3}{8})^2$ i. $(2\frac{3}{8})^2$ j. $(\frac{4}{7})^2$

28.7 Square Root:
What is Square root?
Example:
5 x 5 = 25

i.e., $5^2 = 25$

In words, square of 5 is 25.
Square root of 25 is 5. (Reverse, or converse or opposite or ulta, by definition)

28.7.1 **How to write?**

Take a number, n. To denote (= indicate, identify, point out, say) square root of this number, write:

$\sqrt{\mathbf{n}}$ Thus $\sqrt{1}$, $\sqrt{10}$, $\sqrt{100}$, etc.,

[Some persons shorten these in to $\sqrt{n}$, $\sqrt{1}$ etc i.e., without the ceiling (= top line, *sir – rekha*) avoid this practice. Not good]

Some people write as $\sqrt[2]{n}$, $\sqrt[2]{100}$ etc. This is really correct and accurate. But not necessary. Only when other roots (eg, cube root) are used we need numbers there.
When big quantities are there, use a very long top-line or use brackets or both.
Eg: $\sqrt{10+15}$ or $\sqrt{(10+15)}$, $\sqrt{(a+b)}^{\,2}$, $\sqrt{a^2+b^2}$, or $\sqrt{(a^2+b^2)}$ etc.

28.7.2 How to read?

$\sqrt{n}$. Read it as ' Square root of n' or ' root of n' or simply ' root n'.
If bigger items are written under sq. root:

Eg: $\sqrt{(a^2+b^2)}$ or $\sqrt{a^2+b^2}$
Read it as square root of…$a^2 + b^2$
i.e., give a pause (=time interval, stop and start) after 'root of'. Say the other In one breath (one breath = no stopping, speak continuously) If you say square root of a^2. …Plus b^2, the listener will write as $\sqrt{a}^{\,2} + b^2$.

Exercises: ACTIVITY

A. Let the students make 2 groups. One will read from a list of items. One or two or 3 persons) from the other team will write (as they hear). After 5 items they compare.

B. In A above, reading team becomes writing team & vice versa (= *ulta*)
Sample list:

1. $\sqrt{x}$
2. $\sqrt{(x+y)}$
3. $\sqrt{x} + \sqrt{y}$
4. $\sqrt{x} + y$
5. $y + \sqrt{x}$
6. $\sqrt{x^2+y^2+a^2}$
7. $\sqrt{x}^{\,2} + \sqrt{y^2+a^2}$
8. $x + \sqrt{x^2 + y^2 + a} + a$
9. $\sqrt{x} + 25$
10. $\sqrt{(x+25)}$ You can make your own.

28.8 **Is square root important?**

Students know the answer to this question. Of course, yes. ['Of course' = Surely' certainly, no doubt, without doubt etc]
Ask a history teacher, " Is history important?"
Ask a engish teacher, " Is grammar important? "
Ask a sot (= drunkard) " Is drinking important? "
They will all say, "Of course, yes".

28.8.1 Square root comes in whenever and whereever areas are used. The idea (= concept) of area in almost all branches of science and certainly in engineering. Even in business or economics squares & square roots occur. There they call it ' doubling'.

28.8.2 Exercises:

In section 28.3.1 we made list of squares of numbers from 1 to 10. Using this let us do some problems.

Worked examples:

1. $\sqrt{100}$ =? See the list Ans = 10.

2. $\sqrt{9X9}$ **=**? One can go to $\sqrt{81}$ = 9 (refer to the list). This is not necessary. By definition, $\sqrt{9X9}$ **=** $\sqrt{9}^{2}$ = 9.

3. $\sqrt{3X27}$ =? One method is $\sqrt{3X27}$ = $\sqrt{81}$ = 9.

Another method: $\sqrt{3X27} = \sqrt{3X3X9}$ = $\sqrt{9X9}$ = 9.

28.9.1 Examples:

1. $\sqrt{121}$ =? 121 = 11 x 11 = 11^2

$\sqrt{121}$ = $\sqrt{(11)}^{2}$ = 11 Ans

2. $\sqrt{4x16}$ =? 4 = 2 x 2 = 2^2

16 = 4 x 4 = 4^2

∴ $\sqrt{4x16}$ = $\sqrt{2^2 \text{ x } 4^2}$ = 2 x 4 = 8 Ans

Exercises:

a. $\sqrt{100}$ **=**?

b. $\sqrt{10X10X10X10}$ **=**?

c. $\sqrt{10000}$ **=**?

d. $\sqrt{5X5X25}$ **=**?

e. $\sqrt{9X25}$ **=**?

f. $\sqrt{3X3X16X25}$ **=**?

28.9.2 Worked examples:

1. $\sqrt{121x^{2}}$ **=?** 121 = $(11)^2$ $x^2 = (x)^2$

∴ $\sqrt{121x^{2}} = \sqrt{(11)^2 \text{ x } (x)^2}$ = 11 x x = 11 x

2. $\sqrt{2}$ $\sqrt{2}$ $\sqrt{4}$ $\sqrt{4}$ =?

Ans: By definition $\sqrt{2X2}$ = 2 $\sqrt{4X4}$ = 4

It can be written 4 = $\sqrt{4X4}$ = $\sqrt{(2)}^{2}$ x $\sqrt{(2)}^{2}$

Extending this logic 2 = $\sqrt{\sqrt{2} \text{ x } \sqrt{2}}$ ∴ $\sqrt{2}$ x $\sqrt{2}$ = 2

Thus, LHS = 2 X 4 = 8

3. $\sqrt{125}$ =? 125 = 5 x 5 x 5 = 5^2 x 5

$\sqrt{125} = \sqrt{5^2 \text{ x } 5}$ = $\sqrt{5}^{2}$ x $\sqrt{5}$ = 5 x $\sqrt{5}$ Ans.

Exercises:

a. $\sqrt{100a}^{\,2}$ **=?**
b. $\sqrt{a}^{\,2}\ \sqrt{100}$ **=?**
c. $\sqrt{10000a}^{\,2}$ **=?**
d. $\sqrt{5a \text{ X } 5a \text{ X } 25a^2}$ **=?**
e. $\sqrt{a}$ **x** $\sqrt{a}$ **= ?**
f. $\sqrt{a}$ **x** $\sqrt{a}$ **x** $\sqrt{5}$ **x** $\sqrt{5}$ **= ?**
g. $\sqrt{9a}^{\,2}$ **x** $\sqrt{25a}^{\,2}$ **=?**
h. $\sqrt{3}\ \ \sqrt{3}\ \ \sqrt{a}\ \ \sqrt{a}\ \ \sqrt{25a}^{\,2}$ **=?**

28.10 Approximation method:

Using maggi (= multiplication tables) students can generate a table (= list) of squares.[As in section 28.3.1]. Use this for approximation.

Finding the square root by guessing is ok.

a. $\sqrt{5}$ = ? $2^2 = 4$ $3^2 = 9$

∴ $\sqrt{5}$ is > 2 and < 3 and it is nearer to 2

∴ $\sqrt{5}$ is between 2 & 2.5 $\sqrt{5}$ =2.24 (calculator)

b. $\sqrt{11}$ =? $3^2 = 9$; $4^2 = 16$

$\sqrt{11}$ is between 3 and 4.It is nearer to 3. May be 3.2

Let the students check. $\sqrt{11}$ = 3.32 (calculator)

c. $\sqrt{45}$ = ? $6^2 = 36$ $7^2 = 49$

$\sqrt{45}$ is between 6 & 7 and nearer to 7, >6.5 may be 6.8 check.
$\sqrt{45}$ 6.71 (calculator)

d. $\sqrt{3625}$ Approx = 3600
$\sqrt{3625} > \sqrt{3600}$

$> \sqrt{36}\ \sqrt{100}$

> 6 x 10 >60 Very slightly more than 60 may be 60.01(!)

$\sqrt{3625}$ = 60.2 (calculator)

e. $\sqrt{12345} = x$ x =?
$x > \sqrt{12300}$ $\sqrt{121}$ **<** $\sqrt{123}$ **<** $\sqrt{144}$

> $\sqrt{123}\ \sqrt{100}$ $11 < \sqrt{123} < \sqrt{144}$

> 10 $\sqrt{123}$ May be 11.1

> 10 x 11.1 $\sqrt{123}$ = 11.09 (calculator)

>111 Check. $\sqrt{12345}$ = 111.1 (calculator)

28.11 How to find square root?

28.11.1 Answer to the question above. “Give me a calculator” ok.
Approximation method given in 28.10 above is special to this book. It may not be found in standard books. Since this writer believes in ‘estimations’, it is included here.

28.11.2 Usual methods are:
A. Factorization method
B. Traditional ‘ division method’.

28.12 Factorisation method.
The principle here is to write a given complex number in the form of squares.

Example: $\sqrt{196}$ =? 196 = 4 x 49. Here we have factorised 196 into two factors. Each one is a square.

$\therefore \sqrt{196} = \sqrt{4X49} = \sqrt{2^2 x 7^2}$ = 2 x 7 = 14 Ans.

28.12.1 Factorisation method is important and should demonstrated.

a. $\sqrt{14400} = x$

$\therefore$ 14400
= 100 x 2 x 2 x 2 x 2 x 2 x3 x 3
= 10^2 x 2^2 x 2^2 x 3^2
= $\sqrt{14400}$ = 10 x 2 x 2 x 3 = 120

100 | 14400
2 | 144
2 | 72
2 | 36
2 | 18
3 | 9
3

b. $\sqrt{1440} = y$

$y = \sqrt{1440}$
$= \sqrt{10 X 2 X 2 X 2 X 2 X 3 X 3}$
$= \sqrt{10 x 2^2 x 2^2 x 3^2}$
$= \sqrt{10}$ x $\sqrt{2}^{\,2}$ x $\sqrt{2}^{\,2}$ x $\sqrt{3}^{\,2}$
= $\sqrt{10}$ x 2 x 2 x 3
= 12 x $\sqrt{10}$ = 12$\sqrt{10}$

c. Let students select their own problems & try to solve by factorizing.

28.12.2 **Exercises:**
Factorise and then find sq. root

Example: 1. $\sqrt{12100}$ **=**? 12100 = 121 x 100
= 11 x 11 x10 x 10
= 11^2 x 10^2

$\therefore$ $\sqrt{12100} = \sqrt{11^2 x 10^2}$ = 11 x 1 = 110 Ans
$\sqrt{1210} = \sqrt{11^2 x 10}$ =11 x $\sqrt{10}$ Ans

a. 86436
b. 86436 x^2
c. 36
d. 36a^2b^2
e. 324
f. 324 x $(a + b)^2$
g. 900
h. 900 x a^2 x $(x - y)^2$
i. 676
j. $\frac{676}{a^2}$
k. 3380
l. 2700
m. 162
n. 18
o. 1764
p. 3380a

q. $18a^2b$ r. $\frac{18}{a^2}$ s. $\frac{1764 \times a^2}{b^2}$ t. $\frac{3380 \times a}{b^2}$

28.13 Rules, Tips

28.13.1 Teachers, drill the following:

a. $x \times x = x^2$

b. $x \times x \times y \times y = x^2 \times y^2$

c. $xy \times xy = (xy)^2 = x^2 y^2$

d. $(xyz)^2 = x^2 y^2 z^2$

e. $(\text{Number} \times a)^2 = (\text{Number})^2 \times a^2$

To self – study student: 'drill' means read and write (= do) many times so that you are familiar with all the rules & equations.

28.13.2 Drill:

a. $\sqrt{x} \times \sqrt{x} = x$

b. $\sqrt{x}^2 \times \sqrt{x}^2 = x^2$

c. $\sqrt{(anything)} \times \sqrt{(Something)} = anything$

d. $\sqrt{1025} \times \sqrt{1025} = 1025$

28.13.3 There are formulas for $(x+y)^2$ and $(\text{Number} + a)^2$. Those can be learn't later. At present

$\sqrt{(x+y)^2} = (x+y)$

$\sqrt{(number+a)^2} = (number+a)$

28.14 Exercises: Say √ / X

a. $\sqrt{5} \times \sqrt{3} = \sqrt{15}$ b. $\sqrt{5^2 \times 3^2} = 225$ c. $\sqrt{5} \times \sqrt{3} = \sqrt{8}$

d. $\sqrt{5^2 \times 3^2} = 15$ e. $\sqrt{5} + \sqrt{3} = \sqrt{8}$ f. $\sqrt{5^2 \times 3} = 75$

g. $\sqrt{5+3} = \sqrt{8}$ h. $\sqrt{5^2} \times 3 = 15$ i. $\sqrt{5+3} = 2\sqrt{2}$

j. $\sqrt{5^2 \times 3} = 5\sqrt{3}$

28.15 Traditional method of finding square root – OR – "Square Division method" or "Double Division Method"

28.15.1 Exactly finding square root by traditional method is Not Necessary in this manual. If teachers find some bright students asking for it, here are some:

a.

```
        3.4  |6
      ----------------
  3  | 12.00 |  00
     |  9
      ----------------
 64  |  300
     |  256
      -----------
686  |  4400
     |  4116
      ---------
```

$\sqrt{12} = 3.464$ (calculator)

b.

```
         32
      ----------
  3  | 1024
     |  9
      ----------
 62  | 124
     | 124
      ---------
     |    0
      ----------
```

$\sqrt{1024} = 32$

c.

	25
2	625
	4
45	225
	225
	0

$\sqrt{625}$ =25

Exercises: [Only for advanced students] Do by the above method.

i. $\sqrt{225}$ ii. $\sqrt{324}$ iii. $\sqrt{676}$ iv. $\sqrt{86436}$

v. $\sqrt{14400}$ vi. $\sqrt{1440}$ vii. $\sqrt{2700}$

1. Square root of 5

	2.234
2	5.00 00 00
	4
42	100
	84
443	1600
	1329
4464	27100
	≈17856

$\sqrt{5} \approx$ 2.23

2. $\sqrt{11}$

	3.31
3	11.00 00
	9
63	200
	189
661	1100
	661
	439

$\sqrt{11}$ > 3.31

3. $\sqrt{45}$

	6.708
6	45.00 00 00
	36
127	900
	889
1308	1100 00
	107264

$\sqrt{45} \approx$ 6.708

= 6.71

Can also be: $\sqrt{45}$ = $\sqrt{9X5}$ = 3$\sqrt{5}$

$\approx$ 3 X 2.234 $\approx$ 6.702

4. $\sqrt{12345}$ =?

	111.1
1	1,23,45.00
	1
21	23
	21
221	245
	221
2221	2400
	2221

$\sqrt{12345} \approx$ 111.1

5. $\sqrt{10}$ =?

$$\begin{array}{r|l}
 & 3.162 \\
\hline
3 & 10.00\ 00\ 00 \\
 & 9 \\
\hline
61 & 100 \\
 & 61 \\
\hline
626 & 3900 \\
 & 3756 \\
\hline
632 & 14400 \\
 & 12644 \\
\hline
 &
\end{array}$$

$\sqrt{10} \approx 3.162$

6. $\sqrt{1440}$ =?

$$\begin{array}{r|l}
 & 38 \\
\hline
3 & 1440 \\
 & 9 \\
\hline
68 & 540 \\
 & 544 \\
\hline
 & \approx 38
\end{array}$$

$\sqrt{1440} = \approx 38$

Can also be done as: $\sqrt{1440} = \sqrt{144}$ x $\sqrt{10}$ =12 x $\sqrt{10}$

= 12 x 3.162 = 37.94

==

Chapter - 29 Formulas of Algebra

29.1 Note to the teacher: In this manual, 3 formulas of algebra are selected out as very important. They are:

1. $(a + b)^2$ 2. $(a – b)^2$ 3. $(a + b)(a – b)$

Of these 1 and 2 are combined as $(a \pm b)^2$. Please help the student memorize properly. There may be other formulas, which may be equally important. Such as $(a + b)^3$, $(a + b + c)^2$, $a^3 + b^3$ etc. Ours is a very simple introduction to mathematics.

29.2 Formulas:

$(x + y)^2$ and $(x - y)^2$

These two formulas are so important and so useful in many estimations, it is useful to memorize.

Let the students memorize

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

Now let them ignore these two and memorize only one:

$$\boxed{(x \pm y)^2 = x^2 \pm 2xy + y^2}$$

29.2.1 Exercise:

$(a \pm b)^2$ = $(p \pm q)^2$ = $(u \pm v)^2$ =

$(a + b)^2$ = $(p + q)^2$ = $(u + v)^2$ =

$(a - b)^2$ = $(p - q)^2$ = $(u - v)^2$ =

29.3 Bracket Removal: Multiplication in algebra is already known to the student. Using this knowledge, how to handle brackets is the subject of this paragraph.

a X a = a^2

a X b = ab

a X c = ac

$a(a + b) = a \times a + a \times b = a^2 + ab$
$a(a + b + c) = a \times a + b \times a + a \times c = a^2 + ab + ac$
Now, give many examples, some purely on algebra, some purely with numbers, some both.

29.3.1 Examples:

1. Let us verify a (a+b) = a2 + ab
   Let a=3, b=5
   LHS = 3(3+5) = 3 x 8 = 24

   RHS = $(3)^2 + (3) \times (5)$
   = 9 + 15
   = 24
   = LHS Verified

2. Let us verify a (a + b + c) = a2 + ab + ac
   Let a = 3, b = 5, c = 7
   LHS = 3(3+5+7)
   = 3 x 15
   = 45
   RHS = $(3)^2 + (3)(5) + (3)(7)$
   = 9 + 15 + 21
   = 45
   LHS = RHS (verified)

29.3.1 Exercises: Expand or remove brackets.

x (a + b); x (a – b); $x\ (a^2 – c)$

4 (5 + 10); 4(5342 – 1234); $4(5^2 - 20)$

$2x\ (y + 3)$; $4ax\ (b + 2)$; $ax^2(y - 4)$

29.4 Double Brackets: Let us take an example: $(x + a) \times (x + b)$

$(x+a)(x+b)$
$= x\ (x + b) + a\ (x + b)$
$= x^2 + bx + ax + ab$

How to do $(x+ a)\ (x + a)$
$= x\ (x + a) + a(x + a)$
$= x^2 + ax + ax + a^2$
$= x^2 + 2xa + a^2$

Exercises: Expand

1. $(x + 4)\ (5 + 10)$
2. $(x + 4)\ (5342 – 1234)$
3. $(2x + 1)\ (y + 3)$
4. $(4ax + 1)\ (b + 2)$
5. $(x + 1)\ (a^2 – c)$
6. $(4 – 3)\ (5^2 – 20)$
7. $(ax^2 + 4)\ (y - 4)$

29.5 We have seen that $(x + b)\ (x + b) = x^2 + 2x + a^2$
Now make $a = y$
$(x + y)\ (x + y)$
$= x^2 + 2xy + y^2$
Now make $x = a$ $y = b$
$(a + b)\ (a + b) = a^2 + 2ab + b^2$

Exercises:

1. $(x - a)\ (x - b)$
2. $(x - a)\ (x - a)$
3. $(x - y)\ (x - y)$
4. $(a – b)^2$
5. $(5 – a)\ (5 – b)$
6. $(5 – a)\ (5 – a)$
7. $(5 – y)\ (5 – y)$
8. $(5 – 3)\ (5 – 3)$
9. $(6 – b)^2$

10. Your own questions.

29.6 2 Important Formulas:

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$(a + b)(a - b) = a^2 - b^2$

29.7 Using $(a \pm b)^2$

Worked Examples:

1. We know $(13)^2 = 169$. Now let us do it using the formula

$$\begin{aligned}(13)^2 &= (10 + 3)^2\\ &= (10)^2 + 2(10)(3) + (3)^2\\ &= 100 + 60 + 9\\ &= 169\end{aligned}$$

2. $(499)^2$

$$\begin{aligned}(499)^2 &= (500 - 1)^2\\ &= (500)^2 - 2(500)(1) + (1)^2\\ &= 250000 - 1000 + 1\\ &= 259001\end{aligned}$$

Exercises:

1. $(99)^2$ 2. $(101)^2$ 3. $(49)^2$ 4. $(51)^2$
5. $(1002)^2$ 6. $(505)^2$ 7. $(22)^2$ 8. $(18)^2$
9. $(112)^2$ 10. $(88)^2$

29.8 $(a^2 - b^2) = (a + b)(a - b)$ Formula

Another useful formula is $(a + b)(a - b) = a^2 - b^2$

Same method as before

LHS = $a(a - b) + b(a - b)$
= ………
= ……..
= $a^2 - b^2$
Students can fill up

29.8.1 Example: Find 501 x 499

$501 \times 499 = (500 + 1)(500 - 1)$ Apply Formula
$= (500)^2 - (1)^2$
Here a = 500 b = 1
∴ LHS = 250000 – 1
= 249999

Exercises: Find the values using shortcut formulas

a. 99 x 101 b. 95 x 105 c. 92 x 108 d. 42 x 58
e. 48 x 52 f. 10005 x 9995

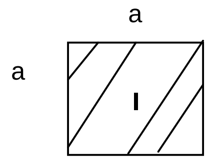
29.9 Some ‘Practical Proofs’

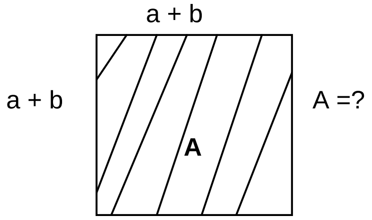
29.9.1 Geometrical ‘Proof’ $(a + b)^2$

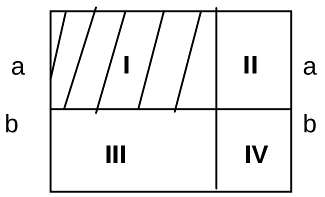
a2 = square = area of a square with side a

$(a + b)^2$ = square = area of a square with side (a + b)

Draw these



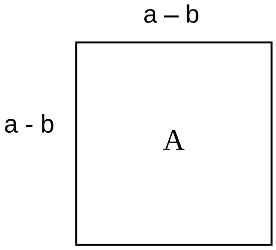


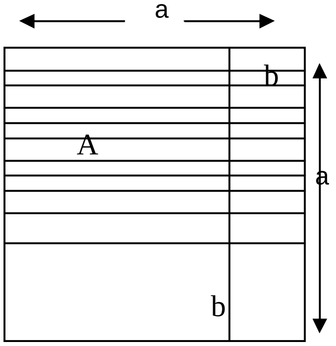


$I = a^2$
II = Area of rectangle = ab
III = Area of rectangle = ab
IV = Area of square = b^2

Therefore A = (a + b) 2
= I + II + III + IV
= $a^2 + ab + ab + b^2$
= $a^2 + 2ab + b^2$

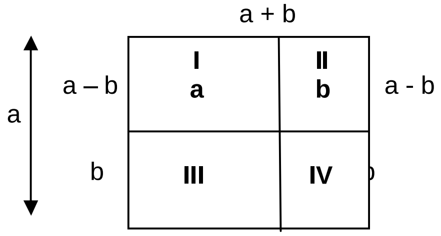
29.9.2 Geometrical Proof of $(a-b)^2$; same as 29.9.1 with $(a-b)^2$



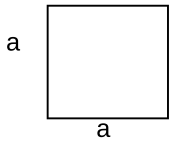


Show that A = I.
Here one has to cut and show.
For this purpose, take two identical pieces (tricky! be careful – b^2 is taken away twice, therefore should put back once.

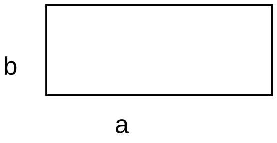
29.9.3 Geometrical Proof of (a + b) (a – b). Do as in the two previous (=earlier, above cases). (Tricky! See III > II by b^2 therefore $(-b^2)$ needed)



29.9.4 For those students who have not yet understood these geometries:

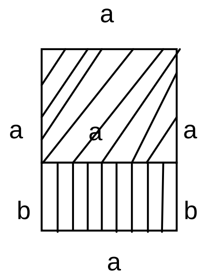


This shape is a square. Its side is a. Its area = a^2
Its area = a^2
= (a x a)

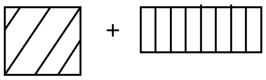


This shape is a rectangle. Its sides are ‘a’ & ‘b’
Its area = a x b

Now see 2 shapes put together (=joined, attached)



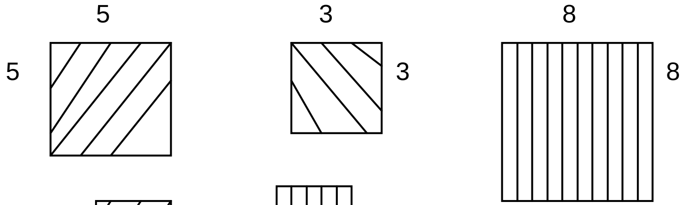
What is the total area?

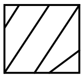
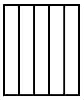


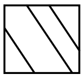
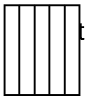
$= a^2 + ab$

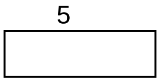
29.10 Activity
All the ideas given in 29.9 can be done by house or classroom activity. To prove $(a + b)^2$ formula. Let a = 5 cm b = 3 cm

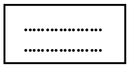
Draw a square of 8 cm (i.e., a + b) on a cardboard. Make 3 pieces (cut neatly).



Place  inside  the big one.

Place  also inside  the big one.

There is still empty space left. Now make 3  2 pieces. Now if

you put  into the big piece, one after the other, you will see it will tightly fit.

29.11 Activity
Cut and paste method of 29.10 can be directly done on a graph sheet. Do the same steps as in 29.10. Find the gaps (& their areas) by counting the small squares (of the graph sheet).

29.12 Activity
Earlier sections showed $(a + b)^{2.}$ Similarly practical constructions can be made for $(a – b)^2$ also. With a little difficulty (a + b) (a – b) also.

Chapter - 30 Assessment - 2

(Assorted random problems of mostly algebra based on chapters 21 to 29).
(Teachers, these can be used for assessment)

30.1 Addition

a. 200 + 20 – 100 –10 b. 201 + 21 – 101 – 11
c. 221 + 21 – 121 – 11 d. 221a + 21 – 121a – 11
e. 221a + 21a – 121a – 11a f. 21a + 1a – 21a – a
g. 20a + a – 19a – a h. 10a + 2a – 11a – a
i. 10a + 2 – 9a – 1 j. 10a + 2 + 1 – 9a –3

30.2 Number Sequence

a. 11, 13, _, _, 19, _, _, 25 b. 10, 12, _, 16, _, _, 22, _,
c. 100, 95, _, _, 80, _, _, d. 60, _, 50, _, 40, _, _,
e. x, 2x, _, _, 5x, _, _, f. 2x, 4x, _, _, 10x, _, _,
g. 50x, 45x, _, _, 30x, _, _, h. 18x, _, 12x, _, 6x, _, _,
i. 2, 4, 8, _, 32, _, j. 2x, _, $8x^3$, _, _,

30.3 Fractions – Equivalence

a. $\frac{2}{3} = \frac{4}{?} = \frac{10}{?}$ b. $\frac{4}{3} = \frac{8}{?} = \frac{20}{?}$ c. $\frac{2x}{3} = \frac{4x}{?} = \frac{10x}{?}$

d. $\frac{2a}{5x} = \frac{4a}{?} = \frac{10a}{?}$ e. $\frac{2}{3} = \frac{?}{3x} = \frac{22}{?} = \frac{?}{15y}$ f. $\frac{a}{a^2} = \frac{a2}{?} = \frac{10a4}{?} = \frac{ka^3}{?}$

g. $\frac{a(a+1)}{2} = \frac{?}{2 \text{ x } 4}$ h. $\frac{a}{b} = \frac{a \text{ x } 4}{b \text{ x } ?} = \frac{a \text{ x } ?}{b \text{ x } (b+1)}$

30.4 Fractions – Simplifying

a. $\frac{75}{100}$ b. $\frac{2 \text{ x } 3 \text{ x } 4}{10 \text{ x } 18 \text{ x } 2}$ c. $\frac{999}{444} \text{ x } \frac{2}{3}$ d. $\frac{3x \text{ x } 25y}{4y \text{ x } 5x}$ e. $\frac{a^2 \text{ x } b \text{ x } c^3 \text{ x } d}{d^2 \text{ x } c^2 \text{ x } b^2 \text{ x } a^2}$

30.5 Fractions – Modifying

a. $\frac{2}{3} = \frac{4}{?} = \frac{10}{?}$ b. $\frac{4}{3} = \frac{8}{?} = \frac{20}{?}$ c. $\frac{?}{5} = \frac{9}{15} = \frac{?}{50}$ d. $\frac{2a}{3} = \frac{4a}{?} = \frac{10a}{?}$

e. $\frac{?}{5x} = \frac{9}{15x} = \frac{?}{50x}$ f. $\frac{2a}{5x} = \frac{4a}{?} = \frac{10a}{?}$ g. $\frac{2}{3} = \frac{?}{3x} = \frac{20}{?} = \frac{?}{15a}$

h. $\frac{a}{a^2} = \frac{a2}{?} = \frac{10a^4}{?} = \frac{ka^3}{?}$ i. $\frac{a\,(a+1)}{1 \text{ x } 2} = \frac{?}{2 \text{ x } 4}$ j. $\frac{a}{b} = \frac{a \text{ x } 4}{b \text{ x } ?} = \frac{a \text{ x } ?}{b \text{ x } (b+1)}$

30.6 Fractions – Addition

a. $\frac{1}{3} + \frac{1}{2}$ b. $\frac{1}{3} + \frac{1}{7}$ c. $\frac{1}{3} + \frac{1}{2} + \frac{1}{7}$ d. $\frac{2}{3} + \frac{3}{4} + \frac{5}{7}$

e. $-\frac{1}{3} + \frac{1}{2}$ f. $-\frac{1}{3} + \frac{1}{2} + \frac{1}{7}$ g. $-\frac{2}{3} - \frac{3}{4} + \frac{5}{7}$ h. $-\frac{2}{3} + 3 + \frac{5}{7}$

i. $\frac{2x}{3} + \frac{3x}{4} + \frac{5x}{7}$ j. $\frac{2}{3x} + \frac{3}{4x} + \frac{5}{7x}$

30.7 Multiplication

a. 4 x 19 b. 14 x 9 c. 99 x 13 d. 8 x 15 e. 8 x 151 f. 18 x 15
g. $(4x)$ x 19 h. (14) x $(9x)$ i. $(99x)$ x $(13x)$ j. 8 x $(15y)$
k. (8) x $(151x\ y)$ l. $(18x)$ x $(15y)$

30.8 Arrange in ascending order:

1. x, $8x$, $3x$, $5x$
2. x, $9x$, $4x$, $6x$ (x=2)
3. x, $9x$, $4x$, $7x$, (x=½)
4. x, x^2, x^3, $2x$ (x=2)
5. x, x^2, x^3, $2x$ (x=½)
6. (a+b), $(a+b)^2$, $(a+b)^3$, $(a+b)^4$ with a=1, b = 2
7. (a+b), $(a+b)^2$, $(a+b)^3$, $(a+b)^4$ with a = 1, b = ½ (Use calculator if you wish)
8. 101, 99, 309, 310, 9
9. 101ab, 99ab, 309ab, 310ab, 9ab
10. 101ab, 99ab, 309ab, 310ab, 9ab with a = 2, b = ½

30.9 Which is bigger?

a. $\frac{x}{4},\frac{x}{3}$
b. $\frac{a}{2},\frac{a}{3}$
c. $\frac{b}{7},\frac{b}{6}$
d. $\frac{2d}{3},\frac{d}{2}$
e. $\frac{2e}{5},\frac{e}{4}$
f. $\frac{2f}{7},\frac{f}{3}$
g. $\frac{11g}{24},\frac{7g}{12}$
h. $\frac{9h}{99},\frac{10h}{11}$
i. $\frac{x}{2},\frac{y}{3}$ with x =1 y =2
j. $\frac{a}{13},\frac{b}{7}$ with b=1, a =2

30.10 Division

a. If 41 x 19 = 779 41 x 18 =
b. If 3 x 18 = 54 103 x 18 =
c. If 3 x 18 = 54 54 ÷ 3 =
d. If 41 x 19 =779 7790 ÷ 41 =
e. (779x) ÷ (41x)
f. (54a) ÷ (18a)
g. (54a) ÷ (18)
h. (7790x y) ÷ (41x)

30.11 Substitution and Solve

a. $x + y = 5$ If x = 3 y = 2
b. $2x - 3y = 0$ If x = 3 y =?
c. If x = 3, y = 2 $2x + 3y$ =?
d. If x = 3, y = 2 $3y - 2x$ =?
e. If x = 2, x^2 =?
f. If y = 3, y^2=?
g. If x^2 = 9, x=?
h. If y^2= 4, y =?

30.12 Solve

a. $\frac{1}{2}x = 41$ x =?
b. $21x = 42$ x =?
c. $41x - 21 = 61$ x =?
d. $41x + 21x - 62$ x =?
e. $61 - 41x = 21$ x=?
f. $21x + 21 = 63$ x =?
g. $40x - 21 = x + 61$ x =?
h. $\frac{2x}{3}=\frac{3}{2}$ x =?
i. $\frac{12}{x}+\frac{13}{x}$ = 5 x =?
j. $\frac{12}{x}+\frac{13}{x}$ = 25 x =?
k. $\frac{12}{x}-\frac{23}{2x}=0.5$ x =?
l. $\frac{7}{a}-\frac{13}{2a}=\frac{1}{2}$ a =?
m. $\frac{b}{13}+\frac{10b}{39}=3$ b =?
n. $\frac{b}{13}+\frac{10b}{39}=\frac{1}{3}$ b =?
o. $y = y^2$ y=?
p. $y^2 - y = 0$ y=?
q. $y(y-1) = 0$ y =?
r. $d^3 = d^4$, $d^4 - d^3 = 0$, $d^3 (d - 1) = 0$ d=?
s. e = 2f, f = 2g, g = 2h, If h=1 e=?

t. e = 2f, f = 2g, g = 2h, If e = 64, h =?

30.13 More Substitution

1. y = 8a + 7b. Find y, if a = 5, b = 2
2. y = 8a + 7b. Find y, if a = $\frac{1}{2}$, b = $\frac{1}{7}$
3. y = 8a - 7b. Find y, if a= 5, b = 2
4. y = 8a - 7b. Find y, if a= $\frac{1}{2}$, b = $\frac{1}{7}$
5. y = 4a – 5b + 1. Find if a = b = 1
6. y = 4a – 5b + 1. Find if a = 3, b = 2
7. y = $a^2 + 2ab + b^2$. Find if a = 5, b = 2
8. y = $a^2 - 2ab + b^2$. Find if a = 5, b = 2
9. y = $(a + b)^2 - (a - b)^2$. Find if a = 4, b = 3
10. y = $(a + b)^2 - (a - b)^2$. Find if a = 2, b = 1

30.14 Addition – Bigger Terms

1. Add $a^2 + 2a + 2$ and $5a^2 - a - 2$
2. $(a^2 + ab + b^2)$ and $(19a^2 - ab + 19b^2)$
3. $(5a^2 + 4a + 3b^2 + 2b + ab)$ and $(5ab + 4b + 3b^2 + 2a + a^2)$
4. $(9a^2 - 8a + 7b^2 - 6b + 5ab)$ and $(9ab + 8b - 7b^2 + 6a - 5a^2)$
5. Add All – (a + b +c), (a – b + c), (a – b - c), (c – a – b), (c – b – a)

30.15 Simplify

1. a x 3 x 5a
2. b x 3 x 5b x a x 6a
3. a x (-3) x 5a
4. a x 3 x (-5a)
5. (-b) x 3 x (5b) (-a) (6a)
6. (b) x 3 x (5b) (-a) (-6a)
7. (ab) x (bc) x (ca)
8. (2ab) (2bc)(2ca)
9. a (b – c) + b (c – a) + c (a – b)
10. $\frac{a}{b} \times \frac{b}{c} \times \frac{c}{a}$
11. $(-\frac{a}{b})(-\frac{b}{c})(-\frac{c}{a})$

30.16 Using Known formulas

1. $(x + 4)^2$
2. $(x - 4)^2$
3. $(a + 2b)^2$
4. $(2b - a)^2$
5. $(a - 2b)^2$
6. Find $(10.2)^2$ [Clue: Use $(a + b)^2$ formula]
7. Find $(9.8)^2$ [Clue: Use $(a - b)^2$ formula]
8. Find (10.2) x (9.8) [Clue: Use (a + b) (a - b) formula]
9. Find (994.5) x (1000.5) [Clue: Use (a + b) (a – b) formula)
10. $(a + b)(a - b)(a^2 + b^2)$
11. Factorise $x^4 - y^4$
12. Factorise $4a^2 - b^2$
13. Factorise $9x^2 - 4y^2$
14. $(x + \frac{1}{x})^2$
15. $(x - \frac{1}{x})^2$

30.17 Some verbal problems (here 'verbal' = given in words)

1. As in May 2009 (Mysore) prices are as follows: 1 kg sugar Rs. 30, 1 kg rice is Rs. 38, 1 coconut Rs. 10. A person buys 5 kg rice 2 kg sugar and 1 coconut. What did he spend?

2. Sum of two numbers is 15. One number is twice (= two times) the other. What are the numbers?

3. Chota ate some idles. Beta ate double of what Chota ate. Daddy ate 4 more than Beta. Mummy ate 1 less than Daddy. Total 14 idles were finished. How many did each one eat?

4. Boys and girls in each section of a school are given:
   Sec A: 30 boys, 12 girls
   Sec B: 28 boys, 12 girls
   Sec C: 32 girls, 8 boys.
   How many students (total) are there in the school? How many boys and how many girls?

5. The ratio of the ages of grandfather and grandchild is 5. 15 years from now, this ratio will become 3. What are their present ages?

6. In one section the number of boys is 9 times the number of girls. The total number of students is 30. How many girls are there?

7. In this problem, 1 girl left and in her place 1 boy joined. Now what is the ratio of boys to girls in the class?

8. In the first year electronic section, the number of girls was twice (=2 times) the number of boys. First year strength (=total number of students) was 30. How many boys, how many girls?

9. In the above electronics section, after annual exam. Class has less number of students. This is because 50% of boys failed (all girls passed). What is the total number of students in the second year? What is the ratio of girls to boys?

10. Akka's weight is 10kg more than that of Thangi. Both together went to a weighing machine and to save money, got on to the machine. Total was 70 kg. can they find out each one's weight?

11. Sum of a number and its square totals to 110. Can you find the numbers? If this total is 30, can you find the numbers? If this total is 2, can you find the numbers?

12. Keep your age a secret. Add 5 multiply by 2. Add 90. Tell me the result. I'll tell you your age. How?

30.18 Substitution in Formulas.

1a. $I = \frac{PTR}{100}$ P=1000, T = 1, R=20 Find I.

1b. In the above A = P + I Find A.

2a. V = R x i
V = 250
R = 1000
I =?

2b. In the above is obtained as amperes ('unit'). How many milliamperes? 1 ampere = 1000 milliampere

3a. F = ma
m = 1
a = 5
F =?

3b. In the above F is force, m is mass. What is the ratio of forces on a Kg mass and a ton mass (for the same a) [ton = 1000 kg]

4a. T = a + (n – 1) d a =1, d=1, n=10, T=?

4b. In the above T was called 10th term. Find 12th term.

5a. $V = u + at$
$u = 0$ $t = 10$ $a = 5$ $V=?$

5b. In the above t is time in seconds; V is velocity (=speed)(after t seconds). What is V after 2 seconds.

6a. $R = M + L \times V$ $M = 5.5$, $L= 0.1$, $V= 8$, $R=?$

6b. In the above, R_1 was for V =8, R_2 was for V =4 (all the others are the same) what is $R_1 – R_2$?

7. $y = mx + c$ substitute in this equation.

1. $c = 0$ $x = 1$ $y =?$ $m = 1$
2. $c= 0$ $x = 4$ $y =?$ $m = 1$
3. $c = 1$ $m = 1$ $x = 1$ $y =?$
4. $c = 1$ $m = 1$ $x = 4$ $y =?$
5. $c = -1$ $m = 1$ $x = 2$ $y =?$
6. $c = -1$ $m = 1$ $x = 0$ $y =?$

8. $y = x^2$, substitute in this equation

1. $x = 0$, $y =?$ 2. $x =1$, $y =?$ 3. $x =3$, $y =?$ 4. $x = 4$, $y =?$
5. $x = -1$, $y =?$ 6. $x =-4$, $y =?$

9. $A = \pi r^2$ Value of $\pi = \frac{22}{7}$

a. Find A, if r = 7cm. What is the unit of A here?
b. Find A, if r = 7 km. What is the unit if A here?

10. $V = \pi r^2 h$ $\pi = \frac{22}{7}$

a. V =? If r = 7cm, h =10cm. V is expressed in what units (in your answer)?
b. V =? If r = 7m, h =10m. V is expressed in what units (in your answer)?

11. $A_1 = a^2$, $A_2 = l \times h$
If a = 50, A_1 =? If l =40, b =60, A_2 =? Which is larger A_1 or A_2?

30.19 Solving Simple Equations.

1. $x + y = 5$ If $x = 3$, $y =?$
2. $x – y = 1$ If $x = 3$, $y =?$
3. Given $x + y = 5$ and $x – y = 1$. Find x and y [Clue: Add (LHS + LHS) = (RHS + RHS)].
4. $x – y =0$ If $x =5$, $y =?$
5. $x – y – 2 = 0$ If $x = 5$, $y =?$
6. $x^2 = 4$, $x =?$
7. $x^2 –4 = 0$, $x =?$
8. $x^2 = a2$, $x =?$
9. $x^2 = (a+5)^2$, $x =?$
10. $x^2 + a^2 + 10a = (a+5)^2$, $x =?$
11. $x^2 + a^2 – 10a = (a-5)^2$, $x=?$
12. $x^2 = y + 1$; $y = 8$, $x=?$
13. $x^2 + y^2 = 20$; $x^2 – y^2 = 12$; $x =?$ $y =?$ [Clue: do as in 3 above]
14. $x^2 – y^2 = 0$ If $x = 5$, $y =?$
15. $x^2 – y^2 – 2 = 0$ If $x = 5$, $y =?$
16. $x^2 – a^2 = b^2$ If $a = 1$, $b = 2$, $x =?$
17. $x^2 + (a - b)^2 = (a+b)^2$ If $ab = 4$, $x =?$ [Clue: Use formulas of $(a+b)^2$ etc and simply

==

Chapter - 31 **Basic Geometry**

31. Activity:
Make use of a dictionary (Nowadays, the 'on-line', Internet in computer, facilities contain 'dictionary' also even on the toolbar). Find the meaning of geometry and fill it up here.

31.1 Some important aspects. Geometry makes use of some words (technical terms). Since geometry is a very old subject, these technical terms have become commonly used words (in all languages of the world). So, many persons do not give much respect (thought, attention) to these. We will mention the more important ones very briefly. They are: point, line, angle, surface, area, plane, volume, shapes etc.

31.2 Point:
It shows a location. Mathematically it has no dimension i.e., it is too small (to occupy space). For our purpose, it should be seen. Points can be identified by numbers or letters. Usually capital letters (of English) are reserved for point. Eg: A, B …. P, Q etc. But O (oh!) is special. It is usually used to show starting point.

Activity:
Students can try to list out the places where points are seen (to have a function). A game can be played. 2 groups say the context and describe. Next chance for the other team. Eg: Team1 – point seen in India Map for Mysore. Team 2 – Ok. Now, Delhi has a bigger point in the same map.
Ok, etc. Some checklist. Starts in the sky/ starting of a rangoli / edges of a sharp object / mark made by a divider / polka dots on a dress / bindi.

31.3 Line:
A large number of points in contact with one another (only in one direction). This kind of geometrical description (another version: "set of adjacent points along one direction") is ok to learn and understand. For practical purposes (such as in science and engineering). "Line is a connecting link between 2 points".

31.3.1 Note for teachers:
Strict mathematicians 'Definition of Line' gives us, ordinary mortals, headache because they have hijacked this term to mean an unending entity (may be in either direction). What we call AB, a line is called by them "line segment". My request to the teachers: "please do not use such accurate statements". Use "ray" only in physics or engineering (not for groups of lines with a common point nor for a single line with direction). "Segment" is a frightening word. Do not use it for AB etc. Call AB as a "portion" or "part" of a much bigger line say CD etc.

[If you so desire, "segment" word can be reserved for circle. Similarly "sector" also].

31.3.2 To the students (Points and Lines). Points are represented by capital letters.

• • • •
P A B • R • S
Q

When your main aim is to draw a line, and points are only needed up to making a line, points need not even have a name.

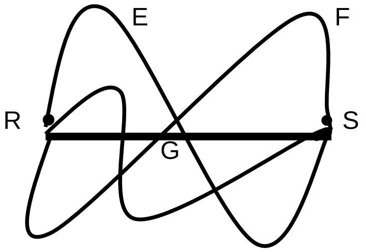
Thus you can use a divider, lightly mark 2 points, join them and erase (or ignore) the points. This is because: The simplest way of joining 2 points is a straight line. It gives you the shortest distance also. If you are given 2 points, you can join them and thus draw a straight line.

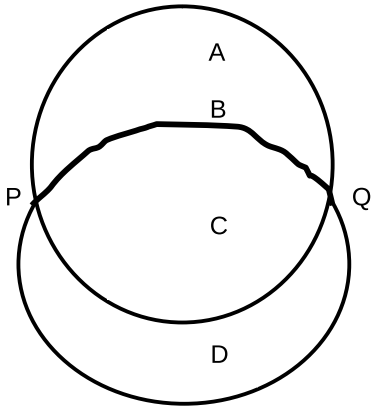
31.3.3 Kinds of lines:

a. Straight Line – you use a scale or a real edge of any object (setsquare, side of a plate or block) to draw.

b. Curved line – any two points could be joined by curved lines also. There is only one straight line passing through 2 points. But there can be many lines between any 2 points.

One Straight Line





Many Curved Lines

One straight line and many funny lines AB, RS are the shortest lengths between A & B, and R and S respectively. [“Respectively” means case by case, one by one, in the same order as given. Thus AB line for A & B points, RS line for R & S points].

31.3.4 Curved lines require some more help to identify. Thus PAQ, PBQ, PCQ, PDQ are curved lines between P & Q (in the figure of section 31.3.3).

You can draw these lines, freehand (i.e. using own skilled hands, like an artist) or you can use “FRENCH CURVES”. These are a set of funny shaped tools with neat, smooth edges. You can use these to draw PAQ, PDQ etc.

RES, RFS, RGS etc are complicated curved lines (some are like waves). To draw them you may have to use 2 or more French curves and different curved edges.

31.3.5 Activity:

A. Bring a set of french curves and let the students play with them.
B. Let each student draw sets of points AB (5 cm apart); PQ (10 cm) RS (15 cm). Let them draw all kinds of lines between the pairs of points.

31.4 How to write points:

31.4.1 Points: Many symbols are used:

• Closed circle or bullet
O Open circle
■ Closed square
□ Open square
▲ Closed triangle
△ Open triangle
❋ Star
X Into symbol
+ Crosswire symbol
⊙ Concentric circle method / bullets eye
⊗ Closed cross
⊕ Closed crosswire

Use minimum size when length is important. Use suitable size when length points themselves (i.e. their location) is important.

31.4.2 Activity:

In maps, many conventions are used.
a. Students can see an atlas and see some.

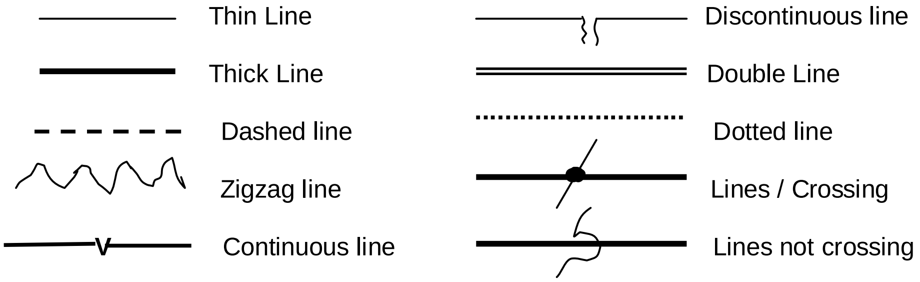
b. Tourist maps have special attractive symbols.

31.5 How to read points:

One point only P• ‘Point P’
Many Points A• B• C•
Points A, B, C or points A, B and C or point A, point B, point C (with ‘and’ if you want) or two point A and B or three points A, B and C etc.

31.6 How to write lines:

31.6.1 Kinds of lines



31.6.2 Activity:

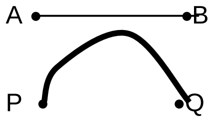
a. Take any map. See different lines given there. Eg: Railways, major roads, minor roads, airlines routes, sea routes etc (use a good atlas).

b. Take an electricity or electronics book. See the drawings there and discuss.

c. Take a few drawings (‘blue-prints”) from the draughtsman’s table and see how he has drawn the lines.

d. See a scale drawing from a civil engineer.

31.7 How to read lines:



Line AB, straight line AB.

Line PQ (in the figure); curved line PQ, Arc PQ

31.8 For Teachers:
[This section is for teachers and self learning students].

31.8.1 Geometry Box:
Many things we take for granted (i.e., we assume that we know – like how to eat or drink). One of these is how to use a geometry box, and each one of its contents. Some students may need help even in this simple thing. So there is an activity at the end of this chapter. Teachers should read that and decide whether his class needs it (to be told in the beginning itself).

31.8.2 How to measure distance:
Distance between 2 points is always the shortest distance between them i.e., the length of a straight line through them. Graduated scales are available only for that purpose. If the points are on paper you can draw the line and measure with a scale or use a divider and go to a scale. If the points are on a machine, instruments remote place etc use some types of calipers available.

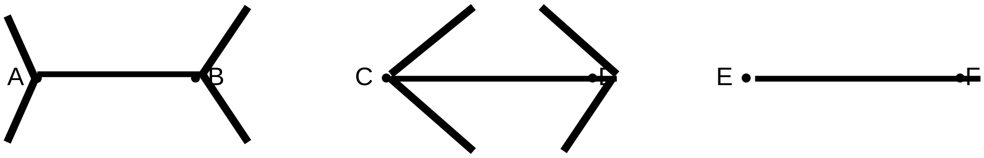
Activity:
Teachers! Kindly take time to show and demonstrate dividers and calipers

31.9 Exercises:

a. Draw lines of length 2 cm; 4 cm; 6 cm.
b. Name the lines AB, PQ, XY.
c. Show 2 points A & B, 2 cm distant.
d. Indicate point p. Mark a point Q, which is 4 cm from point P.
e. The distance between 2 points X and Y is 6 cm. Show these points.
f. Let one student A draw a straight line and write down the length and keep it a secret. Student B should guess the length. If it is (within 20% or so) approximately OK, he can measure and verify.

31.10 Activity: (for fun)

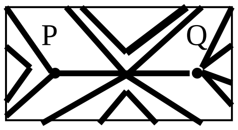
a.

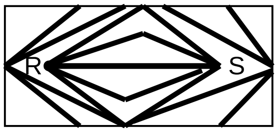


Let students produce this chart (secret AB = CD = EF). Let others guess.

b. Let some charts be AB really greater than CD; some others CD > AB > EF.

c.





Let the students produce charts and (illusions) like these measure PQ, RS.

d. Add you own ideas.

31.11 Exercises and Activities:

31.11.1 Exercises:

1. Draw a line XY. Measure the length both in inches and cm. Write
   Length = Inches = Cm =
2. Using (1) above find 1 inch = …. Cm
3. Let many students do (1) and (2) above.
4. Let the students (or teacher) collect all the results of (2) and make an average value.

31.11.2 Activities:

a. Go and learn from draughtsman how to draw thick lines, thin lines etc. learn how he draws parallel lines.
b. Use 2 setsquares and draw parallel lines passing through various points.
c. Take a fairly large map of India. Measure the distance between Delhi and your place (in cm). Use the scale given in the map to find actual distance.
d. Do (c) above in your own state.
e. Do (c) above in your own city.
f. Learn to measure curved distances using threads or flexible fine wires. Use this to find road or rail distances between 2 points in a map.

31.12 Angles

31.12.1 We saw that many points make a line and lines can be straight or curved.

a. Parallel lines: We have already learnt how to draw parallel lines. Now to define: Two straight lines, which will never meet, are parallel lines.

   Activity:

Teacher can just allow students to actually see or simply imagine where there are parallel lines. Let them make a list.

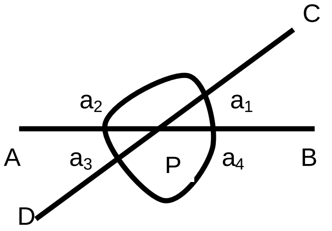
After list is made, cross check if the following items are mentioned: Railway lines, edges of tiles, book, notebooks, floor, ceiling, walls, lines in notebook paper, highway lanes.

b. Intersection: Two lines can cross (= cut) each other. It is called intersection. Two curved line can intersect at many points. Two straight lines can cut only at one point. This called point of intersection.

Activity:
Take two parallel threads; test their strength. Let them intersect at one point; test their strength. Let them interest at many points; test their strength. Go and see twisted ropes and guess why they are twisted. Eg: binding ropes, tug-of-war rope, steel ropes used in cranes, elevators, big machines.

31.12.2 Angles:

When two straight lines intersect, angles are seen. Angle can be understood as a tilt, in relation to another.



In this figure a_1, a_2, a_3, a_4 are the four angles made by the lines AB & CD intersecting at point P.

How to write: angle a1 is written as $\angle CPB$ or $\angle BPC$ (i.e., point of the angle in the middle). Some persons write $C\hat{P}B$ or $B\hat{P}C$.

Exercise:
Write the names of the other angles a_2, a_3, a_4

Types of Angles:

Right Angle: is the most important angle. Any worker should know this. Vertical means straight upwards. Horizontal means "straight down on the ground". The angle between true vertical and true horizontal is the right angle.

Activity:
Let all the students see or imagine right angle and show or name them. Make a list. After this list is made crosscheck if the following have come:
The English letter capital L, edge of a book or notebook, corners of tables, joint of floor and wall or wall and wall, standing at attention, fingers (shown as when we talk of right hand or left hand rule of Fleming in Physics) many joints. [More the messier]

Acute Angle: Any angle less than the right angle.
Obtuse Angle: Any angle bigger than the right angle.

Activity:

1. Indoors, working in Pairs. One person A slowly opens (then closes) a study notebook. Referee asks "stop" student B should say what is the angle [acute, obtuse or right].
2. Outdoors, all in line. Hand exercise (or red cross exercise). Referee says stop. What angle? (i.e., angle between the hand and the body i.e., trunk). If the student says the correct thing, referee can ask everyone to say in chorus. Eg: what angle? OK All

   How to measure angles: Angles are measured in degrees. i.e., unit of angle is degree.

[Caution: The same word is used to measure temperature also. Meaning must be understood according to context (= as per the situation, knowing where it comes].

[To the teachers: Radian as a measure of angle can wait until trigonometry comes].

1 right angle = 90^{o}
2 right angle = 180^{o}

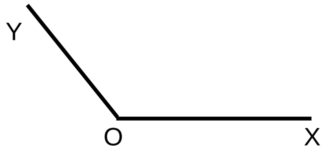
Acute angle = <90^{o}
Obtuse angle = >90^{o}

Angle Measure:

90^{o}

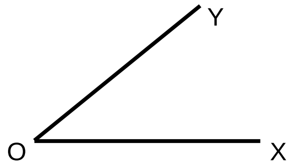
$$\angle XOY = 90^{o}$$

>90^{o}



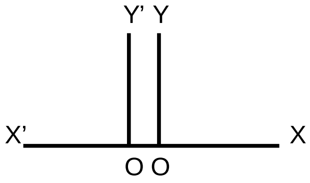
$$\angle XOY > 90^{o}$$

<90^{o}



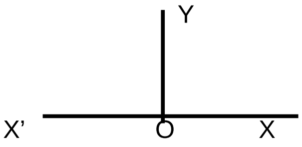
$$\angle XOY < 90^{o}$$

180^{o}



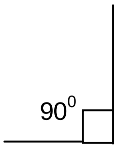
$$\angle XOY + \angle X'OY'$$

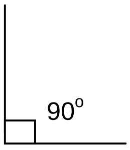
Same as

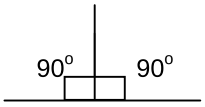


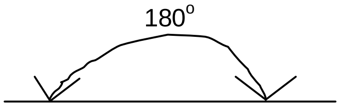
XOX' = 180^{o}

Rule: a horizontal line makes angle of 180 degree. A horizontal line and a vertical line make an angle of 90 degree









[For teachers: This elementary book has no use for terms like complementary and supplementary angles].

31.13 Areas:

Many points (dots) close together makes a line. If they stand touching, a horizontal line may be formed. If they sit on top of the other a vertical line may be formed. Imagine many lines close together. A total area may be formed.

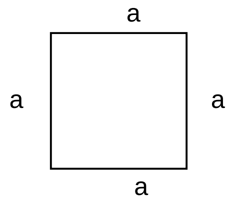
Another way of explaining an area is fencing. You make fence. If the fence goes around and returns to the original spot. You have covered an area.

In real life, areas come in all shapes and sizes. But when man makes, he likes to be neat and methodical. There geometry helps.

Activity:
Take a map of the world. Compare subdivisions of USA, Australia etc. Compare with other continents. Can we make a statement like this? "Natural boundaries are usually irregular (or curved lines). Man-made boundaries tend to be regular (or straight lines).

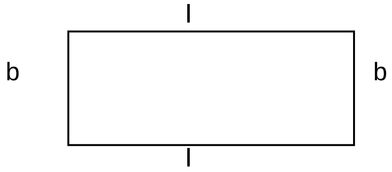
31.13.1 Some geometric figures:

1. Square: Sides equal. All angles are right angles.



Area A = a^2

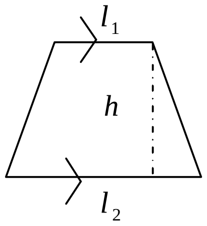
2. Rectangle: All angles right angles opposite sides equal. Longer side is called Length. Smaller side is called Breadth



Area = l x b

[For teachers! Sometimes it is easier for students to understand, if you explain some terms and their meanings. Here you can help. Word 'length' and 'long' are related. Word 'breadth' and 'broad' are related]

3. Trapezium: Two sides are parallel. Area = h x $\frac{l_1+l_2}{2}$



l_1, l_2 = Lengths of parallel sides.
h = vertical distance = height

Two other very important shapes are Triangle and Circle. These are very Important. So separate chapters.

31.14 Any four sided figure. The name given in maths books is Quadrilateral. This literally (= actually, in fact to the letter) means 'Four – Sided'. So we can use when necessary, the term "Four Sided Figure".

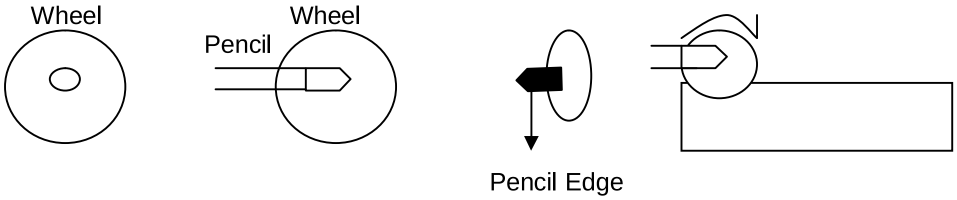
31.15 Many sided shapes. They are called polygon.
[Students! Learn this word. Higher studies require this. Poly means 'many' as in polytechnic, polygamy, polypeptides etc].

31.16 Activity:

a. (Identify the shape). Collect some sketches, photographs, building plans, elevations, some objects, nuts, cut plates, randomly cut cardboard etc.
Let the class identify. To make it interesting, take a piece and ask from a list: Is this a triangle? Is this a square? Is this a circle etc?

b. Extend the idea of (a) to objects around. Eg: cycle spokes, wheels, table, window.

c. Fun activity: Take a circular object with a small hole at the center. Eg: A metal washer, spool of a sewing machine. If you don't find one, make one. Fix a pencil tightly into it. Place this arrangement on a scale (or a thick straight-edge). Ask someone to roll it while you hold the pencil tightly. See what you get.

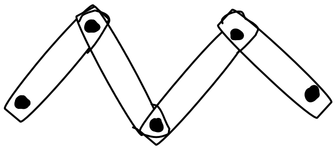


Now a third person holds a cardboard (or paper) touching the pencil's top.
Result: Did you get a straight line? Ok

d. In (c) above make a hole away from the circle. See what you get?
Result: Did you get a wavy pattern? Ok

e. (c) & (d) above can be done with a throw away CD disc also.

f. Get thrown-away ice cream spoons and wash them in soap & dettol water, dry them. Now make as shown. You have to carefully make holes and join at the ends. Play with angles.



31.17 Geometry box: [Teachers! Some students might not have been properly told about the various items and their correct uses. Even if some persons know, it is good to say them, though briefly].

1. Scale – to draw a line, to measure length (even $\frac{1}{16}$ of inch, 1 mm).
2. Setsquares – draw perpendicular (90^o angle) angles 45^o, 30^o, 60^o.
3. Protractor – draw / measure any angle from 0^o to 180^o – least is $½^o$.
4. Compass – with a pencil – to draw circles (arc = part of a circle).
5. Divider – To mark points, to accurately draw lengths, to "Lift" length.
6. Extras – Now a days template are also given along with the above items. At least there are 2 types LETTERS, GEOMETRIC FIGURES.
7. [Teachers can bring calipers and french curves, markers, metal markers, line markers and just mention where they are used].
8. Enterprizing teachers can include in (7) above plumbline and spirit level, mind rafter.

31.18 Exercises:

1. •Q

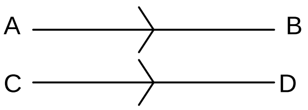
P• •R

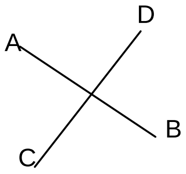
•S

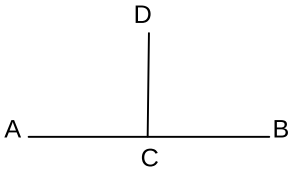
Students can make their own questions like this one.

Guess the lengths PQ, PR, PS and write it down. Now measure and compare. Within 10% 2 marks, 20 % 1 marks, >20% zero.

2. How many lines can you draw (a) starting from a point (b) passing through a point?
3. What are these pairs of lines? (Name)

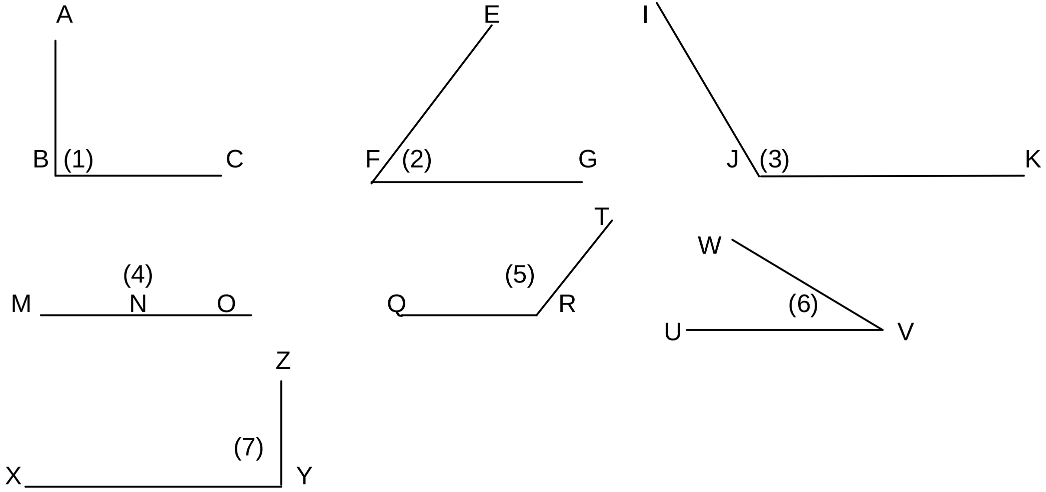






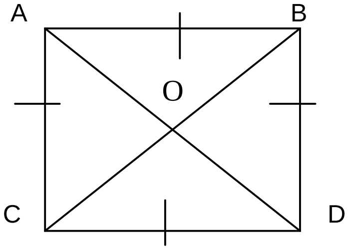
4. What kind of line? (Thick, dotted….)

5. What kind of angles? (Acute etc)



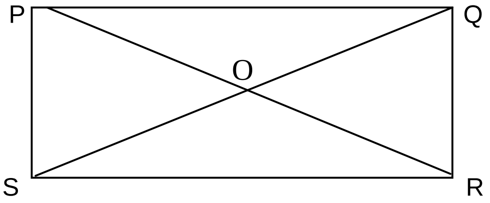
6. In equation 5 above – approximately, how many degrees? (Range is given).
Eg: 90^{o} - (1) 0 - 89^{o} = 90^{o} = 91 – 180^{o} =

7.



a. In this figure, how many angles are there? Choice….
Ans: (a) 4 (b) 8 (c) 12 (d) 16
[Note: 180^{0} is not counted as angle]
b. How many right angles?
c. How many acute angles?

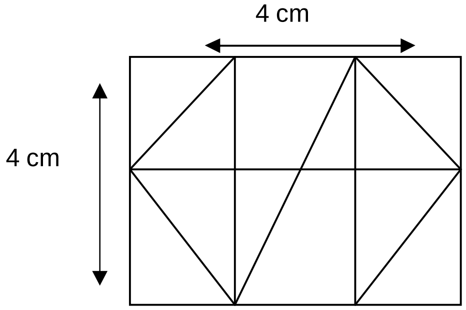
8. Same as (7) but rectangle



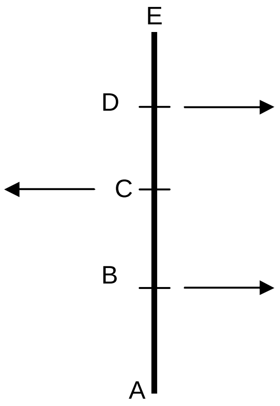
9. Draw a line (any length) AB. At point A draw a line making 60^{o} with AB. At point B draw a line making 60 with AB. Let intersection point B. What did you get? Measure lengths CA and CB. Measure AB. What do you see? Measure angle at C, i.e., $\angle ACB = ?$

10. Draw the figures of (a) Rectangle (b) Square (c) Rhombus (d) Trapezium (e) Circle (free hand drawing).

11. In this Figure, what are the Shapes do you see? How many each?



12.

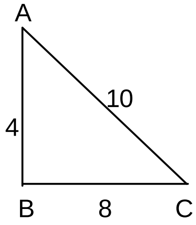


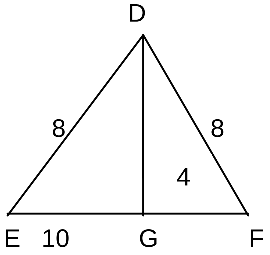
A to E continuous measure B 100, C 200, D 300, E 400.

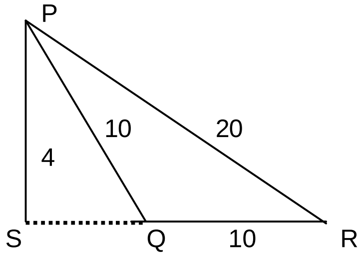
F at B 100
G at D 200
H at C 300 (all in meters)

Complete the figure and find the total area in Sq. m.

13. If (12) is too tough, try this

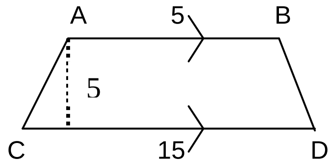


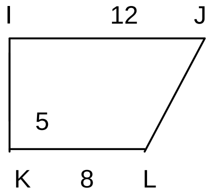


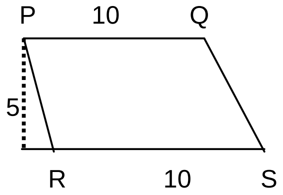


Calculate the areas of ABC, DEF & PQR.

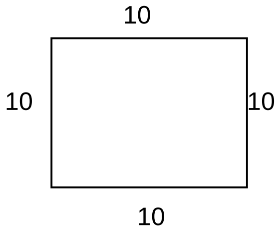
14. Calculate the areas (length are given)

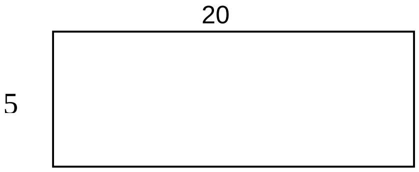






15. Calculate the areas (lengths in meters)





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Chapter - 32 Squares and Area

32. Activity:
Allow the students to consult an English dictionary. Let them write down the meanings of '**square**' first; and then '**cube**'.

32.1 We have already seen in algebra.
If $x = 4$
$x^2 = x \times x$
$= 4 \times 4$
$= 16$

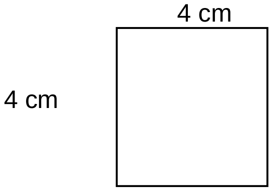
One thing Same thing $\boxed{\cdots}^{2}$

X =

Multiply something by itself or write down twice and multiply

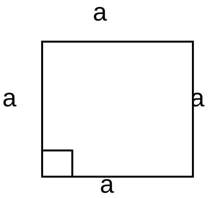
We will not discuss whether algebra used the word first or borrowed it from geometry. We will just appreciate and understand that there is close relationship.

32.2



The name of this shape is **SQUARE**.
Area of the square = 4 x 4 = 16 Sq. cm
(**Sq. cm = Square Centimeter**)

In general Area = a^2



If a is in **cm**, area is in **sq. cm**
If a is in **m**, area is in **sq. m**
If a is in **foot**, area is in **sq. ft**
If a is in **km**, area is in **sq. km**
If a is in **miles**, area is in **sq. miles**

32.3 Graph sheets – Activity:

A. Teachers, show graph sheets.
Let students count bigger squares, smaller squares etc.
Let them verify the formula of 32.2

B. Bring inch graph sheets and bring cm graph sheets. Cut cm graph sheet to fill into 1 inch X 1 inch square on inch graph. Count now the number of cm squares (count small squares). Fill up:

$\therefore$ 1 (inch) ^{2} = ……. (cm) ^{2}

32.4 Aid for approximation: Make a list of squares.

x	x^2
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81

x	x^2
10	100
20	400
30	900
40	1600
50	2500
60	3600
70	4900
80	6400
90	8100

Students, please note that the first list must be memorized. Did you see that the second list can be generated from the first?

Exercise:

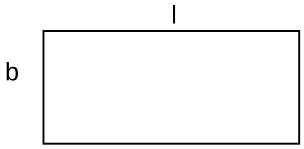
Example: $\sqrt{400}$ =? $\sqrt{4}$ = 2 $\sqrt{100}$ = 10; $\sqrt{400}$ = 2 x 10

$\sqrt{4000}$ =? This is not easy. This cannot be done by $\sqrt{4}$ $\sqrt{1000}$. Instead make it $\sqrt{100}$ x $\sqrt{40}$ = 10 $\sqrt{40}$. $6^2 = 36$, $7^2 = 49$ (see from list1). Therefore $\sqrt{40}$ is >6 and <7 say 6.5. Therefore $\sqrt{4000} \approx 10 \times 6.5 \approx 65$.

Find the approximation:

a. $\sqrt{3}$
b. $\sqrt{5}$
c. $\sqrt{7}$
d. $\sqrt{13}$
e. $\sqrt{90}$
f. $\sqrt{300}$
g. $\sqrt{500}$
h. $\sqrt{700}$
i. $\sqrt{1300}$
j. $\sqrt{900}$
k. $\sqrt{3000}$
l. $\sqrt{5000}$
m. $\sqrt{7000}$
n. $\sqrt{130}$
o. $\sqrt{9000}$

32.5 Rectangles:



Area A = l x b
= Length x breadth

Properties of rectangles:

1. Opposite sides are parallel.
2. Opposite sides are equal (in length).
3. All the four angles are right angles.
   Area formula we have seen earlier

32.6 Practical situations: Activity

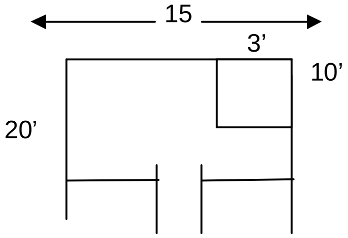
Measurement of area is very important in many fields. In day-to-day life, let the students list down instances. After allowing the students to make the list teacher can check whether at least the following have come in:

a. House site
b. Town area
c. Fields in areas
d. Room sizes
e. Painting surfaces
f. Carpentry
g. Cloth, textile

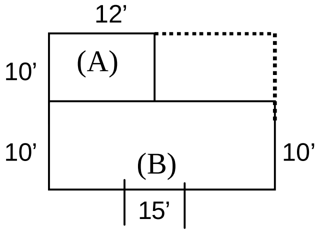
For most of the items discussed in 32.5 above, area can be measured or calculated, based on paper map or plan. These are called scale drawings.

Example:

A. Room plan is given. A small area is left for attached bathroom. Dimensions are given to scale. Room's ground area is to be tile. Tile cost is 50 rupees per sq. ft (including all work). Calculate the cost.



Areas:



Area A = 10' x 12' = 120 Sq. ft
Area B = 10' x 15' = 150 Sq. ft

Total Area = 270 Sq. ft

Cost / Sq. Ft = Rs. 50
Total cost of tiling = 270 x 50 = 13,500

Exercises:

1. In the example (A) given above, calculate the cost for bathroom also.

2. In (1) above, for 5' height all round, walls are tiled. Tile cost is 20 Rs / Sq. ft. Calculate the cost?
3. All the inner walls except the bathroom are to be painted. Room height is 10 ft. Contractor takes Rs. 150 / sq. ft. Calculate the cost of painting?
   [Clue: Forget ceiling; assume doors also as area (over estimate is ok).

Graphical Method: Area

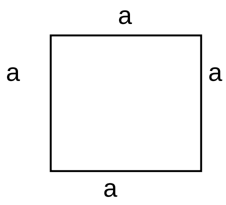
Activity:

For any shape, regular or irregular, graph sheet method is easy & versatile. Counting of small (millimeter) squares (if the size in centimeters). Count Cm square if the shape is very large. This method is very useful to actively involve students in verifying the different formulas. Let some other students do as above for squares, rectangles, rhombuses, trapezium any four sided figure. Some other do for various sized circles.

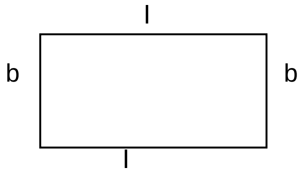
For irregular shapes, drawing on the graph sheet helps. (Scale drawing is an art worth learning. Students of any discipline can learn this. It is quite useful).

32.7 Perimeter (Also called circumference):

32.7.1



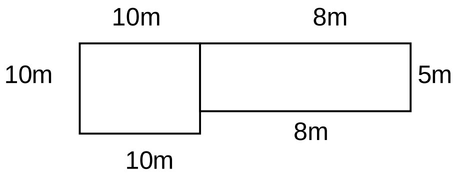
Perimeter of a Square = 4a



Perimeter of a rectangle = l +b +l +b
= 2 (l +b)

32.7.2 Exercises:

A. Find the perimeter of a square of side 10 meters?
B. Find the perimeter of a rectangle of sides 5 m and 8 m?
C. In (A) & (B) above, assume these are sites (for house). Steel fencing is to be done. Fence cost is quoted by contractor as Rs. 50 per running meter. What are the costs of fencing for (A) and (B)?
D. What is the cost if the site was as given: [Clue – one fence in overlapping boundary is enough].



32.8 Exercises:

A**.** A site has an area of 2500 sq. ft. It is square shaped. Draw the map. (Show lengths).
B. In (1) above area is only 2400 sq. ft. One side measured was 40 ft and the shape is a good rectangle. Draw the map.
C. In (1) & (2) above calculate the cost of fencing [Rs. 20 per running foot].
D. Do (1) & (2) in meters.
E. Suggest a simple method of finding the perimeter of irregular shapes.
   [Clue: Go back to 31.11.2 (f) and see how one can use thread of wires].

Chapter - 33 Triangles

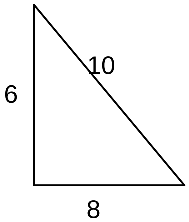
33.1 Its name itself is descriptive. Funny thing is it is called 'Trilateral' (or trisides) in many of (= Indian) languages. Liberally meaning 'a figure haying 3 angles' or in the other 'a figure having 3 sides'. This statement is true of any polygon. Properties of a triangle:

a. 3 sides
b. 3 angles
c. Sum of 3 angles = 180^0
d. Sum of 2 sides always > third side.
e. Area = ½ x b x h
Where b = base
h = height

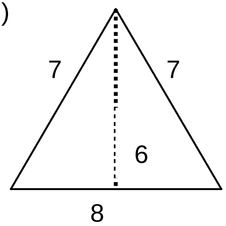
33.2 Exercises

a. Calculate the areas of triangle:

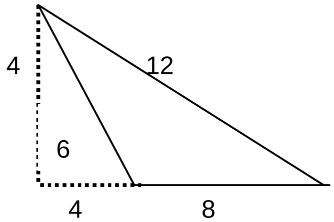
(a1)



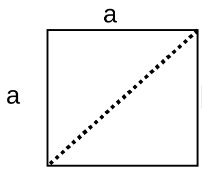
(a2)



(a3)



b. Using formulas show that the area of a square = a2

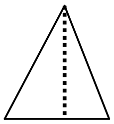


[Clue: Make into 2 triangles]

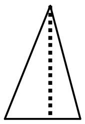
c. Same as (b) but rectangle area = I x b. [Clue: Use only area of triangle formula].
d. Do the same as (c) for a Trapezium.

33.3 Activity: Area of Triangles:

Let the students take different shapes and sizes of triangles. Measure the base and height. Use formulae for area. Measure area by graph method.



1



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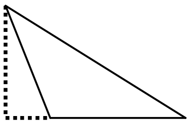

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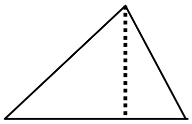

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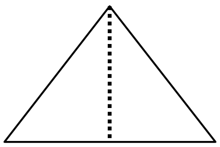

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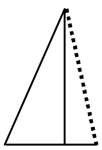

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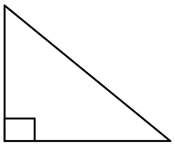


7

S. No	Base cm	Height cm	Area by formula	Count of small squares	Area by graph
1					
2					
3					
4					
5					
6					
7					

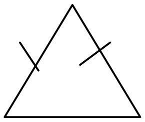
33.4 Types of triangles:

Equilateral - All sides equal, ∴ all angles equal, ∴ each angle = 60^o
Isosceles – 2 sides equal, ∴ 2 angles equal
Right angled – one angle is 90^o, ∴ special properties.
Acute angled – All angles are less than 90o. [∴ equilateral, isosceles are all in this].
Obtuse angled – one angle is > 90^o
Scalene triangle – Any ordinary triangle

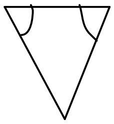
Exercises:

I. Name these triangles (i.e., say to which category each one belongs):

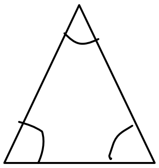
A.



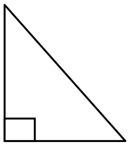
B.



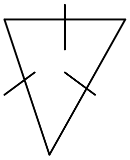
C.



D.



E.



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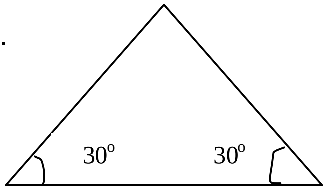
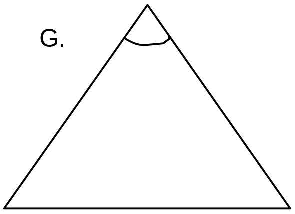
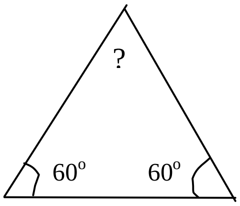


G.

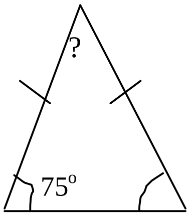


II. Find the third angle (shown by?)

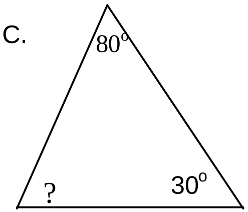
A.



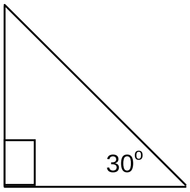
B.



C.



D.



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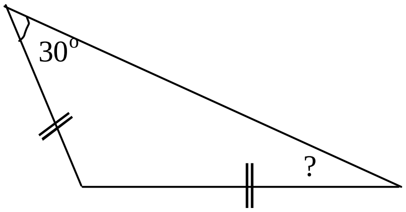
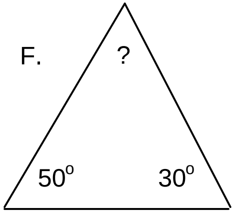
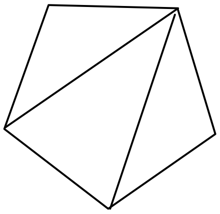


F.



III. Show that any polygon can be divided into triangles. Worked examples:



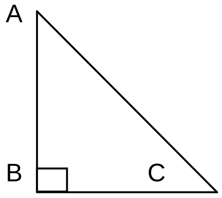
Pentagon = 5 sided figure
Number of triangles = 5 – 2 = 3

A. Quadrilateral B. Hexagon (Hexa = Six) C. Octagon (Octa= 8)

33.5 Right angled triangle:
This is very special. It is seen in every practical situation.

Pythagoras theorem says, if you know 2 sides of a right angled triangle, you can calculate and find out the third one.

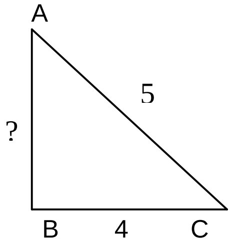
Thus



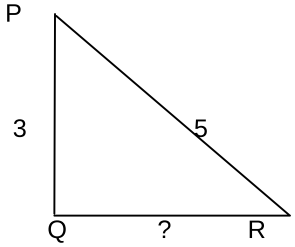
AC2 = AB2 + BC2
(AC is the longest side, of course)

Exercises:

1. 2 sides of a right angled triangle (rat) are 3 cm and 4 cm. Calculate the third?

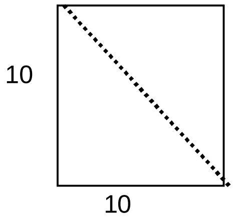


AB =?



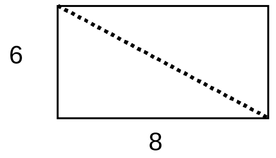
QR =?

2.



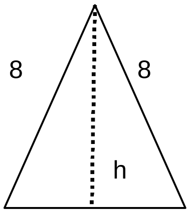
Calculate the diagonal of the square?
Calculate the other diagonal also? Are they equal?

3.



Calculate the diagonal of the rectangle?
Calculate the other diagonal also?
Are they equal?

4.

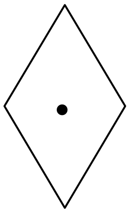


Calculate the height h.
Then calculate the area of triangle.

Chapter - 34 Circles

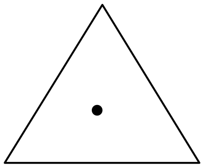
34.1 Is it not strange! Zero and Circle are similar! Something more strange:
Fix a rope with a nail at one end. Rotate the other end (rotate = go round and round) what do you get?

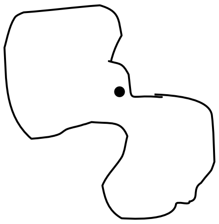
Take a stick: Fix in the middle. Hold the middle and rotate. What do you get?

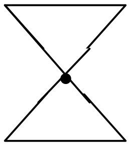


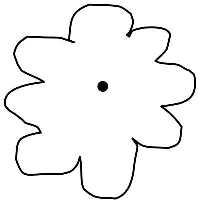
Take a shape like this fix it at the dot. Rotate what do you get?

Do the same with: (any of these):









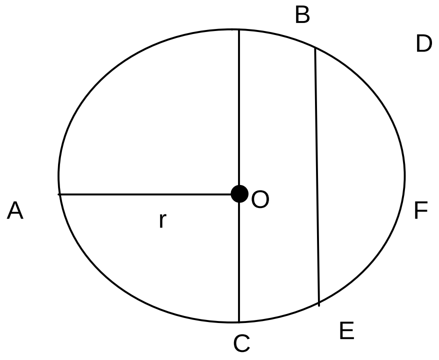
What?

Always a circle.

34.2 Parts of a circle

O CENTER
OA = radius = r
BOC = diameter = d
DE = chord
DFE = arc

(ABDFECA) = Circumference
= C (Our notation)

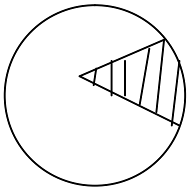


34.2.1 In any circle:
Diameter = 2 x radius i.e, d = 2 r
Diameter is the biggest chord. Circumference is directly related to radius.
C = 2Π r
= Π d where $\Pi = \frac{22}{7}$

[Π is a constant (= it has a fixed value) It is approximately $\frac{22}{7}$ or 3.14]

Area of a circle, A = Πr^2

34.3 Sector



Sector of a circle is the area covered by 2 radii and an arc of the circle. (Radius – singular. Radii – plural).

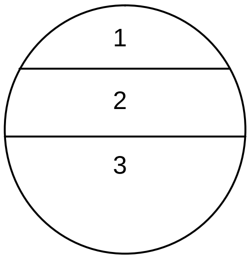
[Imagine pizzas, pies (cake like eatable famous in England), cakes, cut into equal pieces].

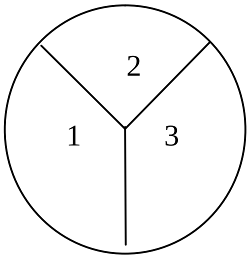
Activity:

1. Imagine a one by 4 dosa (or chapatti) how will you cut?

2. Imagine the same 1/3. How will you cut?

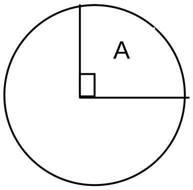
Cut paper





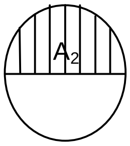
Which is better?

34.3.1 Area of a sector: See this figure:

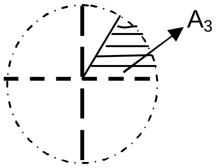


$$\text{Area } A_1 = \frac{1}{4} \text{ (area of circle)}$$

$$= \frac{A}{4}$$

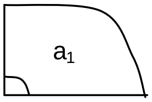
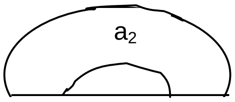
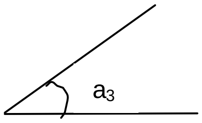


$$A_2 = \frac{A}{2}$$



$$A_3 = \frac{1}{2} A_1 = \frac{1}{2} \text{x} \frac{A}{4} = \frac{A}{8}$$

Now find the angles and compare:

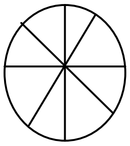
 $a_1 = 90^o$	 $a_2 = 180^o$	 $a_3 = 45^o$
$\frac{a_1}{360} = \frac{1}{4}$	$\frac{a_2}{360} = \frac{1}{2}$	$\frac{a_3}{360} = \frac{1}{8}$
$\frac{A_1}{A} = \frac{1}{4}$	$\frac{A_2}{A} = \frac{1}{2}$	$\frac{A_3}{A} = \frac{1}{8}$

Conclusion: Area of sector is in the same ratio of angle of sector.

Thus

$$\boxed{\frac{\text{Sector area}}{\text{Circle area}} = \frac{\text{Sector angle}}{360^0}}$$

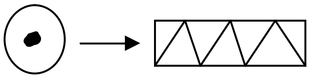
34.3.2 Exercises:

1.  Circle made into 8 equal parts. What is the sector angle?

2. A cake (circular shape) is to be shared by 6 people. How will you cut it?

3. Count the spokes in the Ashoka Chakra (of Indian flag) and then calculate the angle between 2 adjacent spokes. [Spoke = wire (or rods) in the middle of a wheel (eg: cycle wheel)].

34.4 Activity:
Take cardboard (or thick paper). Draw a circle. Cut into 16 or 24 equal sectors. Arrange them to make an approximate rectangle.
Knowing that circumference = $2 \Pi r$
You can prove Area of a circle = Πr^2

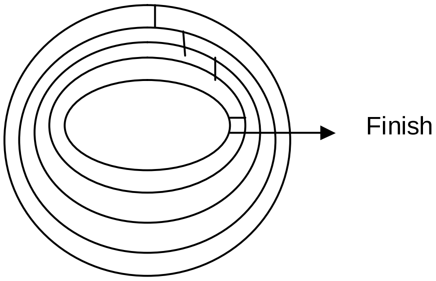
[i.e.,  Circle converted to rectangle]

34.5 Exercises:

1. Find the area of a circle 7 cm radius?
2. Area of a circle = 28.3 sq.m. What is its radius? [Clue: $28.3 \approx \frac{198}{7}$ $\Pi = \frac{22}{7}$].
3. In (1) & (2) what are the diameters?
4. If diameter increases to 3 times, what happens to the area?

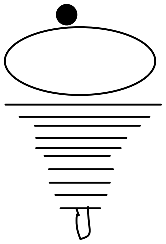
34.6 Exercises:

1. Tell me why?

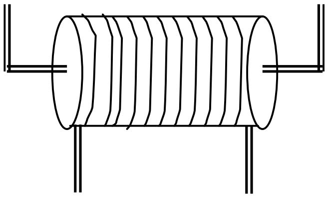


In a multitrack (many running paths) athletic field, for a 200 or 400 m. race the starting points on the lanes (=paths) are different. But finish line is the same why?

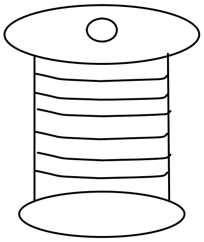
2a. Take a top (=spinning toy) count the groves (=cut lines on its sides) and thus guess how much string you will need.



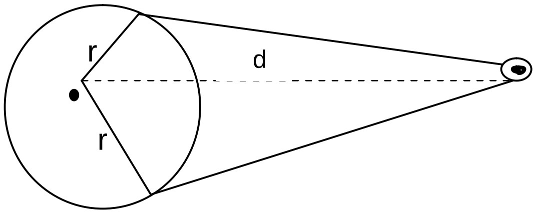
2b. In villages a wheel is used for drawing (=taking out) water from a well. Counting the grooves, can you find how many meters or rope is needed?



3. This is a cable roll. Can you guess the length of the cable in this roll?



4. A cycle wheel is 24” diameter. How much distance is covered in 10 revolutions?
5. 1 tin of paint was needed to paint a circular area of some diameter. If double the diameter area is to be painted. How many tins will be needed?
6. Which child is cleverer? The one who was happy with 2 dosas of a plate size or the one with a larger plate (double the diameter) only one dosa?
7.



Two shafts are connected by a belt. Can you calculate the length of the belt. d = 1m, r = 0.4 m

[Clue: assume the second roller has negligible radius. Assume the belt is like a tangent to the shaft (right angle is shown) approximate value is Ok].

==

Chapter - 35 Cubes and Volume

35. Cube and Volume:
Let the students see a dictionary, just as they did for ‘square’.
Let them get both the algebra meaning and the geometry meaning.

35.1 We have already seen in algebra that Cube is defined as “a number multiplied 3 times”. Thus $x^3 = x \text{ X } x \text{ X } x$

Let the students create 1 to 9 and their cubes:

$1^3 = 1$
$2^3 = 8$ etc

$\boxed{\text{One thing}} \text{ x } \boxed{\text{Same thing}} \text{ x } \boxed{\text{Same thing}} = \boxed{}^{3}$

How to write
$(10)^3$ or 10^3 3 should be small
$(x)^3$ or x^3 3 should be small

If more quantities are there bracket is a must (=necessary)

$10 + 2^3$ wrong $(10 + 2)^3$ – Right
$(x + y^3$ wrong $(x + y)^3$ - Right
$10 \text{ x } 2^3$ wrong $(10 \text{ x } 2)^3$ - Right
xy^3 wrong $(x \text{ X } y)^3$ or $(xy)^3$ - Right

How to read:
$(10)^3$ or 10^3 as ‘ten cube’ [cubed not necessary]
$(a)^3$ or a^3 as a cube
$(10+2)^3$ as (10+2) whole cube [(10+2) cube wrong]

$(a+b)^3$ as (a+b) whole cube ['whole' saying is necessary]

35.2 Activity:
Let the students fold paper / cardboard & make cubes of different sizes. If uniformly many students make 1 cm side shapes, you will get many small cubes. Assemble them to make bigger cubes. Demolish those to prove the formula.

Volume of a cube = a^3
Where 'a' is the side of the cube.

If available, Rubik's cube can also be used. May be one can count the various colours.

35.3 Three Dimensions:
Introduce the concept of 3 dimensions.
One dimension: length – cm, m, km etc
Two dimensions: area- cm^2, m^2, km^2 etc
Three dimensions: volume – cm^3, m^3 etc
Area can be area of a rectangle A = l X b sq. cm
Where l = length in cm b = breadth in cm

It can also be written as A = a X b. Where a & b are the lengths of two adjacent sides of a rectangle.

A box (not exactly a cube) has a volume V

V = l b h Where l = length b = breadth h = height

If a, b, c are the sides of a box (the right word is **Parallelopiped** – should you use it?).
V = a b c

To demonstrate this also do as in 35.2 above. Cut the box into small **'CUBES'** and count the total number of cubes.

35.4 Exercises:

Formulas: for a cube $V = a^3$ (a = side).
For an oblong box V = lbh where l = length, b = breadth, h = height

1. What is the capacity of a square tank of 2m side and 2 m height?
   [$1m^3$ = 1000 liters]
2. If the tank was 1m by 2m and 4m height. What is its capacity?
3. A lorry is 10ft wide 20ft long and sand height can be 5ft. Is one lorry sand enough to cover a 40' x 60 ' site to a height of 1 ft?

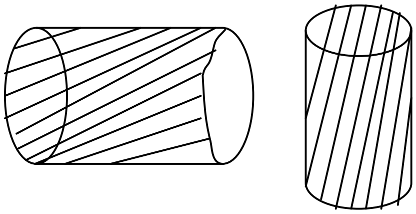
35.5 Activity with liquids:
The idea given in 35.2 above could be demonstrated using liquids (water). Take a plastic bag (called 'cover' in Mysore), big enough. Make a nice cardboard box whose inner dimensions are 10 cm X 10 cm X 10 cm. The height can be exactly 10 cm or even more up to 15 cm. Now fit the plastic 'cover' inside this cardboard box. Fit it tightly using wires or small sticks along the corners, such that the plastic also will be in the form of a 'box'. Take a 1-liter water bottle and gently empty the 1-liter of water into this plastic lined box. Measure the depth of water. Verify if the formula works.

Do the same as above. Here let the box be of known length and breadth. After pouring measure the height of water.

35.6 Other Volumes: Cylinder
Cylinder is a figure whose cross section is a circle.

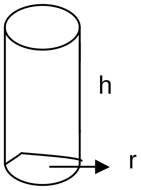
Solid cylinder: Full of same material.

Eg: roller, road roller, rod, wire, pillar, tree (approximately), some crawlers (worms)! Some fat persons (approximately).



Hollow Cylinders: (Hallow means nothing inside)

Eg: Drum, pipe, conduits, some gutters, penstock pipes.



Area of the base of the cylinder = πr^2 .
Where r = radius

Volume of the cylinder= $\pi r^2 h$ h = height or = $\frac{1}{4}\pi d^2 h$

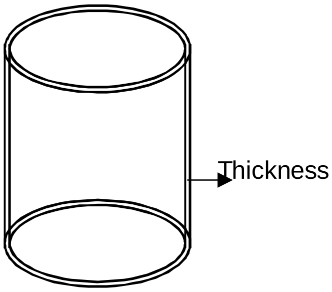
35.7 Activity:

Let the students do the same as in 30.5 above. They can make as given there. But not necessary. They can find all kinds of cylindrical glass, tins, many utensils for the experiment. Let them verify: Volume = (area of the base) X height = $\pi r^2 h$
Where r is the radius, h is the height and π is a special constant.

Exercises:

1. Plastic water tanks are sold in markets outer measurements are made. Its circumference is 156 cm. Height is 50 cm. what is the maximum capacity in liters [1000 cc = 1 liter cc = cubic centimeter].

2. In (1) above, the tank is made up of very thick plastic. Thickness is 1 cm. Calculate the actual capacity of the tank. [Clue: 1 cm on both sides are less for diameter-one side for radius]. [1 cm taken for easiness].



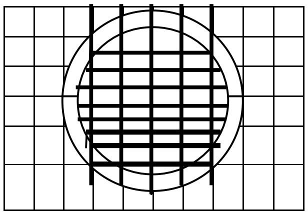
3. A community water tank is designed to serve 1000 family. A family needs 500 liters / day. Tank must have 2 days supply. Design a cylindrical tank of suitable size.

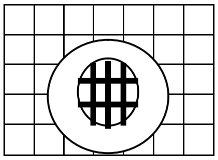
35.8 Activity: Water pipes

a) Let half of the class do 35.7. Let the other half go in search of long tubes of different diameters. Let them take 1 liter water and fill up the tubes and measure the length up to which water fills up. Make up the table as below:

S. No	Inner radius of the tube r	Area πr^2	Length of water h	Volume $\pi r^2 h$
1				
2				
3				
4				

b) Many tubes (plastic ones) may not be circular. They might have become mis-shaped to oval shape. In such case (including good circular ones) easier method of measuring area:





Tightly place the tube on a graph sheet and get the outline as shown. Count the small squares in the inner area. (Shown as dark).

Now the table:

S. No	Count of small squares	Area in (A) sq. cm	Length of water in cm (l)	Volume A X I
1				
2				
3				
4				

35.8 Exercises:

1. Water is coming out of a dam through a big circular pipe of dia 2m. The speed of water coming out is 1 meter / min. how much water will be drained in 1 hour.
2. Water tap is kept open. Tap dia is 1 cm. Speed as (1) above. How much water is wasted in (a) 10 minutes (b) 1 hour?

35.9 Activity: Converse of water pipes:

Let some students do a variation of the experiment given in 30.7.3. Instead of filling 1 liter of water into a pipe, take whatever length & shape of pipes available. Find area as given in 30.7.3 (b). Measure total length. Fill up the tube with water. Now empty the water in a measuring jar (see note below). Find the volume in cc. Make table:

S. No	Count squares	Area in (A) cm^2	Length cm (l)	Volume A X I	Measuring jar value
1					
2					
3					
4					

35.10 Activity:

Measuring jars or cylinders.
Teachers, show the students, if you can get hold of a regular measuring jar. Explain the capacities.

For fun they can measure many things:

a. The volume of a normal tea or coffee cup they use.
b. How much their Tiffin box can hold.
c. How much ink an old fashioned fountain pen can hold.
d. How little is a ballpoint refill's capacity.
e. Competitions on how much can one drink non-stop.
f. How much can one's mouth hold?
g. What is the volume of one's urine at a time?

If a measuring cylinder is not readily available, make one.
a. Using throw away bottles (plastics ok)
b. Using thick pipes (metal better or thick plastic > 2 mm)

In all the above teachers can impress upon the students the concepts of calibration, measurement etc and their importance.

35.11 Exercises:

1. Collect all the formulas given in this chapter.
2. Make your own formulas for the Total surface area of (1) cube (2) box (3) Cylinder (open both ends, open top, closed both ends).

===

Chapter - 36 Measurements

36. Measurement: Measurements are important in science, engineering, commerce and any day-to-day transactions. They all involve some kind of simple maths.

36.1 Units of measurements:
Three basic measurements (also called ‘dimensions’ in physics) are length, mass & time.

36.2 Length:
Old system: inch, foot, yard, furlong, and mile.
New system: millimeter, centimeter, meter and kilometer.

(Teachers, conversion from one to other is very important. Even today in all civil & architectural drawings 3.1 meter is used. Why this funny .1 etc? Convert it to feet & see).

Your geometry box is the first step in measurement. Students start measuring length with a scale (called “foot – rule” in the British days; called inappropriately “straight-edge in USA). A scale usually has inches on one edge and centimeters on the other edge. You can measure a given length either in inches or cms [of course in fractions of inches or in millimeters] scales come in 2 sizes 6 inches or 12 inches. Next we see a tape in a tailor shop this usually has 36 inches (some have just 1 meter). Many tapes may be only in inches or only in centimeters. But tapes are available with inches on one side and cms on the reverse. They even have different colours for the sake of convenience.

36.2.1 Length related quantities:
Area – area is measured in length squared. Sq. cm or $(cm)^2$ or cm^2

Sq. ft is used very much in real estate (land measure) etc. it is also used by carpenters, civil maistries and others.

Acre is a measure of land. Hectare is now a days used in offices. Sq. km is for large areas like forests or huge cities etc.

Volume:
Volume also can be calculated using formulas for some well-known shapes.

Volume is given in cm^3 (some times cc, m^3 etc (cubic …., ….).

(Sometimes timber etc are sold in cft (i.e. cubic feet)

In the case of liquids, volume is expressed in liters = 1000 cc.
In such a case 1 cc = 1 ml also.

36.3 Mass:
In the old system: pound and ton. They are still being used.
In the new system: microgram, milligram, gram, kilogram, quintal and METRIC TON [are current].

Weight:
(For simple understanding, mass = weight) weight is measured in kg, gm etc.)
Quintal is used for grains and some commodities.

36.3.1 Length, volume, weight are all being used in commerce for thousands of year. So, many local systems also exist.

36.3.2 Weight-related quantities:

Density:
This quantity considers both the weight and volume at the same time.

It is $\text{Density} = \frac{\text{weight}}{\text{volume}}$

When weight is in grams and volume is in cubic centimeter, density is in gm / cc.
When weight is in kilograms and volume is in cubic meters, density is in kg / m^3.

Density of water:
It so happens that 1 cc of water weighs 1 gm (or) 1 liter of water weighs 1 kg.
[Perhaps, scientists manipulated definitions to come to this figure].

$$\therefore \text{Density of water} = \frac{\text{Weight of 1cc of water}}{\text{Volume of 1 cc of water}} = 1 \text{ gm / cc}$$

It is the same as 1 kg / liter

Specific Gravity (SG):
It is the density of substance as compared to water. But density of water = 1. Therefore SG of a substance is the same as its density. Only difference is density has units specific gravity is a ratio i.e., a number.

36.4 Time:
In this field, old system continues.
Second – minute – hour – day – week – month – year.

Just to feel the magnitude of the numbers let some bright students complete a chart (as the one given below:

	Sec	Minute	Hour	Day	Week	Month	Year
Sec	1						
Minute	60	1					
Hour	3600	60	1				
Day			24	1			1/365
Week				7	1		
Month				30	4	1	
Year				365	52	12	1

36.4.1 Time related quantities:

Speed or Velocity:

Distance traveled per unit time is speed. [For this book, there is no difference between speed & velocity]. Speed can be expressed in cm/sec, m/sec, km/hr, miles/hr etc.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Acceleration:
This has become a common word, since almost all teenagers drive some vehicle sometime. They use accelerator to feel great or to show off!

Acceleration is defined as rate of change of velocity.

$$\text{Acc} = \frac{\text{Change in Speed}}{\text{Time}}$$

[Time here is the duration in which speed changed].

Some explanation:

If you push accelerator, speed increases.
If you apply brake speed decreases.
[Decrease in speed is also called Deceleration].

36.5 Quantities Big and Small

36.5.1 **WE NEED BOTH**: We need big amounts sometimes and small amount some other times. We may need a ton of sand and can only afford a few grams of gold. We may use a liter of milk and a few spoons of sugar. Thus, a millimeter is important in its own region. A kilometer may have its own significance in some other place. So, we need to measure, talk about and measure both in small and big units.

36.5.2 Numbers Big and Small:
For increasing order, we have names viz unit (=one), ten, hundred, thousand, million, billion etc. These can be written as 1(=10^o), 10^1, 10^2, 10^3, 10^6, 10^9 etc. For fractions there are no names. But can be written as .1 (10^{-1}), .01 (=10^{-2}), .03 (10^{-3}) etc

36.5.3 Units Big and Small:
Increasing side has names like unit, kilo, mega, giga. These are 10^o, 10^3, 10^6, 10^9 respectively (= in that order). Unlikes numbers, decreasing side also is given names. i.e., fractions of units also have their special prefixes (prefix = a term or label attached to a word, before the word). Thus unit, milli, micro, nano, pico are 10^o, 10-^{3}, 10-^{6}, 10-^{9}, 10-^{12} respectively.

36.5.4 Table:
Attach the prefix in front of the unit. Thus nano second, microgram, millimeter etc. On the bigger side, kilogram, megaton etc.

	Multiplying Factor
Pico	$\frac{1}{10^{12}} or 10^{-12}$
Nano	$\frac{1}{10^9} or 10^{-9}$
Micro	$\frac{1}{10^6} or 10^{-6}$
Milli	$\frac{1}{10^3} or 10^{-3}$
Centi	$\frac{1}{100} or 10^{-2}$
Deci	$\frac{1}{10} or 10^{-1}$
	1
Deca	10
Kilo	1000 or 10^3
Mega	1000000 or 10^6

Students can start with

Gram or Liter or Meter or Second

Or any other unit they know.

36.5 Activity:

1. Length, mass, time are basic quantities. Many others are derived from these. Conversion tables are very important. It is nice to show to students some standard reference books containing these conversion tables.
   (Let students make their own general conversion tables of big & small quantities).
2. How many centimeters = 1 meter? How many millimeters = 1 meter?
3. In measuring meters, can you have an accuracy of 1% (i.e., errors less than 1%).
4. In measuring cms, if you want 1% accuracy what can you do? Discuss.
5. It may be interesting to see (a) a chemistry laboratory balance. (b) balance and weights used by gold dealers.

Chapter - 37

Graphs - A

37. Graphs are representation of two different things related to each other.

Some real life situations changing with time – They can be seen as a sequence. Can also be shown in a graph. Eg: Age Vs. Height

Those items which can be given a number (expressed quantitatively) can be shown in a graph.

37.1 Some simple illustrations

a. For Fun 1 2 3 4 5 …….

```
        0
      0 0
    0 0 0
  0 0 0 0
0 0 0 0 0
```

Show abacus if you like

b. Same idea as shown in a simple example.

Person	Has
A	2
B	4
C	6
D	8
E	10

```
            0
            0
         0  0
         0  0
      0  0  0
      0  0  0
   0  0  0  0
   0  0  0  0
0  0  0  0  0
0  0  0  0  0
A  B  C  D  E
```

c. Such representations are called by some: pictographs. We can see these in Newspaper articles, some economics books etc.

37.2 Graph Sheets:

A paper with ready made squares makes drawing graphs easier.

Activity:

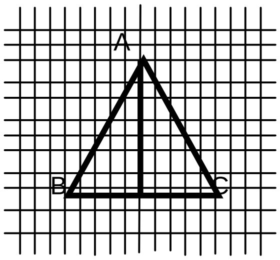
Bring graph papers and discuss lengths.

37.2.1 Area estimation:

Graph sheet is a great way of estimating area. We have done it earlier as activities and exercises. It is nice to do them once again.

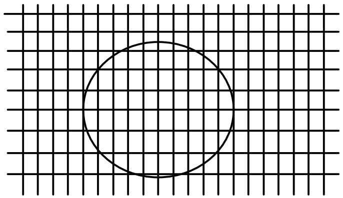
Activity & Exercise:

A.



Count the small squares inside ABC (less than ½ square is 0. More than ½ =1). From graph itself measure the lengths AC & AD (in small square). Use formula and verify your counting.

B.



Do as done for triangle above.

C. A & B above can be done by each student taking different sizes of triangles & circles.

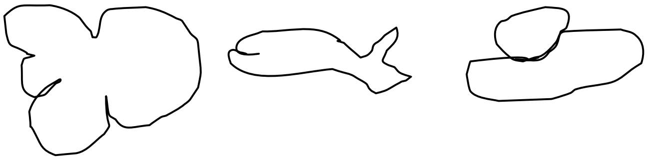
37.2.2 Some office procedures and secretarial practices:

Carbon paper: Typist in every office knows how to use a carbon paper. Students also can learn this and it will be useful. For us, for learning elementary maths also.

Tracing paper: This is also very useful for copying figures and maps. It helps to make a copy (approximately reproduce) a drawing, map, plan etc.

Scanner: This is a computer accessory, available in modernized offices. This is the best way of reproducing any figure. It has additional advantages of a computer; such as size reduction or enhancement.

37.2.3 If you want to measure the area of an irregular shape, you can use a graph sheet.

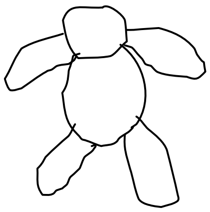


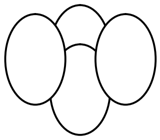
Irregular Shapes

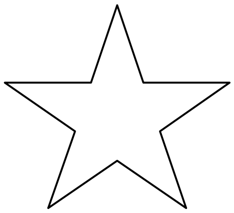
Method: Use any of the methods given in 37.2.2 above and then reproduce the shape on a graph sheet.

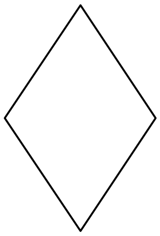
Exercise:

Find the area of the given figures:









[Clue: Tracing paper is the best]

37.3 Area (surface) – Activity:

37.3.1 Suppose you want to know the surface area of a tough surface (for painting purpose, or for any other reason). Tough surface can be shape wise extremely irregular, or it is connected, erected, embedded etc into a structure or having surfaces not accessible (=reach) for measurement. The following method is using a graph sheet is quite useful.

A. Spread a thick (=non stretching) plastic sheet, or cardboard, or paper on to the surface (call this 'transfer sheet'). Poring this transfer sheet to graph paper draw the outline and count squares on the graph sheet.

B. If the transfer sheet is too big, you can cut them into small manageable size pieces and count each piece and then add them all up.
C. In (B) above, if there is symmetry (i.e., if you can fold into smaller piece) count the folded version and multiply suitably.

37.3.2 Exercises and Activity:

Take one small object (Group A) and one large object (Group B). Find surface area by graphical method.

Group A: Pencil, pencil box, book notebook, belt, ribbon, phone – dial – surface, base of a cup/saucer/plate.

Group B: Table top, windows door, door barrel, surface of a grouted (=fixed) machine, seat of a motorbike, car (outside), steering wheel (of car, bus, truck), TV, Radio.

37.3.3 Strip method:

For group (b) paper strip method is very powerful. Keep a large number of paper strips of equal width (1” or 1 cm). Cover the area fully with these strips one strip just touching the next. Carefully take out one by one, numbering each one of them serially.

A. Now count of each strip – area using graph paper – Add them all up.
B. If some strips are too long, fold them by 2, 4 or 8 and count. After counting multiply by the folding factor.

Exercise:

Do the exercise of 37.3.2 Group B, once by earlier method, next by strip method and compare.

37.3.4 Perimeter – strip method:

We saw just now how paper strips can be used for estimating area. The same could be used for assessing lengths (perimeter of irregular shapes) .

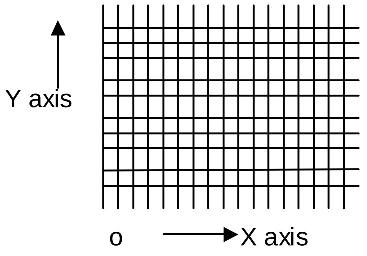
Exercise:

Use strip method (same as string method) to measure circumference / perimeter of bicycle wheel, cycle pedal, cycle axle, bicycle chain length, car tyre, steering wheel, boiler, barrel top, any big pipe. Body measurement (tailors work).

[Strip, string, tape are all ok. In fact cloth & metal tapes are often used and carried by engineers].

37.4 X, Y Axis:

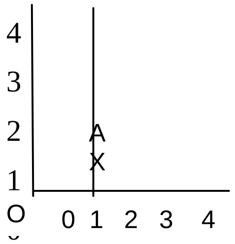
A graph sheet has graduations (divisions) on both sides horizontally and vertically. You can choose a horizontal line as a reference line. It is called X axis. The starting point of X axis is the origin o. At o, if you draw a vertical line, this is called Y axis.



O is origin.
At o value of X axis is 0.
At o value of Y axis is 0
Therefore o is called (0,0).

37.4.1 Activity:

Take a graph sheet choose O near the left side bottom corner. Mark it O (0,0). Draw X axis,Y axis. Mark 1, 2, 3, 4 ….. on right side. Mark 1, 2, 3, 4 ….. on vertical side. A point P (x_1, y_1) on the graph sheet is at a distance of x_1 from O on the x-axis and at a distance of y_1 from O on the y-axis.



A here is (1,1) (x_1, y_1).
(1, 1) are called coordinates.
(x_1, y_1) values are called coordinates of that point.
X_1 is called x coordinates.
Y_1 is called y coordinates.

Given below H (0, 2) its x coordinates is 0 and its y coordinates is 2.

Exercise:

a. Mark	B(2, 2)	C(3, 3)	D(4, 4)	
b. Mark	G(0, 1)	H(0, 2)	I(0, 3)	j(0, 4)
c. Mark	P(1,0)	Q(2, 0)	R(3, 0)	S(4, 0)

37.5 Lines on Graph:

Exercise: First do the exercise of 37.4:

a. Take a scale and see whether B, C, D fall on straight line. If so, join. [If no, you have made a mistake, go back to 37.4 and do it right].
b. Join GHIJ
c. Join PQRS
d. Describe the line a, b, c. in other words, what do you know about them?
e. Draw any vertical line on your graph sheet. Take 4 or more points on this line. Write their coordinates.

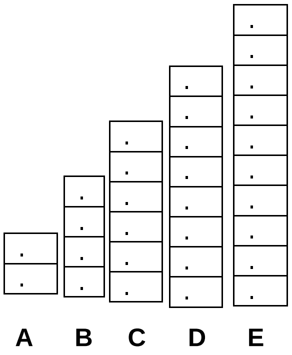
37.6 Pictographs

37.6.1 Pictographs on a graph sheet are neater and easier to read. Statistical data can be nicely shown on pictographs. Pictographs are also called Bar-Graphs or Bar-Charts. These do not even need any graph papers. [Go back to section 37.1 and see what we started with]. Let us put the same data on the bar graph.

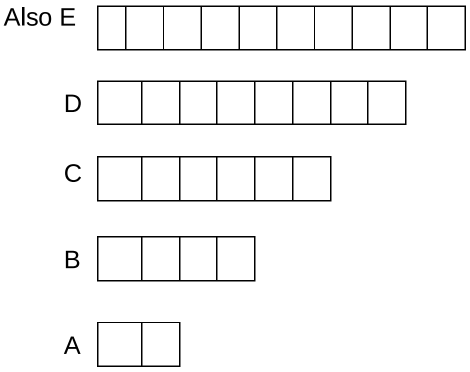
Person (X)	Age (Y)
A	2
B	4
C	6
D	8
E	10

This can be done in 2 ways.

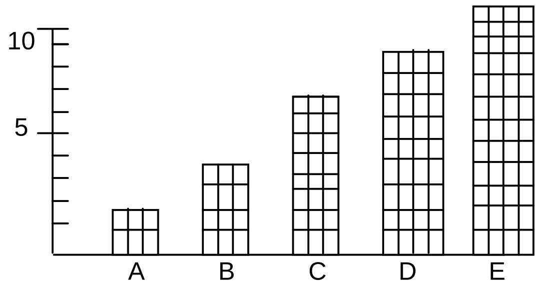
A.



B.



C. If you put them in X & Y axis the same will look like this:



37.6.2 Advantages of pictographs:

"Standing" pictographs as shown in (A) above are useful to visually see "high" and "low" values of an item.

"Horizontal" pictographs as shown in (B) above are nice in an article describing some aspects to be compared. These are easy on the eye and do not disturb a flowing matter. Thus these are seen in newspaper articles, reports etc.

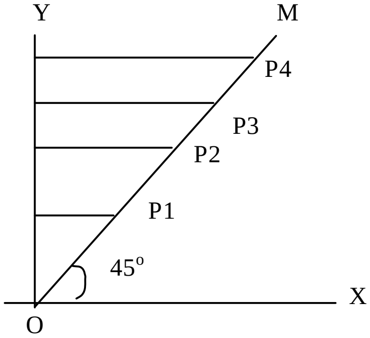
Bar-graphs as shown in (C) above are informative and quantitative. They can be used for 'discrete' quantities (i.e., unconnected items) or 'continuous' quantities. Thus bar graphs can show "variations" of a quantity with some parameter (Eg: item). They can also show maximum, minimum etc. For this reason they are useful in sciences, commerce and economics subjects.

37.7 Lines:

Go back to 37.5 exercise. We can discuss the answers now. Start with the reverse of the work done there.

37.7.1 a. Do this exercise. Let OM be drawn at 45^o to origin. Let coordinates of P1, p2, P3… be measured and tabulated.

	X	Y
0	0	0
P1		
P2		
P3		
⋮		



[Clue for those who need it: 45^o line is not difficult to draw. Use your set square].

b. Let another group to do as follows:

S. No	X	Y
1		
2		
3		
4		

Given Y = X. Let them give any integer values to X. Find Y. Tabulate.
Let (X1, Y1) be plotted as P1
Let (X2, Y2) be plotted as P2 etc
Join the points. Let the two groups compare their work.

37.7.2 [Note for teachers: y = x is the equation given above. X is the 'independent variable' i.e., item for which you give any value you want. Y is the dependent variable'. Value of y is not your will or wish. It depends on the value given to x. It has to be calculated. The author hopes you could convey this idea to the students. Please do not write any table of numbers to be copied].

37.7.3 Exercises:

1. $y = x$
2. $y = 2x$
3. $y = 3x$
4. $y = 4x$

After understanding these thoroughly, you can do:

5. $y = \frac{1}{2}x$ 6. $y = \frac{1}{3}x$ 7. $y = \frac{1}{4}x$ 8. $y = 0.1x$
9. $y = 0.4x$ 10. $y = 0.5x$ 11. $y = 0.8x$

[Help for those who have not yet started: say (12) y = 5x. First make a table. Start:

S. No.	x	y
1.	0	
2.	1	
3.	2	
4.	4	

S. No. 1: $y = 5x = 5 \text{ x } 0 = \underline{0}$
S. No. 2: $y = 5 \text{ x } 1 = \underline{5}$
S. No. 3: $y = 5 \text{ x } 2 = \underline{10}$
S. No. 4: $y = 5 \text{ x } 4 = \underline{20}$

These underlined values will go to next column.

With allotting values to x only. Now calculate y. The next step of the table will be like this:

S. No.	x	y
1.	0	0
2.	1	5
3.	2	10
4.	4	20

Next step

S. No.	x	y	(x, y)
1.	0	0	(0,0)
2.	1	5	(1, 5)
3.	2	10	(2, 10)
4.	4	20	(4, 20)

The last column is to be plotted

37.8 Plotting:
Marking points on a graph. Using the (x, y) coordinates of these points is called plotting. This is the first step in drawing graphs correctly. This what we did earlier (sec 37.4).

Exercises:

I. Plot these points. See what figure you get:

A (0, 8) B (6, 8) C (0, 7) D (6, 7) E (0, 6) F (6, 6) G (0, 5) H (6, 5) I (0,0)

II. For this exercise you should have both left side and right side of X axis so let your origin be in the center of the graph sheet.
RHS is +1, +2, +3 ………
LHS is –1, -2, -3, …..

A (-5, 10) B (5, 10) C (-7, 8) D (7, 8) E (-6, 8) F (6, 8) G (-6, 0)
H (-1, 0) I (1, 0) J (6, 0) K (-1, 4) L (1, 4)

Join AB, CD, AC, BD, EG, FH, HK, IL etc. What did you get?

III. Fun activity:

Students can make their own puzzles like I and II above and challenge the others to guess or plot.

37.9 Line y = mx + c

37.9.1 We have drawn y = x
Now let us take y = x + 1. How to do it. Give x different values. Calculate y. [Students! You should know substitution. If you are weak in this, go back and learn. No copying. No one will cook for you. You have to prepare what you need].

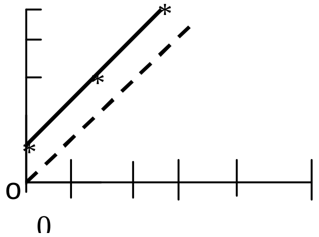
Let x = 0, then y = x +1 = 0 +1 = 1
Let x = 1, then y = x + 1= 1 +1= 2
Let x = 3, then y = x + 1= 3 +1= 4

Make a table.

	x	y	(x, y)
1.	0	1	(0, 1)
2.	12	(1, 2)	
3.	34	(3, 4)	

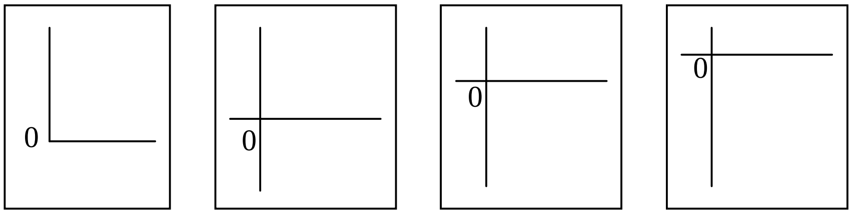
Plot these points.

Make a graph. Does it look like this?
Dotted line is our old friend y = x

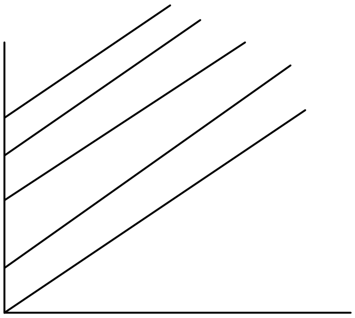


39.9.2 Y = x and y = x +1 plots were done. They look similar lines and parallel. This is because they are the same, only the starting point is different. If you shift the X axis a little higher, you will get dotted line coinciding with the thick line.

37.9.3 Activity:
The work given below will explain the concept of the earlier sections. This requires Do and Learn effort.
Make a group of 5 students. Give them a previously prepared set of 4 graph sheets. Prepare them as follows. One person is a coordinator.



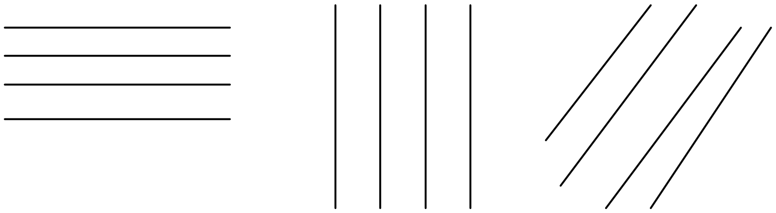
Let each of them draw the graph of $y = x$. Let the leader put them all together. (Transparent / translucent graph will be better).



Show that they are all parallel. Tell them what is ‘C’ in $y = mx + C$.

Other groups can be given other slopes

37.10 Slopes:

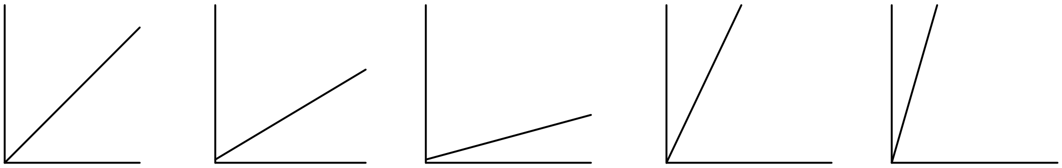


Look at these sets of lines. Each one set consists of parallel lines. This is the common quality of these 3 sets of lines. But one set of horizontal; another is a set of vertical lines. The third is a set if slanted lines. This is called slope.

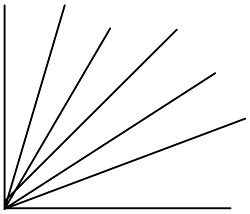
Mathematicians have no problems with 3 sets. For them (1) and (2) are the extremes of the slope. Thus (1) has zero (2) has infinite and (3) has some finite quantity of slope.

37.11 Go back to 37.7.3 Let each students bring his / her own graph and a leader assemble them.

Generate various graphs by different groups. Now put them all together



Now put them all together



(For this, transparent graph sheets – on tracing paper will be useful).
Now discuss this.

We got straight lines of different slopes equation of the first one was y = x.
i.e., if you write y = ☐ x x

For the first line ☐ was equal to 1.

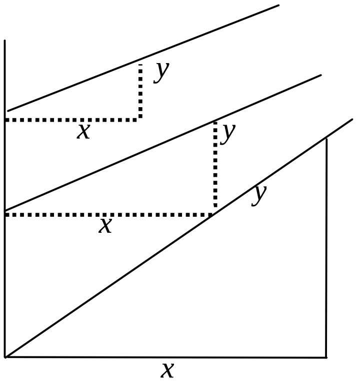
For the other lines ☐ was different.

We can write it as $y = mx$ where m is the slope. For each m, we got one new line. If m was more we got a line of more slope.

37.12 Line $y = mx + C$

Now explain $y = mx + c$. To do this, use all the graphs made above, and draw parallel graphs, by shifting the origin of the y axis.

Using a graph sheet, one can easily measure the slope. (Teachers, DO NOT USE trigonometry at this stage. If the whole class is very bright you can say slope = tan θ. No sine or cosine – not needed).



Let the students measure the slope (not angle) of each line **slope =** $\frac{y}{x}$**.**

Say $y \,\&\, x$ can be measured at any convenient place. (It will be the same because the graph is a STRAIGHT LINE).

37.12.1 Lines & statements:

It is now the opportune time to explain statements (in science / social) subjects) like:

a. y increases linearly x or

b. y is directly proportional to x or

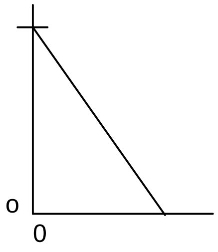
c. y increases as x increases. Graphical form helps here. a, b, c all mean the same thing. A graph with a positive slope also means the same.

37.12.2 We saw just now graphs and statements.

Similar statements:

a. y decreases as x increases or

b. y is inversely proportional to x . These deserve to be shown in a graph.



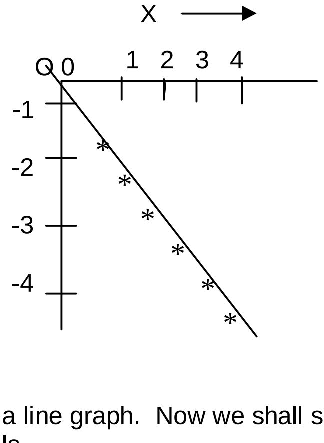
Nature of the graph. AT x = 0. It has y value
x > 0, y decreases.

y = x was seen by us earlier

y = - x really looks like given here.

It starts from zero and goes negative.

[Note to teacher: Just a mention of the above is enough. No more necessary. Real situation in science and engineering Are more like the earlier graph, with a negative slope].

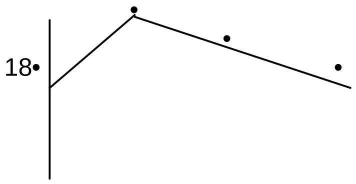


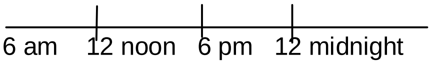
37.13 Data as a line graph:

In 37.12 we saw an equation of the form y = mx + c shown as a line graph. Now we shall see some data, which may change continuously or at some intervals.

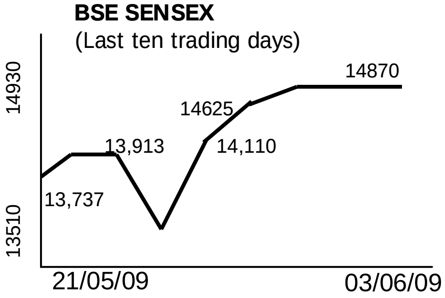
37.13.1 Temperature data at Mysore.

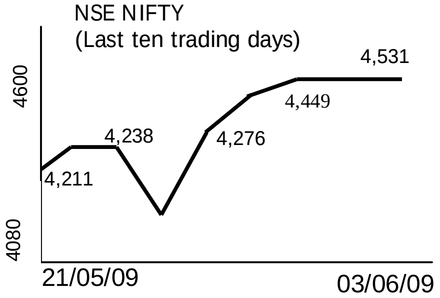
6 am 18^{o}C 12 noon 32^{o}C 6 pm 28^{o}C 12 midnight 20^{o}C





37313.2 From newspaper- Line Graph quantity changes with time.





37.14 Pie charts or sector graphs: Here the base scheme is not a square, no x or y axis. There is nothing changing. No dependent or independent variable. It is only representation of a given data, which has many parts.

37.14.1 Example: Household expenditure. In a family, assume a total income of 8000 rupees. Expenditure is put into some categories.

Food	Rs. 2000
Rent	Rs. 2000
Education	Rs. 1000
Transport	Rs. 1000
Fuel	Rs. 500
Entertainment	Rs. 500
All others	Rs. 1000

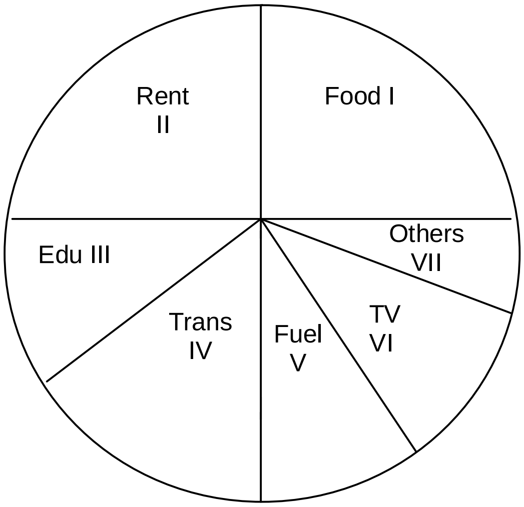
The total expenditure of Rs. 8000 can be put into a circle.
As we have seen earlier ("cake cutting"), the best method of dividing a circle into equal part is by sector method. For this purpose we use angle (at the center of the circle).

A circle has 360^o angle. In our example Rs. 8000 occupies (=covers) 360^o angle. Rs. 1000 will occupy $\frac{360}{8} = 45^0$. Let us now covert the expenditure (=spending) categories into a table of angles.

S. No.	Category	Rs.	In Angles	
I	Food	2000	$\frac{2000}{8000}$ x 360	= 90^o
II	Rent	2000		= 90^o
III	Education	1000		= 45^o
IV	Transport	1000		= 45^o
V	Fuel	500	$\frac{1000}{8000}$ x 360	
VI	TV	500		= $22\frac{1^0}{2}$
VII	All others	1000	$\frac{500}{8000}$ x 360	

				$= 22\frac{1^0}{2}$ $= 45^o$
	Total	8000		360^o

Put them all in a circle:



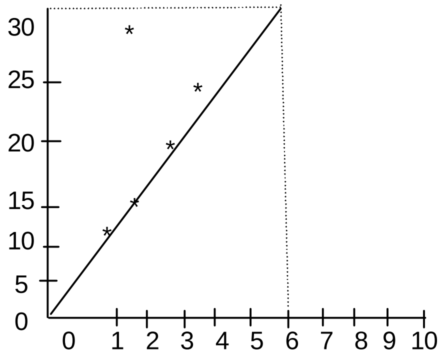
This is the pie chart or the pie graph or sector graph of this family budget.

[PIE – a cake –like eatable].

37.15.1 'Maggi' on a graph

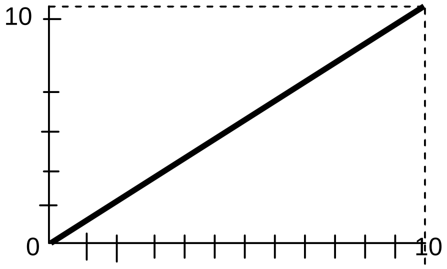
5 x 1 = 5
5 x 2 = 10
5 x 3 = 15
5 x 4 = 20
5 x 5 = 25
5 x 6 = 30

5 x 10 = 50
Let us put y = 5x



On the same lines any 'maggi' can be read out from a 45^o line.

Such graphs are sometimes called Ready Reckoners.



X axis will be 1 to 10. Y axis also be 1 to 10 (same scale).
Y axis will change as you wish. i.e., for 5 maggi it was 5, 10, 15 ……. For 7 it will be 7, 14, 21 …….

[For teachers; students can skip this].

37.15.2 Why always Maggi “tables” not “graphs”? The answer is very simple. Our education has always been geared towards “memorization”. Graphs cannot be memorized. Tables can be and they are made in order to memorize. This over insistence on memory (and less on thinking & logic) is the reason for the notoriety of mathematics and the ensuing enmity towards it].

37.15.3 Fun Activity:

A. Make your own “maggi” scales.
Take a difficult number like 13, 17 or 19. Write down 13, 26, 39 …… 130 at 1 cm intervals (=distance, space between two items). Write 1 2 3 ….. 10 at the same spacing. Put one of them in the x axis and the other on the y-axis of a graph paper. Draw a near 45^o line. You got your 13th “maggi”.

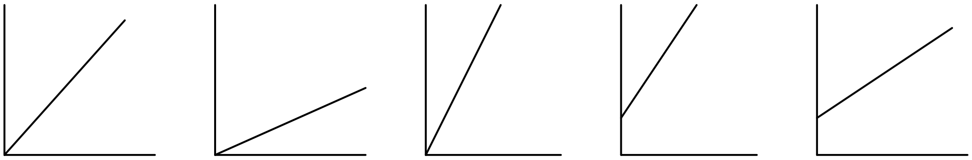
B. Like (A) above you can have a series of numbers while one axis (say y axis is fixed as 1, 2 …. To 10) the x axis can be replaced by these patties (patti-strip). You can “read out” your “maggi”.

C. Activity: One graph sheet could be cut and used to make 5 to 10 scales. Let the students make using carboard, wood, plastic or metal backing.

Some students with tailoring skill can do the activity to make measuring tapes – with suitable cloth, ribbon etc.

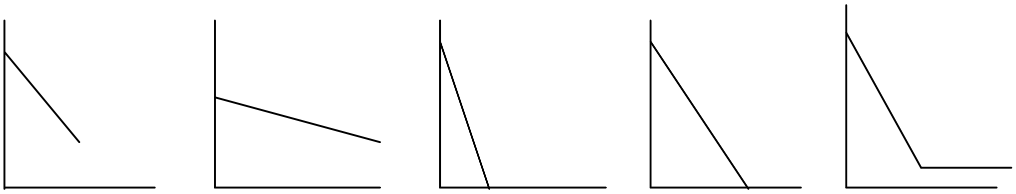
37.16 Describing the nature of a graph:
Teachers! Discuss graphs of the following types first.

a.



Important words: Linear, starting from, ….., small slope, large slope (=steep), increase etc.

b. Now go to



These were:

a. Ascending straight-line graph.
b. Descending straight-line graph. The words increases, decreases to describe.

We have seen that mathematical equations express an idea clearly and without ambiguity. Graphical representation clarifies information **qualitatively** and sometimes, when data is available, **quantitatively** also.

Chapter - 38 Graphs - B

38. This chapter contains some problems and suggested activities. Students should have read the chapter on graphs before attempting to solve.

38.1 Activity:
Teachers! Show the students all possible types of graphs. Let the students bring many examples either by imagining or by looking up at newspapers, books etc (give homework). AFTER they bring the results, check whether some of the following are included:

a. Population growth with time.
b. Increase in weight, height etc from birth to adulthood.
c. Growth of microorganisms.
d. DC & AC voltage / current.
e. Pendulum oscillations.
f. Gold price over 50 years etc.

38.2 Activity: Same as 38.1 but different source.
Teachers, ask students to bring a few old newspapers. Discuss the graphs given there. If you are lucky, you will get 2 or more of the following:

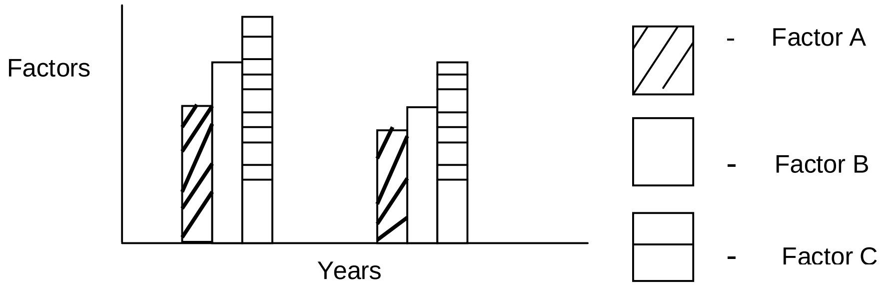
a. Different booths, voters list.
b. Different districts and their population.
c. Population of a city over the last decades.
d. Population of India.
e. Many census data.
f. Daily, monthly variation of stock markets.
g. Monthly variation of one company's share.
h. Prices of commodities.
i. Temperature data
j. Rainfall data.
k. Heights of water in reservoirs etc.

38.3 Exercise: State how (&which quantities) you will show the following in graphical form.

a. Stages of a building from site to finish.
b. Stages of a product from start to finish.
c. Students can make their own drawings.

38.4 Activity:
Take many factors about a city (E.g. Mysore) one can collect data about many items for the past years. Try.

Population, number of houses, number of schools, number of vehicles, number of sick persons (hospital), amount of pollution, amount of water consumption etc. Make a table and try to explain.
Now use the same data to make a multiple bar chart.



You can rightly claim that the graphical representation makes understanding the subject simpler and closer.

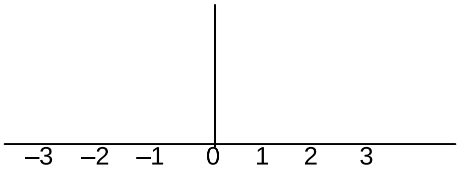
Qualitative nature of graph is shown here. Students should collect real data and draw. Only then he/she will get the total benefit.

38.5 Graph in two quadrants:

Introduce negative in the x-axis. Take a reference time as the 0 on X. Go before also.

E.g.:

	Price or some variable
3 days ago	
2 days ago	
Yesterday	
Today	
Tomorrow	



38.6 Pictograph or Bar Charts or Pie Chart or Line Graph – decision making:

I. Population or some states of India: (all numbers in millions) (Source Manorama year book 2001).

UP – 140
Maharashtra – 80
West Bengal – 70
Andra Pradesh – 66
Tamil Nadu – 56
Karnataka – 45
Kerala – 30
Nagaland – 1.2

Show these on a bar chart in the ascending order (can be pictograph also with number written there itself).

II. Some mother tongues (= languages) and the number of persons (in millions) given below (approximated).
(Source: year book 2001)

Hindi	500
Bengali	200
Urdu	100
Punjabi	100
Telugu	75
Tamil	75
Marathi	75
Kannada	50
Gujarathi	50

Put these in (a) a bar chart
(b) a pie chart

In making a pie chart we have made some assumption (i.e., we have neglected something) what is it?

III. Temperatures recorded by a nurse on a patient A's chart is as below:
7 am 98.6^{o}F, 10 am 98.4, 2 pm 99, 3 pm 99, 4pm 99.5, 6 pm 102, 7 pm 100, 8 pm 99^{o}F, 9 pm 98.6^{o}F. Draw them on a line graph.

IV. Blood pressure (BP) is recorded in mm of Hg as a set of two points at the same time. They are called systole and diastole.

Patient A

Time	BP	
	Systole	Diastole
7 am	120	80
10 am	110	70
2 pm	120	80
6 pm	145	90
8 pm	120	80
Plot these also on the same graph		

V. Student can make his own data and plot it to see his own progress.

Subject	Marks Obtained		
	8th std	9th std	10th std
I	-	-	-
II	-	-	-
III	-	-	-
IV	-	-	-
V	-	-	-
VI	-	-	-

Fill in all the data and make 3 line graphs.

38.7 Variables and Equations on Graphs:

A. Draw the graph of $y = x^2$

B. Draw the graph of $y = 2x$

C. Draw the above on the same graph sheet.

D. The square of a number and double the number are equal. Which is that number? Find by graphical method or show that there is one number whose square and double the number are the same. [Clue: (C) is the same as (D)].

E. The sum of 2 numbers is equal to 3 and their difference is 1. Find the numbers (This can be solved by drawing graphs).

F. Solve graphically:
   a. $x + y = 5$ and $2x – 3y = 0$
   b. $x + y = 6$ and $x – y = 4$

G. Draw graphs of equations.
   a. $y = 0$
   b. $x = 0$
   c. $y = x$
   d. $y= 2x$
   e. $y = 2x + 1$
   f. $y = (x + 1); (x+ 2); (x + 3); (x + 4)$
   g. Draw a line and find equation.

H. Using (a) and (b) and (f) above answer:
   1. What is the equation of x-axis?
   2. What is the equation of y –axis?
   3. What is the equation of horizontal line at a distance of 10 cm from x-axis?

38.8 Drawing and Areas

1. Using the method of joining points, whose (x, y) coordinates are to found, give set of points, which will make some figure. For this, draw first:
   a. a outline of a building (elevation)
   b. a flag (alone or on a pole)
   c. a series of hills
   d. any drawing like fish, saw

2. Using scale drawing method find the area of:
   a. a bench

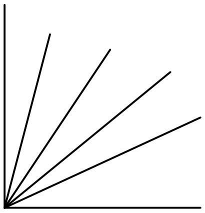
b. a blackboard
c. plan of a site
d. plan of a building
e. area of Karnataka (you need a map with scale given).

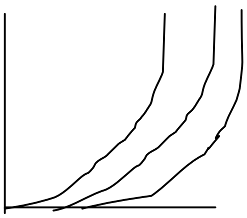
3. A. Four points are given O(0,0), P(2,2), Q(4,2), R(2,0). Join OPQR and say what kind of figure you got.
   B. O(0,0), P'(-2,2), Q'(-4,2), R'(-2,0). These are 4 points of a quadrilateral (=Four sided shape). Draw this area on the same graph.
   C. A(1,1), B(-1,1), C(-1,-1), D(1,-1) join ABCD and say what shape?
   D. Count the areas of (A), (B) and (C) which is the biggest? (or, are they equal?

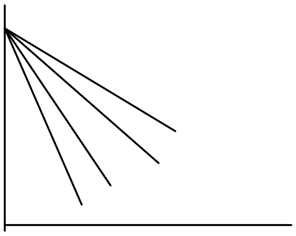
38.9 For studious students

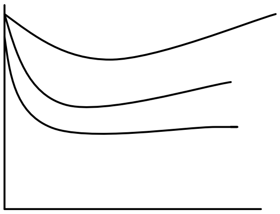
A. Draw and show ascending, descending, saturation etc.

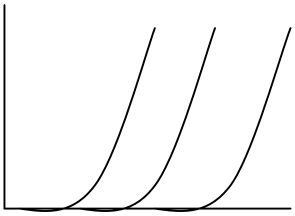
Clue: Examine different shapes of curves and then decide which is what, and then draw your own.

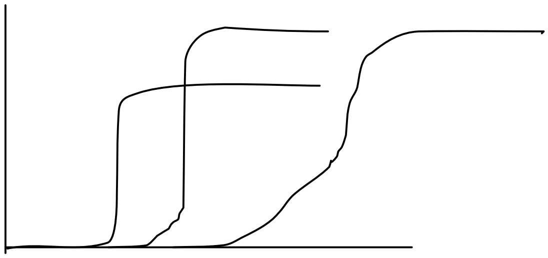








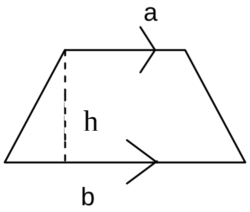




B. Show a waveform (Clue: Waves go up and down. Therefore we need top and bottom of x-axis to show a wave).

C. Just show how $y = x^2$ will look like (clue: $(+x)^2$ is +ve; also $(-x)^2$ is +ve. So, y-axis will be +ve, x-axis will have +ve and –ve].

D. Area of a trapezium is given by the formula.]



$$A = \frac{1}{2} h (a + b).$$

1. Show this formula is true for a trapezium of a = 4 b= 6 and h = 5 cm

(Graphical method).

2. Prove by graphical method that this formula is generally true (i.e, for any values of a, b & h).

E. Square Roots:

$\sqrt{1}$ = 1=1

$\sqrt{2}$ = 1.414 $\approx$ 1.4

$\sqrt{3}$ = 1.732 $\approx$ 1.7

$\sqrt{4}$ = 2 =2

$\sqrt{5}$ = 2.236 $\approx$ 2.2

$\sqrt{6}$ = 2.449 $\approx$ 2.5

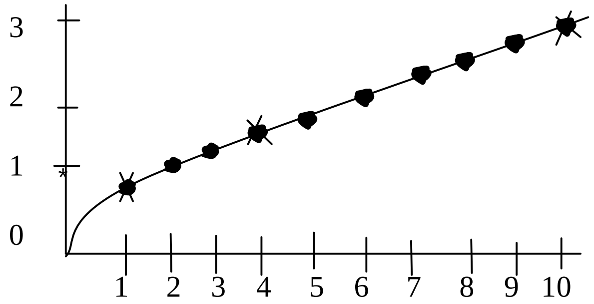
$\sqrt{7}$ = 2.646 $\approx$ 2.6

$\sqrt{8}$ = 2.828 $\approx$ 2.8

$\sqrt{9}$ = 3 =3

$\sqrt{10}$ = 3.162 $\approx$ 3.2

Students can plot these on a graph sheet and use these for future $\sqrt{\ }$ calculations.



F. Graph in (e) above has its limited use. The student can extend it to $\sqrt{20}$ =…. $\sqrt{30}$ = ….. up to $\sqrt{100}$ = 10. Plot all these on one graph. This graph (plus graph in (e) above) together could be used to find square root of any number.

Try: $\sqrt{3}$, $\sqrt{5}$, $\sqrt{10}$ using (E) above.

$\sqrt{30}$, $\sqrt{50}$, $\sqrt{40}$, $\sqrt{88}$ using (f) above.

G. Studious student can do the reverse i.e., plot $y = x^2$ using the data given below and use that graph to find square root.
Fill up:

x	1	2	3	4	5	6	7	8	9	10
Y	1	..	..	..	..	..	..	..	..	100
(x, y)	(1,1)	..	..	..	..	..	..	..	..	(10, 100)

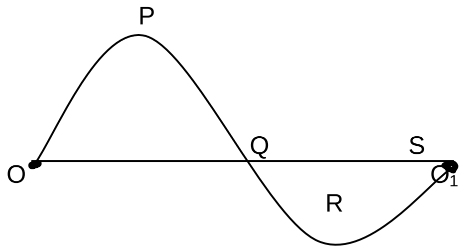
Fill up this table:

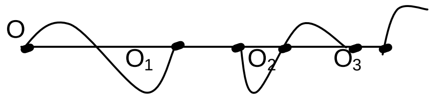
Plot the (x, y) coordinates and get 10 points on the graph. Draw a smooth curve passing through ALL the 10 points.

H. Using the above graph i.e., $y = x^2$, find $\sqrt{3}$, $\sqrt{5}$, $\sqrt{10}$. Verify your answer with (a) calculator (b) graph of (E) above.

I. Using (G) find $\sqrt{50}$, $\sqrt{40}$, $\sqrt{88}$ etc. Verify using (f) above and a calculator.

J.





This figure is a WAVE. S is another O (say O_1). From S the wave repeats itself. In the next figure O_1, O_2, O_3 are shown. Some portions are missing. Can you fill it?

38.10 For all students making of pie charts:
A. Students create their own data and draw:

S. No.	Subject	Marks Obtained
1	-	-
2	-	-
3	-	-
4	-	-
5	-	-
6	-	-
	Total	

Which is the subject in which you are:
a. Strong [Clue: Largest area: Sector with angle more than 60^0 to 90^o
b. Very weak [Clue: Sector angle < 60^o (is 40^o & Less)]
c. So – so, average [clue = sector angle around 60^o]

B. Students can find the total income of the family and make one pie graph. Average expenditure by different categories in another pie graph.

Income

S. No.	Earning Member	Rs.
1	A	-
2	B	-
3	C	-
4	Others	-

Expenditure

S. No.	Category	Rs.
1	Rent	_
2	Food	-
3	-	-
4	-	-
⋮		

==

Chapter - 39 Problems in Geometry

39. Given below are questions, activities, problems, worked examples, demonstrations – based on the earlier chapters on geometry. Some new content also is introduced in a very simple format.
[Note for teachers: please treat this chapter also as equally important. If some problems appear to be difficult, not in sync with the general simplicity of this whole manual please help the students. Clues, hints etc are given everywhere].

39.1 Construction of triangles

39.1.1 Draw a triangle having sides 5 cm 8 cm and 3 cm.
[Hint: take 8 cm as base. Use compass & cut arcs. This will look easier to do than taking the smaller ones. But in principle anything can be the base].

39.1.2 Same as above, but sides 5, 4, 3.
[Hint: Try smaller one of the sides as base. You will see some special angle].

Calculate the area in the above [Hint: you have to draw a perpendicular and measure height. 39.1.2 can be done in 2 ways, one as above and the other without a perpendicular].

39.1.3 Draw a triangle of base 8 cm and height 5 cm.
[Clue: you can draw as many as you can (=infinite number). Help: Height 5 cm can be obtained by any point on a line parallel to the base line (at distance of 5 cm)].

39.1.4 In the above another side is 5 cm. Draw the triangle.

39.1.5 a. Construct an equilateral triangle (= all sides equal) of side 6 cm.
b. 12 cm
c. Measure their areas. Find the ratio of areas.

39.1.6 a. Construct an isosceles triangle (=two sides equal) with base (third unequal side) 10 cm. How many can you draw?
b. In (a) above the other sides are 8 cm.
c. In (a) above one angle is 45^0.

39.1.7 Construct right angled triangle:
a. Sides 6 cm, 8 cm, and 10 cm.
b. 3 cm, 5 cm, one right angle
c. 3 cm, 4 cm, one right angle.
d. One 90^o, second 30^o, one side 4 cm.
e. How will you draw a triangle whose hypotenuse (=side opposite to the right angle) is given? In all the above, the method and how many are important.

39.2 Squares and Circles

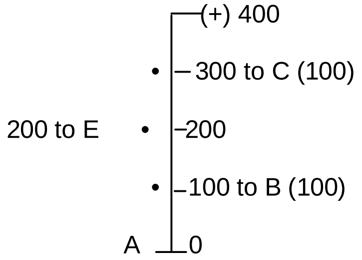
39.2.1 Fold a paper to make a square and cut off a square. How many lines of symmetry could you see?
[Line of symmetry: If you fold one portion over another and it exactly overlap (=covers up), the crease (=Line seen, groove made) is called the line of symmetry].

39.2.2 a. Given the area of a square, is there a unique figure (unique = only One)? Or, can these be many?
b. In (a) above, how about a rectangle.
c. In (a) above, how about a circle?

39.2.3 a. Draw concentric circles of 1 cm apart, the last one having 5 cm radius.
b. In (a) above 4 persons run on tracks (created by the annular space= space between an outer circle & inner circle). They have the same speed. How many times the inner most runner go around, in the time taken by the last runner to make one round.
c. In long distance running race (eg: 10 km race) all runners run in a bunch (= group, close together). In short distance races (eg: 100 metre dash) each one is given a track. Even in 400 m. race in an oval path, tracks are allotted? Why this difference?

39.3 Drawing

39.3.1 Scale Drawing:
a. Mysore to Bangalore distance is 150 km (assume). Mysore to Mandya 60 km. Mandya to Maddur 20 km. Bidadi in only 10 km from Bangalore. Draw a scale map of the road assuming the road is a straight road.
b. In (a) above assume Mysore to Maddur is direct East and Maddur to Bangalore is Northeast. Draw a new scale maps.
c. Given below is a scale map of a land. If the land – cost is Rs. 1000 per sq. m. Find the total market value of this land.



All in meters.
Draw to scale 1 cm = 10 meters and from the drawing find the total area of the land.

d. Draw a map of any of the rooms of your building to scale; including windows and doors.

39.4 a. What is a line of symmetry? Can you use symmetry to simplify area measurement.
b. Calculate the area of a "Sector" and hence find the total area circle.

39.5 Lines and solving for X:
a. Draw any line on a graph sheet. Do some measurements. Find the Equation? [Hint: y = mx + c. m is slope = $\frac{y}{x}$, c is when x = 0].
b. Solve using graph: 2x – y = 2; x + 2y = 21
c. A number plus 10 is equal to the square of the number minus 10. Solve this by algebra and by graph.

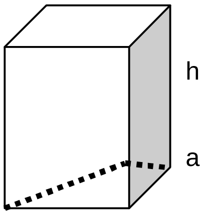
39.6 Coordinates:
a. On a map A is (0, 0) B is (4, 4). In what direction is B, with reference to A?
b. In (A) above, place C is exactly the same distance as AB but in the opposite direction. What is the direction of C w.r.t A and its coordinates?
c. Point (D) is 8 units north of C, what are the coordinates of D.

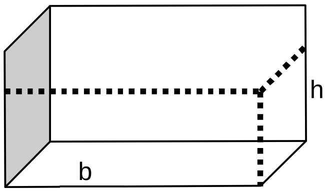
39.7 Types of Graphical Representation:
A. Many options are there to represent the given data. What are your options?
B. Get answer to (A) first. Then, decide which option is best suited for the following:
1. Budget proposal of India under different categories of income.
2. Same of expenditure.
3. Budget allotment for education over the past 10 years.
4. Rainfall per day in August of a year and the level of water in a dam during the same time.

39.8 Volumes: Box (=Parallelopiped), tank

a. Volume of a cube V = a^3
A pit of 2 m x 2 m square was dug up to 2 m depth. How much mud came out? If this mud is loaded on a truck of width 1.6 m and length 5 m, will it be enough?
If yes, how many pits can the lorry take?
If no, how many lorries a pit will need?
b. A water tank is a cube of 1 m x 1 m x 1m. How much water, in liters, it will store. If the family needs 200 liters / day how many days a tank full will supply?
c. A syntax water tank is almost cylindrical in shape? A small tank is 1 m dia and 1.2 m height. What is its capacity?
d. Which is bigger, capacity – wise, (B) or (C)?

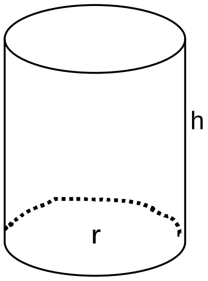
Help: Cube: V =a^2h Box, Tank: V =lbh





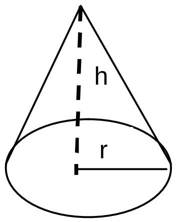
a l

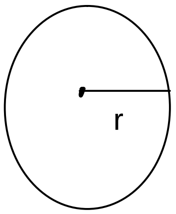
Cylinder: V = $\Pi r^2 h$ $\Pi = \frac{22}{7}$ or 3.14



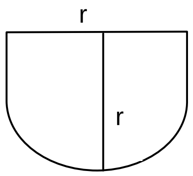
39.9 Volume:

Cone: V = $\frac{1}{3}\Pi r^2 h$ Sphere: V = $\frac{4}{3}\Pi r^3$





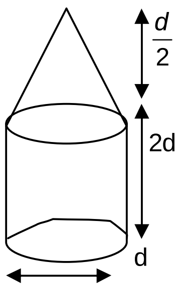
Hemisphere: V **=** $\frac{2}{3}\Pi r^3$



a. When sand is unloaded from a lorry it falls into a cone shape. A truck of 1-5 m width and 4 m length and 1 m height of sand unloads. What are the dimensions of the cone of sand on the ground?
[Help: Assume diameter of cone is 3 m and calculate height using formula:
lbh = $\frac{1}{3}\Pi r^2 h$ where l = 4 m, b = 1.5m, h = 1m, 2r = 3m, h =?].

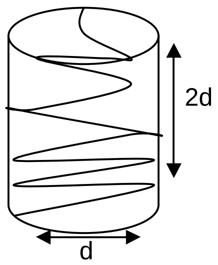
b. In (A) above assume the cone of sand finally takes the shape where the height is half the diameter. Now, find the height?

c. Traditionally solids (eg rice, sugar etc) used to be given in a cylinder measure. It used to be up to the height it will take.



Now smart fellow give only up to the top level.

How much more profit the smart seller gets?



[Hint: Take same assumption as in (B) above]

% profit is independent of the radius but depends on heights. Assume the height is double the diameter].

Sl. No.	Symbol	Meaning	How to read
1	+	Addition	a + b a plus b
2	-	Subtraction	a – b a minus b
3	X	Multiplication	a x b a into b
4	÷	Division	a ÷ b a divided by b
			a/b a by b
[Caution: Do not read a/b as a over b or b over a. Misunderstanding is possible]			
5	=	Equal to	a= b a equal to b
6	>	Greater than	a > b a greater than b
7	<	Less than	a < b a less than b
8	≈	Approximately equal	a ≈ b
9	≥	Greater than or equal to	a ≥ b
	≰	Not less than	a ≮ b
10	≤	Less than or equal to	a ≤ b
	≯	Not greater than	a ≯ b
11	≠	Not equal to	a ≠ b
12	$\sqrt{\ }$	Square root of	$\sqrt{a}$ Root a, Square root a, root of a
13	$\sqrt[n]{\ }$	nth root	$\sqrt[n]{a}$ nth root of a
14	$\sqrt[3]{\ }$	Cube root	$\sqrt[3]{a}$ cube root of a
15	±	2 values	$\pm a$ Plus or minus a
16	$(\ )^n$	Index n times multiplication	$a^{n;}$ a to the power n; a raised to n
17	$\sum$	Serial addition $\sum_n = n + (n-1) + \ldots + 3 + 2 + 1$	$\sum_n$ Sigma n
18	$\angle$ or !	Serial Multiplication $\angle n$ = n!= n x (n – 1) x … x 3 x 2 x 1	$\angle n$ or n! Factorial n
Computer Version			
1	+	Addition	a + b
2	-	Subtraction	a - b
3	*	Multiplication	a * b
4	/	Division	a/b
5	**	Exponent	a**n =a^n

Commonly Used Symbols

Roman Numerals

This is the old system of writing numbers. This is the Roman system, before the use of zero (0). Roman system is used even today for numbering pages, chapters of a book, writing the year in some buildings, to show sub section etc. In these situations they are used as cardinal numbers (Eg: integers). Other wise they have gone out of use i.e. they are not useful any more for arithmetical operations. Like addition, multiplication etc. But they are very much in use as ordinal numbers.
(i.e, numbers to show rank, position etc).

Eg: I = first, 1st
II = second, 2nd
III = third, 3rd
IV = fourth, 4th
XXV = twenty fifth, 25th

Roman Numbers:
I = 1, one

V = 5, five
X = 10, ten
L = 50, fifty
C = 100, hundred
D = 500, five hundred
M = 1000, thousand

A bar over a letter shows the value multiplied by 1000. Thus D = 500 x 1000
2 rules of writing Roman numbers:

1. If a letter is immediately followed by a letter of equal or lower value. The two values are added.
2. If a letter is immediately followed by one of greater value, the first is subtracted from the second.

Thus	II	= 1 + 1 = 2	(by rule 1)	III	=1 + 1 +1 = 3	(by rule 1)
	IV	=5 – 1 = 4	(by rule 2)	VI	=5 + 1 = 6	(by rule 1)
	VII	=5 + 2 = 7	(by rule 1)	VIII	=5 + 3= 8	(by rule 1)
	IX	=10 – 1 = 9	(by rule 2)	XI	=10 + 1 = 11	(by rule 1)

XX = 20	XL = 40	CXVI = 116	MCMXLVII=1949
XV = 15	CM = 900	MCXX = 1120	MMX = 2010
VI = 6	XLVIII = 48	MCMXIV = 1914	MMIX =2009

$\overline{M}\ \overline{C}\ \overline{X}$ = (1000 x 1000) + (10 x 1000) + (100 x 1000)
= 1000000
+ 10000
+ 100000

1110000

Usually capital letters (upper case) are used. Sometimes smaller letters (lower case) used mean the same. Cardinal or Ordinal meaning depends on the context**.**

Source: Random House Dictionary of the English Language.

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Chapter - 40 Random Assorted Problems

40.1 a. Write in words: 1, 11, 111, 101, 1001, 1011, 9909
b. Write in words both in Indian System and International System: 101001, 110010, 987654321, 87654321, 7654321, 706050403

c. Fill in the blank:

MERA BANK

Pay Self Rs. 22033450…………………………………..
In words ……………………………………………………..
……………………………………………………………………

XXXXXXXXXXXXXX

40.2 a. Write in Roman number system: 5, 15, 50, 55, 95, 90
b. Write in modern system: II-X-MDCCCLXIX, XV-VIII-MCMXLVII, XIV-VI- MMIX.
[Hint: Go to appendix of this manual]
c. Write the ordinal number: Eg: Rank VI = 6th Rank: I Prize, III Place, XXII Person

40.3 a. Expand as in Eg: 1234 = 1 x 1000 + 2 x 100 + 3x 10 + 4 x 1

$= 1 \text{ x } 10^{3} + 2 \text{ x } 10^{2} + 3 \text{ x } 10^{1} + 4 \text{ x } 10^{o}$

1. 89, 789, 6789, 56789, 456789
2. 809, 7008, 60709, 50601, 406100

b. Expand as in Eg.

$.987 = \frac{9}{10} + \frac{8}{100} + \frac{7}{1000}$

$= 9 \text{ x } (.1) + 8 \text{ x } (.01) + 7\text{x } (.001)$

$= 9 \text{ x } 10^{-1} + 8 \text{ x } 10^{-2} + 7 \text{ x } 10^{-2}$

1) .123, 0.103, .045, .9805

Example $123.45 = 1 \text{ x } 10^{2} + 2 \text{ x } 10^{1} +3\text{x } 10^{o} +4 \text{ x } 10^{-1} \text{ x } 5 \text{ x } 10^{-2}$

2) 8.75, 78.75, 608.07, 608.107

40.4 a. 123 + 456 =? b. 123 + 456a =? c. 1+ 2 + 3 + 5 =?
d. a + 2 + 3 + 5 =? e. a + 2 + 3a + 5 =? f. 456 – 123 =?
g. 456a – 123a =? h. 1 – 2 – 3 + 5 =? i. –a –2 -3 + 5a =?
j. a – 2 – 3a – 5 =?

a. $\frac{246}{123} - \frac{456}{278} = ?$ b. $\frac{24a}{12} - \frac{45a}{15} + a = ?$ c. $\frac{24a}{12a} - \frac{45}{15} + 1 = ?$ d. $\frac{4abc}{2ab} - \frac{9c}{3} + C = ?$

e. $\frac{1}{(x+a)(x+b)}[\frac{15(x+a)(x+b)}{5} - 4(x+a)(x+b) + \frac{(x+a)^2(x+b)}{(x+a)}] = ?$

40.4 Work with fractions:

a. $\frac{2}{3} = \frac{?}{6} = \frac{10}{?} = \frac{?}{33x}$ b. $\frac{3}{7} = \frac{3+7}{7+3}$ True or False c. $\frac{3}{7} = \frac{3 \text{ x } 7}{7 \text{ x } 3}$ True or False

d. $\frac{3}{7} + \frac{4}{7} = \frac{7}{14}$ True or False e. $\frac{3}{7} + \frac{4}{7} = \frac{7}{7}$ True or False

f. $\frac{3+4}{7 \text{ x } 2} = \frac{3}{7} + \frac{4}{2}$ True or False g. $\frac{3a}{7a} + \frac{4b}{7b} - \frac{5c}{7c} = ?$

h. $\frac{3a(x+y)}{7a} + \frac{4b(x+y)}{7b} - \frac{5c(x+y)}{7c} = ?$ i. $\frac{1}{2} \text{ x } \frac{2}{3} \text{ x } \frac{3}{4} \text{ x } \frac{4}{5} = ?$

j. $\frac{a}{2b} \text{ x } \frac{2b}{3c} \text{ x } \frac{3c}{4d} \text{ x } \frac{4d}{5a} = ?$ k. $(\frac{a}{2b} \text{ x } \frac{2b}{3c} \text{ x } \frac{3c}{4a}) - (\frac{1}{2} \text{ x } \frac{2}{3} \text{ x } \frac{3}{4}) = ?$

40.5 Codes:

a. If A =1, Z = 26 Find the name: (4)(18). (18)(1)(10) (11)(21)(13)(1)(18) Famous film personality in Kannada!
b. If A = 2, Z = 27 find the name given below? Who is a good type setter and office worker: (15) (10) (19) (14) (2) (18) (2)
c. Given that "Taj Mahal" is in (26) (19)(8)(26)" write a coded message to read "MEET ME AT TEN".
d. There is a sentence written in code. Code is: Instead of a, e, i, o, u code is u a e i o. Solve: "MARU UBBU TARU NUNU". For those who still need help:

Another code a e i o u → e i o u a makes the same as.

Now try "MIRE EBBE TIRE NENE"

e. Substitute in place of 'A', the letter 'B' and similarly others."VHKK XNT AD LX EQHDMC?"

f. Make your own code: Eg: Z = 1 ….. A = 26 or A → C B → D etc. Write in code: "ENGLISH IS EASY. I LOVE ENGLISH"

40.7 Substitution:

a. "15 years ago, Raja was a baby. Today Raja is a boy. Tomorrow Raja will be an engineer". In these sentences, substitute your name instead of Raja. If you like replace "an engineer" by a/an '▭"

b. Essay on ▭

▭ was a great person. He/She was born in ◯. Even in childhood, his/her parents and teachers saw his/her future greatness. He / she became the best in his / her chosen field. Even today ▭ is remembered.

Substitute ▭ by a big person's name.

◯ by a place. You got an essay.

Make a similar "pattern" for history (Eg: A great king) or for a leave letter.

c. 1. If $x = 23$ $2x = ?$ $2x - 23 = ?$
2. If $x = 4$ $x^2 - 4x = ?$ $2x^2 - 5x = ?$
3. If $a = 2, b = 5$ $5a + 2b = ?$ $5a - 2b = ?$
4. If $a = 1, b = 4, c = 3,$ $ax^2 - bx + c = ?$
5. If $a = 1, b = 4, c = 3$ and $x = 1$ $ax^2 - bx + c = ?$
6. If $a = 1, b = 4, c = 3$ and $x = 3$ $ax^2 - bx + c = ?$

40.8 Rule of Three:

a. Basava has 2 acres and 20 guntas of land. If land cost is Rs. 5 lakhs / acre. How much will he get? [Clue: 40 guntas = 1 acre].
b. 1 sq ft land in a big city goes for Rs. 1000. If a person wants to buy a site 30' x 40', how much money?
c. If $10a = 60$, $a = ?$
If $x + 2x = 34$, $x = ?$
If $x = 5$, $x^2 = ?$ $x^3 = ?$
d. If a box of 10 pencils cost Rs. 22. What is the cost of each pencil? What is, for 4 pencils? If 10 pens cost Rs. 60, each pen's cost =? In 1 hour chakri cycled 8 km. She rested for a while. Again cycled for 2 hours. What is the total distance traveled?
e. A worker finished a job in 5 days. In how many days will 5 workers finish the same job?
f. A microorganism (bacterial cell etc) splits into 2 cells in 1 hour. Each one of these splits into 2 cells in 1 hour. How many in 8 hours? If there were 100 bacteria at night in a person's mouth, how many in the morning (say after 8 hours).
g. Pagoda tree is very special. It gives 2 new branches at every branching point. This happens every 3 months. How many branches in 3 years?

40.9 Fractions, Decimals, Percent:

a. Express fractions into decimals and percent:
$\frac{1}{2}, \frac{1}{4}, \frac{3}{8}, \frac{1}{3}, \frac{3}{4}, \frac{8}{9}, \frac{22}{7}$
b. Express into decimals and fractions:
10%, 50%, 80%, 100%, 120%, 500%, $8\frac{1}{3}$%, 12½%, 18%, $33\frac{1}{3}$%, 75%.
c. Express decimals into % and fractions:
.1, .01, .15, .5, .8, .9, .75, .125, .33, .66, .05, 1.5, 1.125, 5.0
d. Let us share this 50/50. The vote was split into 50/50. There is a 50/50 chance of getting it. Express these in more mathematical terms.
e. Some admissions were shared 60/40 between 2 colleges P (private) and G (Government). If there were a total of 1200 seats, how many each one of P & G got?

f. In a test the maximum marks was 10. Maximum obtained in class was 8. Basava got 2. What is the % of the top student? What % did Basava get? If the same were evaluated for 25 what were the percentages?

40.10 Business

a. In a small scale buying and selling activity, how will you calculate percent gain or loss? [Help: make any reasonable assumptions].

b. Describe a few situations of 'No gain No loss'. Many times, even if the selling price is the same as buying price, there is loss. Why is it so?

c. Students of a certain age innately understand profit and loss. It is unfortunate that our mathematics textbooks introduce these ideas even before children reach the age of ten. At that age it is difficult to introduce the concept of buying bulk quantities at wholesale price and selling smaller quantities at retail price.
Students of this manual, the author hopes, understand some simple business transactions (=trading, selling and buying). Such students can ask their relatives and friend about their trade (=small business). Write about profit & loss. Some hints:

Name / nature of business: _____________
How much bought: __________________
Buying price: ___________________
Any other expenditure: _______________
Total money put in this: ______________
Selling price: ______________________
Total sales money: _________________

Therefore
Profit = Total sales – Total Expenditure
Loss = Total Expenditure – Total Sales

40.11 Percent

1. Student A passed SSLC with 330 marks. Assuming total maximum marks is 600, calculate his average % marks. What is the minimum % marks for a pass and the total? If the mother tongue is for maximum of 125, what are the new numbers?

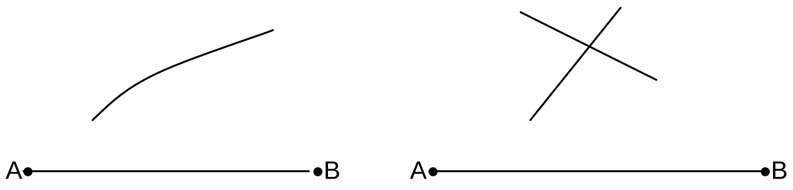
2. A department store had announced 10% discount on all items. A customer bought the following: Textiles Rs. 800, provisions Rs. 700, electronic items Rs. 4000, others Rs. 500. Calculate how much he had saved. Tax (vat) is charged on the final billed amount. If tax is 10%, did he save anything at all?

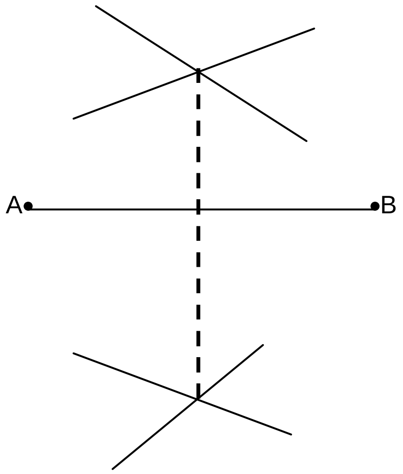
3. A person started with a capital of Rs. 1000. If he wants to get a profit of 20%. What should be his sales? He gives a commission of 10% to a seller. What will be the vendor's prices?

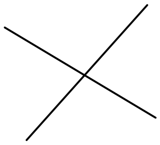
4. A saree in Surat costs Rs. 200. It comes to Mumbai. From there it comes to a wholesale dealer at Mysore. From him a retailer buys and sells. At every stage 10% profit margin is available. What is the final retail price of one saree at Mysore?

40.12 Constructions

40.12.1

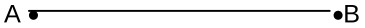






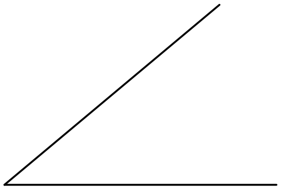
Describing the construction given above. Say what have achieved. [Can you PROVE your statement? Prove means showing what you have done is correct].

40.12.2



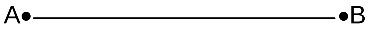
Angle of 90^o is needed at points A & B. Describe how you will do it (using only compass).

40.12.3



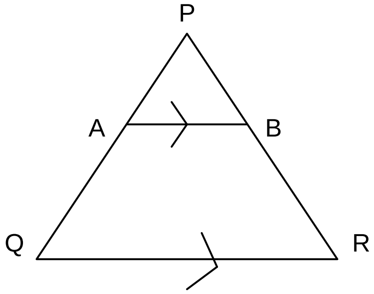
An angle is given. Show how you can bisect it (i.e., make it into 2 equal parts).

40.12.4



How will you divide the line AB into 3 equal parts, 7 equal parts, n equal parts?

Hint: Theorem says $\frac{PA}{PQ} = \frac{PB}{PR}$ if AB ıı QR. Use this property].



40.13 Business

1. Bought a quintal of something for ___ . Sold at ____ per kg. Profit or loss and percent? Make many examples. Let the students make the questions.
2. Add an extra factor of transport cost (and sellers salary if you like). Make questions. In the above, instead of assuming any selling price, turn the questions in to: If the seller wants a profit of 10% what should be the sale price? Let students work in groups. Let different groups take different percent profit margin starting from 10% to 20, 30 and even 100%. Let them work out.
3. Take any two of the items discussed above and add many other factors mentioned. Work out the sale price if a profit of 20% is to be achieved.
4. To (3) above, add retail dealers margin. He will get 10%, not on the cost price, but on his cost price. Teachers, try to work out this.
5. Take a vegetable vendor. Fill in the blanks:

Cost of a basket of 100 kg	------
Auto charges	------
Total Expenses	------

Cost of 1 kg (buying price) = $\frac{\boxed{}}{100}$

Selling price = $\frac{\boxed{}}{\text{KG}}$

Profit =?

6. In (5) above, if the vendor wants to have 25% profit. What will be sale Price/kg?
7. In (6) above, if 20% is wasted (i.e., 20 kg lost) what should be per kg selling price?

40.14 Borrowing

1. Go to a bank. Find out how much rate of interest on money kept in (a) Current account (b) Saving bank account.
2. In (1) above, do some more work. Find about FD and RD (FD fixed deposit, RD recurring deposit).
3. Loan of 10 rupees. Every week 1 rupee interest. How much do you pay at the end of 1 year? What is the rate of interest? Anything more than 50% (interest / annum) is called exorbitant (= too much, very bad). Is this exorbitant or not. Do such things happen in your area?
4. Bank gives loan for home-building at 18%. Maximum of 10 lakhs. How much interest should be paid per month?
5. In (4) above, you can only pay maximum of Rs. 1000 per month. How much loan amount can you take?

40.15 Puzzles:

1. A mother's age is 3 times daughter's age. After 10 years this ratio will become 2. What are their ages?
2. Mr. G had some money. Doubled it by gambling.
   Gave away Rs. 2. Again doubled it by gambling.
   Gave away Rs. 2. Again doubled it by gambling.
   Gave away Rs. 2. He had Rs. 10 left
   What did he have in the beginning?
3. Mr. H did the same thing. But he gave away Re. 1 every time. What did he start with? (He had Rs. 18 at the end).
4. Ajji is twice mummy's age. Mummy is twice beti's age. Beti is twice Beta's age. Beta is twice Munni's age. Munni is 5 times baby's age. If baby is just one year old, what is Ajji's age.
5. In 20 years time Ajji will be a grand old lady. At that time and now: Fill up

	Now	**Later**
Ajji's age/ Beti's age		
Ajji's age/ Beta's age		
Ajji's age/ Munni's age		
Ajji's age/ Baby's age		

40.16 Equations

1. $x + y = 22$ $y = 10$ x =?
2. $x + y = 24$ $x - y = 4$ x =? y = ?
3. $2x + y = 35$ $2y + x = 40$ Solve
4. $x^2 - y^2 = 9$ $y = 4$ x =?

5. $x^2 - y^2 = 9$ $x^2 + y^2 = 41$Solve
6. Find x, given that x - 4 = 0
7. Find x, given that x - 3 = 0
8. Find x, given that (x - 4) (x - 3) = 0
9. Find x, given that x (x - 4) = 0
10. Find x, given that x (x - 4) (x - 3) = 0
11. $\frac{2x}{7} = 22$ x =?
12. $\frac{7x}{2} = 98$ x =?
13. $\frac{2x}{7} + \frac{7x}{2} = 120$ x =?

14. $x^2 + 7x + 2 = x^2 - 7x + 16$ x =?
15. $x^2 + xy + y^2 + 7x + 2 = x^2 + xy + y^2 - 7x + 16$ x =?
16. $x^2 + abc + a^2b + 7x + 2 = x^2 + abc + a^2b - 7x - 1$ x =?

40.17 1. Using setsquare draw an angle of 45^0
2. Do (1) above using only compass.
3. Draw 90^o angle and bisect it. Bisect this also.
4. Draw an equilateral triangle whose
   a. Sides (are equal, of course) are 10 cm.
   b. Angles are 60^0
   [Show that question (b) was set up by a person who wants to pull your leg (=make fun of you)].
5. 4(b) examiner wrote some more questions:
   a. Draw a triangle whose sides are 10, 5, 5 cms.
   b. Draw a triangle whose sides are 8, 4, 3 cms.
   c. Draw a right angled triangles whose angles are 90^0, 70^0, 30^0
   d. Draw a right angled triangle whose sides are 5 cm, 5 cm and 8 cm.
6. A friend of the examiner above (let us call him Mr. Khota Sikka) had set up some questions of his own.
   a. Draw a rectangle of sides 3, 4, 5, 6 cms.
   b. Draw a rectangle of sides 10, 5, 10, 8 cm
   c. Draw a square of side 10cm inside a circle of diameter 10 cm.
   d. Draw a circle of radius 5 cm inside an equilateral triangle of side 10 cm.

40.18 a. Prove (or just say with some reason and confidence) that the following questions are asked by Mr. Khota Sikka:
1. Solve $x + y = 5$ and $2x + 2y = 10$
2. Solve $x + y = 0$ and $x - y = 0$
3. Solve $x^2 - y^2 = 0$ and $x = 2y$
4. a + b + c + 2 = a + b + c – 2; (a + b + c) =?

b. Mr. Khota sikka's student is chhota sikka. Obviously his notebooks are full of mistakes. Some are here: can you correct him?

a. $\frac{15}{5} = ?$

$$\begin{array}{r} 21 \\ 5\overline{)15} \\ \underline{10} \\ 5 \\ \underline{5} \\ 0 \end{array}$$

$\therefore \frac{15}{5} = 21$

b. 15 x 25 = 0

```
  15
x 25
------
   75
  30
 -----
  105
 -----
```

$\therefore$ 15 x 25 = 105

c. x - 10 = 15 find x

x - 10 = 15

x =15 – 10 =5

d. $\frac{x}{10}$ =10 x =? $\frac{x}{10}$ = 10 x = $\frac{10}{10}$ =1

40.19 Shri. Andazwala does not mind approximate answers to some difficult problems.

1. Given $\sqrt{25}$ = 5; $\sqrt{49}$ = 7; $\sqrt{37}$ =? ; $\sqrt{30}$ =?
2. $\sqrt{10} \approx$ 3.0 and $\sqrt{3} \approx$ 1.8, $\sqrt{30}$ =?
3. $(10)^2$ = 100 and $(11)^2$ = 121, $(10.5)^2$ =?
4. a. Give the formula $(a + b)^2$ =
   b. Find $(102)^2$, $(401)^2$, $(1001)^2$
5. a. $(a – b)^2$ =?
   b. Find $(49)^2$, $(98)^2$, $(999)^2$
6. a. (a+b) (a-b) =?
   b. (105) x (95); (147) x (53); (99 x (101)
7. Given that 1234 x 12 = 14808
   Find (1) 1235 x 12
   (2) 1234 x 13
   (3) 1235 x 14
8. Those who find (7) above quite tough try this first and then go back.
   Given that 13 x 7 = 91
   Find 14 x 7 =?
   13 x 8 =?
   14 x 9 =?

Answers and Solutions

[Numbers refer to paragraphs. Solutions are full answers or extensive answers. These are given where methods and steps are as important as the results. Thus chapters 1 to 7 get answers in details. For the others, only answers are given].

Chapter – 1

1.3.4 55, 56, 59, 61, 64, 65, 76, 89, 91, 104

1.3.5 160, 170, 190, 220, 255, 260, 270, 1110

Ex I.1 a. 7 b. 77 c. 777 d. 707 e. 7007 f. 7077
g. 70777 h. 7007 i. 707000 j. 7070007

Ex I. 2
- a. Naluvaththelu – Forty Seven
- b. Eighty Seven
- c. One hundred and seven
- d. Three hundred and fifty nine
- e. Eight thousand eight hundred eighty nine
- f. Sixty six lakhs sixty six thousand six hundred eight nine
- g. Sixty six thousand six hundred eighty nine
- h. Five lakhs sixty seven thousand eight hundred ninety one.
- i. One crore twenty three lakhs forty five thousand six hundred seventy eight.

Ex I. 3 a. 101>99 b. 801>799 c. 49999> 5001 d. 100001>99899

Ex I.4 a. 6989>7444 b. 609<906 c. 769<771 d. 9898989<10203040

Ex I.5 a. 112<121<201 b. 999<1001<9821 c. 41099<101012<110012
d. 7667766<20202021<191919191

Ex I.6 a. 201>121>112 b. 999<1001<9821 c. 41099<101012<110012
d. 191919191>20202021>7667766

Ex I.7
a. 1<3<4<5<8 ; 8>5>4>3>1
b. 11<13<14<15<18 ; 18>15>14>13>11
c. 10<30<40<50<80 ; 80>50>40>30>10
d. 5<65<85<125<325 ; 325>125>85>65>5
e. 1<11<101<111<1001<100001 ; 100001>1001>111>101>11>1
f.190<889<898<900<989<998<1989;1989>998>989>900>898>889>190
g.1002<1020<1112<1120<1201<1220;1220>1201>1120>1112>1020>1002

Ex. I.8 Oral: a. (28+1) Twenty nine, thirty, thirty one, thirty two = 32
(2) (3) (4)
b. 131 c. 104 d. 104 e. 105 f. 1240 g. 107 h. 12391

Ex. I.9
a. 98 + 9 = 108 – 1 = 107 b. 89 + 8 = 99 – 2 = 97
c. 184 + 7 = 194 – 3 = 191 d. 178 + 8 = 188 – 2 = 186
e. 12345 + 9 = 12355 – 1 = 12354 f. 12346 + 8=12356 -2=12354

Ex. I.10

a. 99
+1

100

$$\left.\begin{array}{l} 99 = 90 + 9 \\ \;\;1 = \quad\;\; 1 \end{array}\right\} = 90 + 10 = 100$$

b. 87
+3

90

$$\left.\begin{array}{l} 87 = 80 + 7 \\ \;\;3 = \quad\;\; 3 \end{array}\right\} = 80 + 10 = 90$$

c.
$$\begin{array}{r} 76 \\ +3 \\ \hline 79 \\ \hline \end{array} \qquad \left.\begin{array}{l} 76 = 70 + 6 \\ 3 = \quad 3 \end{array}\right\} = 70 + 9 = 79$$

d.
$$\begin{array}{r} 65 \\ +4 \\ \hline 69 \\ \hline \end{array} \qquad \left.\begin{array}{l} 65 = 60 + 5 \\ 4 = \quad 4 \end{array}\right\} = 60 + 9 = 69$$

e.
$$\begin{array}{r} 43 \\ +36 \\ \hline 79 \\ \hline \end{array} \qquad \left.\begin{array}{l} 43 = 40 + 3 \\ 36 = 30 + 6 \end{array}\right\} = 70 + 9 = 79$$

f.
$$\begin{array}{r} 10099 \\ + \quad 1 \\ \hline 10100 \\ \hline \end{array} \qquad \left.\begin{array}{l} 10099 = 10000 + 90 + 9 \\ \quad 1 \qquad\qquad\qquad 1 \end{array}\right\} \begin{array}{l} = 10000 + 90 + 10 \\ = 10000 + 100 \\ = 10100 \end{array}$$

g.
$$\begin{array}{r} 20087 \\ + \quad 3 \\ \hline 20090 \\ \hline \end{array} \qquad \left.\begin{array}{l} 20087 = 20000 + 80 + 7 \\ \quad 3 \qquad\qquad\qquad 3 \end{array}\right\} \begin{array}{l} = 20000 + 80 + 10 \\ =20000+90 \\ = 20090 \end{array}$$

h.
$$\begin{array}{r} 30076 \\ + \quad 3 \\ \hline 30079 \\ \hline \end{array} \qquad \left.\begin{array}{l} 30076 = 30000 + 70 + 6 \\ \quad 3 \qquad\qquad\qquad 3 \end{array}\right\} \begin{array}{l} = 30000 +70 + 9 \\ = 30000+79 \\ = 30079 \end{array}$$

i.
$$\begin{array}{r} 40065 \\ + \quad 4 \\ \hline 40069 \\ \hline \end{array} \qquad \left.\begin{array}{l} 40065 = 40000 + 60 + 5 \\ \quad 4 \qquad\qquad\qquad 4 \end{array}\right\} \begin{array}{l} = 40000 + 60 + 9 \\ = 40000 + 69 \\ = 40069 \end{array}$$

j.
$$\begin{array}{r} 50043 \\ + \quad 36 \\ \hline 50079 \\ \hline \end{array} \qquad \left.\begin{array}{l} 50043 = 50000 + 40 + 3 \\ \quad 36 \qquad\qquad 30 + 6 \end{array}\right\} \begin{array}{l} = 50000+70 + 9 \\ = 50000 + 79 \\ = 50079 \end{array}$$

Ex. I.11

a.
$$\begin{array}{r} 99 \\ +3 \\ \hline 102 \\ \hline \end{array} \qquad \left.\begin{array}{l} 99 = 90 + 9 \\ 3 = \quad 3 \end{array}\right\} = 90 + 12 = 102$$

b.
$$\begin{array}{r} 87 \\ +5 \\ \hline 92 \\ \hline \end{array} \qquad \left.\begin{array}{l} 87 = 80 + 7 \\ 5 = \quad 5 \end{array}\right\} = 80 + 12 = 92$$

c.
$$\begin{array}{r} 176 \\ + \ 6 \\ \hline 182 \\ \hline \end{array} \qquad \left.\begin{array}{l} 176 = 170 + 6 \\ 6 = \qquad 6 \end{array}\right\} = 170 + 12 = 182$$

d. $$\begin{array}{r} 465 \\ +\ 7 \\ \hline 472 \\ \hline \end{array} \qquad \left.\begin{array}{rr} 465 = 400 + 60 + & 5 \\ 7 = & 7 \end{array}\right\} = 400 + 60 + 12 = 472$$

e. $$\begin{array}{r} 843 \\ +38 \\ \hline 881 \\ \hline \end{array} \qquad \left.\begin{array}{rr} 843 = 800 + & 40 + 3 \\ 38 = & 30 + 8 \end{array}\right\} \begin{array}{l} = 800 + 70 + 11 \\ = 800 + 81 \\ = 881 \end{array}$$

f. $$\begin{array}{r} 10099 \\ +\ \ 3 \\ \hline 10100 \\ \hline \end{array} \qquad \left.\begin{array}{rr} 10099 = 10000 + 90 + & 9 \\ 1 & 1 \end{array}\right\} \begin{array}{l} = 10000 + 90 + 10 \\ = 10000 + 100 \\ = 10100 \end{array}$$

g. $$\begin{array}{r} 10087 \\ +\ \ 5 \\ \hline 10092 \\ \hline \end{array} \qquad \left.\begin{array}{rr} 10087 = 10000 + 80 + & 7 \\ 5 & 5 \end{array}\right\} \begin{array}{l} = 10000 + 80 + 12 \\ =10000+92 \\ = 10092 \end{array}$$

h. $$\begin{array}{r} 1000176 \\ +\ \ 6 \\ \hline 100182 \\ \hline \end{array} \qquad \left.\begin{array}{rr} 10000176 = 100000 + 100 + 70 + & 6 \\ 6 & 6 \end{array}\right\} \begin{array}{l} =100000 +100+82+12 \\ =100000+100 +82 \\ =100182 \end{array}$$

i. $$\begin{array}{r} 100465 \\ +\ \ 7 \\ \hline 100472 \\ \hline \end{array} \qquad \left.\begin{array}{rr} 100465 = 100000 + 400 + 60 + & 5 \\ 7 & 7 \end{array}\right\} \begin{array}{l} =100000+400+60+12 \\ =100000 + 400 + 72 \\ = 100472 \end{array}$$

j. $$\begin{array}{r} 100843 \\ +\ \ 38 \\ \hline 100881 \\ \hline \end{array} \qquad \left.\begin{array}{rr} 100843 = 100000 + 800 + & 40 + 3 \\ 38 & 30 + 8 \end{array}\right\} \begin{array}{l} =100000+800+70+11 \\ =100000 + 800 + 81 \\ = 100881 \end{array}$$

Ex I.12

a. $$\begin{array}{r} 11 \\ 29 \\ 38 \\ \hline 78 \\ \hline \end{array}$$ b. $$\begin{array}{r} 12 \\ 28 \\ 37 \\ 55 \\ \hline 132 \\ \hline \end{array}$$ c. $$\begin{array}{r} 14 \\ 27 \\ 35 \\ 55 \\ \hline 131 \\ \hline \end{array}$$ d. $$\begin{array}{r} 17 \\ 24 \\ 34 \\ 56 \\ \hline 131 \\ \hline \end{array}$$ e. $$\begin{array}{r} 10011 \\ 10029 \\ 10038 \\ \hline 30078 \\ \hline \end{array}$$ f. $$\begin{array}{r} 10012 \\ 10028 \\ 10037 \\ 10055 \\ \hline 40132 \\ \hline \end{array}$$

g. $$\begin{array}{r} 1014 \\ 1027 \\ 1035 \\ 1055 \\ \hline 4131 \\ \hline \end{array}$$ h. $$\begin{array}{r} 917 \\ 924 \\ 934 \\ 956 \\ \hline 3731 \\ \hline \end{array}$$

==

Chapter – 2

2.1

1 + 4 = 5 and 5 – 4 =1 and 5 – 1 = 4
2 + 3 = 5 and 5 – 3 =2 and 5 – 2 = 3
3 + 2 = 5 and 5 – 2 =3 and 5 – 3 = 2
4 + 1 = 5 and 5 – 1 =4 and 5 – 4 = 1
5 + 0 = 5 and 5 – 5 =0 and 5 – 0 = 5

2.2.3

$$\begin{array}{r} 8 \\ -1 \\ \hline 7 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -2 \\ \hline 6 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -3 \\ \hline 5 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -4 \\ \hline 4 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -5 \\ \hline 3 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -5 \\ \hline 2 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -6 \\ \hline 2 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -7 \\ \hline 1 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -8 \\ \hline 0 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -0 \\ \hline 8 \\ \hline \end{array}$$

2.3.1

$$\begin{array}{r} 5 \\ -0 \\ \hline 5 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ -1 \\ \hline 5 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ -2 \\ \hline 5 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ -3 \\ \hline 5 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ -4 \\ \hline 5 \\ \hline \end{array} \quad \begin{array}{r} 55 \\ -10 \\ \hline 45 \\ \hline \end{array} \quad \begin{array}{r} 66 \\ -20 \\ \hline 46 \\ \hline \end{array} \quad \begin{array}{r} 77 \\ -30 \\ \hline 47 \\ \hline \end{array} \quad \begin{array}{r} 82 \\ -42 \\ \hline 46 \\ \hline \end{array} \quad \begin{array}{r} 99 \\ -44 \\ \hline 55 \\ \hline \end{array}$$

2.3.3

$$\begin{array}{r} 123 \\ -122 \\ \hline 1 \\ \hline \end{array} \quad \begin{array}{r} 123 \\ -112 \\ \hline 11 \\ \hline \end{array} \quad \begin{array}{r} 246 \\ -135 \\ \hline 111 \\ \hline \end{array}$$

2.4.1

$$\begin{array}{r} 2 \\ -1 \\ \hline 1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -2 \\ \hline 0 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline 11 \\ \hline \end{array} \quad \begin{array}{r} 12 \\ -2 \\ \hline 10 \\ \hline \end{array}$$

2.5 Subtraction Grid

-	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1		0	1	2	3	4	5	6	7	8
2			0	1	2	3	4	5	6	7
3				0	1	2	3	4	5	6
4					0	1	2	3	4	5
5						0	1	3	23	4
6							0	1	2	3
7								0	1	2
8									0	1
9										0

Ex. II.1

a. 25 – 4 = 24 23 22 21 = 21
(1) (2) (3) (4)
b. 16 – 3 = 13 (in 3 steps)
c. 17 – 5 = 12 (in 5 steps)
d. 29 – 2= 27 (in 2 steps)
e. 30 – 6 = 24 (in 6 steps)
f. 10025 – 4 = 10021 (in 4 steps) [see 1 & in 1 step]
g. 20016 – 3 = 20013 (in 3 steps) [see 2 & in 2 step]
h. 30017 – 5 = 30012 (in 5 steps) [see 3 & in 3 step]
i. 50029 – 2 = 50027 (in 2 steps) [see 4 & in 4 step]
j. 99930 – 6 = 99924 (in 6 steps) [see 5 & in 1 step]

Ex. II.2 Oral Method

a. 25 – 6 = (25 – 10) + 4 = 15 + 4 = 19
b. 16 – 7 = 7 + 3 = 9
c. 17 – 9 = 7 + 1 = 8
d. 29 – 8 = 19 + 2 = 21
e. 30 – 9 = 20 + 1 = 21
f. 10025 – 6 = 10019 [see 1 & in 1 step]
g. 20016 – 7 = 20009 [see 2 & in 2 step]
h. 30017 – 9 = 30008 [see 3 & in 3 step]

i. 50029 – 8 = 50021 [see 4 & in 4 step]
j. 99930 – 9 = 99921 [see 5 & in 5 step]

Ex. II.3

a. $\begin{array}{r} 987 \\ -\ 76 \\ \hline 911 \\ \hline \end{array}$ b. $\begin{array}{r} 987 \\ -\ 43 \\ \hline 933 \\ \hline \end{array}$ c. $\begin{array}{r} 654 \\ -\ 42 \\ \hline 612 \\ \hline \end{array}$ d. $\begin{array}{r} 109 \\ -\ 107 \\ \hline 2 \\ \hline \end{array}$ e. $\begin{array}{r} 20128 \\ -10117 \\ \hline 10011 \\ \hline \end{array}$

f. $\begin{array}{r} 17 \\ -\ 6 \\ \hline 11 \\ \hline \end{array}$ g. $\begin{array}{r} 26 \\ -\ 13 \\ \hline 13 \\ \hline \end{array}$ h. $\begin{array}{r} 34 \\ -\ 12 \\ \hline 22 \\ \hline \end{array}$ i. $\begin{array}{r} 9823109 \\ -9823107 \\ \hline 2 \\ \hline \end{array}$ J. $\begin{array}{r} 138 \\ -107 \\ \hline 31 \\ \hline \end{array}$

Ex. II.4

a. $\begin{array}{r} 987 \\ -\ 88 \\ \hline 899 \\ \hline \end{array}$ b. $\begin{array}{r} 931 \\ -\ 43 \\ \hline 888 \\ \hline \end{array}$ c. $\begin{array}{r} 654 \\ -\ 65 \\ \hline 589 \\ \hline \end{array}$ d. $\begin{array}{r} 107 \\ -\ 88 \\ \hline 19 \\ \hline \end{array}$ e $\begin{array}{r} 20117 \\ -\ 10128 \\ \hline 9989 \\ \hline \end{array}$ f. $\begin{array}{r} 27 \\ -\ 18 \\ \hline 9 \\ \hline \end{array}$

g. $\begin{array}{r} 56731 \\ -\ 12343 \\ \hline 44388 \\ \hline \end{array}$ h. $\begin{array}{r} 123654 \\ -\ 12365 \\ \hline 111289 \\ \hline \end{array}$ i. $\begin{array}{r} 987107 \\ -987088 \\ \hline 119 \\ \hline \end{array}$ J. $\begin{array}{r} 7117 \\ -6128 \\ \hline 989 \\ \hline \end{array}$

Ex. II.5 **Same as Ex. II.4**

==

Chapter – 3

3.2

a. $\begin{array}{r} 5 \\ +\ 4 \\ \hline 9 \\ \hline \end{array}$ b. $\begin{array}{r} 18 \\ +\ 4 \\ \hline 22 \\ \hline \end{array}$ c. $\begin{array}{r} 215 \\ +\ 4 \\ \hline 219 \\ \hline \end{array}$ d. $\begin{array}{r} 215 \\ +14 \\ \hline 229 \\ \hline \end{array}$ e. $\begin{array}{r} 215 \\ +114 \\ \hline 329 \\ \hline \end{array}$ f. $\begin{array}{r} 3215 \\ +114 \\ \hline 3329 \\ \hline \end{array}$

g. $\begin{array}{r} 3215 \\ +\ 2215 \\ \hline 5430 \\ \hline \end{array}$ h. $\begin{array}{r} 1234500678 \\ +\ \ 234005121 \\ \hline 1464505799 \\ \hline \end{array}$

3.3

a. $\begin{array}{r} 6 \\ +\ 4 \\ \hline 10 \\ \hline \end{array}$ b. $\begin{array}{r} 86 \\ +\ 4 \\ \hline 10 \\ \hline \end{array}$ c $\begin{array}{r} 96 \\ +\ 4 \\ \hline 100 \\ \hline \end{array}$ d. $\begin{array}{r} 886 \\ +\ \ 4 \\ \hline 890 \\ \hline \end{array}$ e. $\begin{array}{r} 996 \\ +\ \ 4 \\ \hline 1000 \\ \hline \end{array}$ f. $\begin{array}{r} 9886 \\ +\ \ 4 \\ \hline 9890 \\ \hline \end{array}$

g. $\begin{array}{r} 9886 \\ +\ \ 14 \\ \hline 9900 \\ \hline \end{array}$ h. $\begin{array}{r} 9886 \\ +\ 104 \\ \hline 9990 \\ \hline \end{array}$

3.6.2

a. $\begin{array}{r} 12345 \\ -\ \ 2345 \\ \hline 10000 \\ \hline \end{array}$ b $\begin{array}{r} 12345 \\ -\ \ 2234 \\ \hline 10111 \\ \hline \end{array}$ c. $\begin{array}{r} 12345 \\ -\ 11345 \\ \hline 1000 \\ \hline \end{array}$ f.

3.6.3

a. $\begin{array}{r} 8765 \\ -\ 7965 \\ \hline 798 \\ \hline \end{array}$ b. $\begin{array}{r} 12345 \\ -\ \ 3456 \\ \hline 8889 \\ \hline \end{array}$ c. $\begin{array}{r} 1234 \\ -\ \ 235 \\ \hline 999 \\ \hline \end{array}$

Ex. III.1

1		One
21		Twenty one
331		Three hundred thirty one
4331		Four thousand three hundred thirty one
54441		Fifty four thousand four hundred forty one
654321	= 6,54,321	Six lakh fifty four thousand three hundred twenty one.
	= 654,321	Six hundred fifty four thousand three hundred twenty one.

7654321 =76,54,321 Seventy six lakh fifty four thousand three hundred twenty one.
=7,654,321 Seven million seventy six lakh fifty four thousand three hundred twenty one.

87654321 = 8,76,54,321 Eight crores seventy six lakh fifty four thousand three hundred twenty one.
= 87,654,321 Eighty seven million six lakh fifty four thousand three hundred twenty one.

987654321= 98,76,54,321 Ninety eight crores seventy six lakh fifty four thousand three hundred twenty one.
= 987,654,321 Nine hundred eighty seven million six lakh fifty four thousand three hundred twenty one.

Ex. III.2
9
89
889
9889
9911
900001
990001
6500004
7060005
4000000001
400000001

Ex. III.3 a. 1 lakh b. 10 lakhs c. 100 lakhs d. 10 millions e. 1000 millions f. 100 crores h. 10000 lakhs

Ex. III.4 a. 600 b. 600 c. 100 d. 0 e. 674 f. 664 g. 564 h. 70 i. 60 j. 100 k. 90 l. 80 m. 590 n. 589 o. 577 p. 79

Ex. III.5 a. 32000010 b. 32000000 c. 31999990 d. 31999980
e. 31998990 f. 31988990 g. 31888990 h. 32000042
i. 31889248 j. 31688990

Ex. III.6 (+,+), (+,+), (+,+), (+,-),(+,-), (+,-), (-,+),(-,+), (+,-), (-,+),(+,-), (-,+), (+,-),(-,+),(+,-), (-,+),(-,+) (+,+), (-,+)

Chapter – 4

4.6 Multiplication grid :

X	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	24	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

4.10b

X	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

Ex. IV.1	a.	123	(123) x 3
		X 3	= (100 + 20 + 3)3
		-------	= 300 + 60 + 9 = 369
		369	
		--------	
	b.	812	(812) x 3
		X 3	= (800 + 10 + 2)3
		-------	= 2400 + 30 + 6 = 2436
		2436	
		--------	
	c.	922	(922) x 4
		X 4	= (900 + 20 + 3)4
		-------	= 3600 + 80 + 8 = 3688
		3688	
	d.	202	(202) x 5
		X 5	= (200 + 2) 5
		-------	= 1000 + 10 = 1010
		1010	
	e.	123123	(123123) x 3
		X 3	= (100000+ 20000 + 3000 + 100 + 20 + 3)3
		----------	= 300000 + 60000 + 9000 + 300 + 60 + 9= 3688
		369369	= 300000
			+ 60000
			+ 9000
			+ 300
			+ 60
			+ 9

			369369
			----------------
	f.	812312312	(812312312) x 3
		X 3	= 2400000000
		--------------	30000000
		2436936936	6000000
			900000
			30000
			6000
			900
			30
			6

			2436936936
	g.	9222121	(9222121) x 4
		X 4	= 360000000
		--------------	8000000
		36888484	800000
			8000
			400
			80
			4

			36888484

```
h. 102102102                 (102102102) x 5
    X          5             = 500000000
   ----------------             00000000
   510510510                     1000000
                                  500000
                                   00000
                                   10000
                                     500
                                      00
                                      10
                               -------------------
                                 510510510
```

Ex. IV.2

```
a.  1 2      8 x (12)        = 8 (10+2)
   X 8                       = 80 + 16
  ------                     = 96
   9 6
 -------

b.  2 2      8 x (22)        = 8 (20+2)
   X 8                       = 160 + 16
 -------                     = 176
   176
 -------

c.  3 5      9 x (35)        = 9 (30+5)
   X 9                       = 270 + 45
 -------                     = 315
   315
 --------

d.  9 8 7 6  5 x (9876)      = 5 (9000+800+70+6)
    X      5                 = 45000
 ------------                   4000
   49380                         350
 -----------                      30
                                --------
                                49380

e.  9 8 7 6  2 x (19752)     = 2 (9000+800+70+6)
    X      2                 = 18000
 ------------                   1600
   1s9752                        140
 -----------                      12
                                -----------
                                19752

f.  9 1 4 3  2 x (9143)      = 2 (9000+100+40+3)
    X      2                 = 18000
 ------------                  200
   18286                        80
 -----------                     6
                              -----------
                               18286

g. 9 1 4 3   3 x (9143)      = 3 (9000+100+40+3)
    X     3                  = 27000
 ------------                    300
   27429                         120
 -----------                       9
                               ----------
                               27429
```

```
h. 2 3 4 5          4 x (2345)        = 4 (2000+300+40+5)
     X   4                            = 8000
 ------------                            1200
   9380                                   160
 -----------                               20
                                       -----------
                                         9380

i. 2 2 2 8          6 x (2228)        = 6 (2000+200+20+8)
   X    6                             = 12000
 ------------                             1200
   13368                                   120
 -----------                                48
                                         --------------
                                         13368
```

Ex. IV.3 Writing is important

```
a.   3 4            34 x 12                  b.     2 3         23 x 33
   X 1 2            = 34 (10 + 2)                 X 3 3         = 23 (30 + 3)
    ------          = 340                         ------        = 690
     68              + 68                           69           + 69
    340             ----------                     690         ----------
   -------             408                        ------         759
     408                                           759
```

Similarly do others: (c), (d), (e), (f), (g), (h), (i), (j)

```
Ex. IV.4  a.  56789 x 11     = 56789 (10 + 1)      = 567890
                                                       56789
                                                    -------------
                                                      624679
                                                     -------------
          b.  56789 x 111    = 56789 (110 + 1)     = 6246790
                                                         56789
                                                    -------------
                                                      6303579
                                                    -------------

          c.  56789 x 1112   = 56789 (1110 + 2)    = 63035790
                                                          56789
                                                          56789
                                                        -------------
                                                          6449368
                                                        -------------
          d. 56789 x 12 = 56789 (11 + 1)       = 624679
                                                   56789
                                                 -------------
                                                  681468
                                                 -------------
          e. 56789 x 123= 56789 (120 + 3)      = 6814680
                                                   170367
                                                  -------------
                                                   6985047
                                                  -------------
          f. 56789 x 1234= 56789 (1230+ 4)     = 69850470
                                                    170367
                                                     56789
                                                --------------------
                                                 70077626
```

Ex. IV.5

```
a.  (12) x (6789) = 6789
                     X 12
                   ----------
                    13578
                    67890
                   -----------
                    81468
```

```
b. (123) x (6789) = (120 + 3) (6789)
                  =  81468
                     20367
                 ------------------
                     835047
```

c. (1230) x (6789) = See b Ans: 83504700

d. (12300) x (6789) = See b or c Ans: 83504700

==

Chapter – 5

5.1.2

```
a. 2 x 5 = 2
         + 2
           2
           2
           2
         ----
          10
         ----
```

```
b. 2 x 5 = 5
         + 5
          ----
           10
          -----
```

```
c. 8 x 3 = 8
         + 8
           8
         ----
          24
         -----
```

```
d. 3 x 8 = 3
         + 3
           3
           3
           3
           3
           3
           3
         ----
          24
         -----
```

```
f. 123 x 3 = 123
           + 123
             123
             ----
             369
             -----
```

```
e. 17 x 4 = 17
          + 17
            17
            17
          ----
            24
          -----
```

```
g. 123456 x 3 = 123456
              + 123456
                123456
              -----------
                370368
              ----------
```

```
h. 98765 x 4 = 98765
              +98765
               98765
               98765
             -----------
               395060
             ------------
```

[Students may see that 9 x 9 multiplication grid helps here. i.e., 6 added 4 times = 4 x 6 = 24 can be used while adding the same number many times].

```
I.  29 x 9 can be done by writing 9 times or
    29 x 9 = (29 x 10) – 29 x 1 = 290
                                  -29
                                 -----
                                  261
                                  ----
```

5.4.2 Num = Numerator Den = Denominator

Ratio	Fraction	Num	Den
15 ÷5	15/5	15	5
9 ÷3	9/3	9	3
21 ÷7	21/7	21	7
16 ÷4	16/4	16	4

Ratio	Fraction	Num	Den
36 ÷9	15/5	15	5
24 ÷6	9/3	9	3
12345 ÷25	12345/25	12345	25
9676 ÷ 8	9676/8	9676	8

5.4.3

Ratio	Fraction	Quotient	Remainder
15 ÷5	15/5	3	0
16 ÷5	16/5	3	1
18 ÷5	18/5	3	3
9 ÷ 3	9/3	3	0
8 ÷ 3	8/3	2	2
10 ÷3	10/3	3	1
12345 ÷ 25	12345/25	493	20
12375 ÷ 25	12375/25	495	0
9676 ÷ 8	9676/8	129	4
9672 ÷8	9672/8	129	0

5.8.1 a. 13 x 1 = 13
13 x 2 = 26
13 x 3 = 39
13 x 4 = 52
13 x 5 = 65

Seeing this

a1) $\frac{65}{13} = 5$ a2) $\frac{52}{4} = 13$ a3) $\frac{26}{2} = 13$ a4) $\frac{39}{13} = 3$

b. Students own question and answers

c. 137 x 17 = 2329. $\therefore \frac{2337}{17} = 137$

d. (13579) x 24 = 325896 $\therefore \frac{325896}{13579} = 24$ $\frac{325896}{24} = 13579$

5.8.3 a. Same as 5.8.1(a)

a1. $\frac{67}{13} = \frac{65+2}{13}$ = 5 + Remainder 2

a2. $\frac{50}{4} = \frac{52-2}{4}$ = 13 – (2 remaining) = 12 + 2 remainder

a3. $\frac{25}{2} = \frac{26-1}{2}$ = 13 – (1 remaining) = 12 + 1 remainder

a4. $\frac{42}{13} = \frac{39+3}{13}$ = 3 + (3 remaining) = 3 + 3 remainder

b. Students own question and answers.

c. 13 x 17 = 221 $\frac{228}{13} = \frac{221+7}{13}$ = 17 + (7 remainder)

d. 13 x 17 = 221 $\frac{228}{17} = \frac{221+7}{17}$ = 13 + (7 remainder)

e. (1234) (56789) = 70077626

(e1) $\frac{70077626}{56789} = 1234$

(e2) $\frac{70077628}{56789} = \frac{70077626+2}{56789}$ = 1234 + [2 remainder]

(e3) $\frac{70087626}{56789} = \frac{70077626+10000}{56789}$ = 1234 + [10000 remainder]

5.9.3 a. $\frac{92345676}{2} = 46172838$ b. $\frac{92345676}{4} = 23086419$

c. $\frac{92345676}{5} = 18469135 + [1\text{ remainder}]$ d. $\frac{92345676}{6} = 15390946$

e. $\frac{92345676}{7} = 13192239 + [3\text{ remainder}]$ f. $\frac{92345676}{8} = 11543209 + [1\text{ remainder}]$

g. $\frac{92345676}{9} = 10260630 + [6\text{ remainder}]$ h. $\frac{92345676}{10} = 9234567 + [6\text{ remainder}]$

i. $\frac{124}{6} = 20 + [4\text{ remainder}]$ j. $\frac{125}{5} = 25$ k. $\frac{126}{7} = 18$ l. $\frac{121}{11} = 11$

5.9.6 a. Add all digits i.e., 9+2+3+4+5+6+7+6 = 42 div by 3 yes
b. Last 2 digits 76. It is div by 4 yes. Therefore 92345676 is div by 4 yes.
c. by 5 – No.
d. Div by 2 & 3. Therefore div by 6 – yes.
e. Div by 4 and 3. Therefore div by 12 – yes.
f. by 9; $\frac{42}{9}$ - No. Therefore 92345676 div by 9 – No.
g. 9 + 3 + 5 + 7 = 24 and 2 + 4 + 6 + 6 = 18; div by 11 – No.

5.9.7 i. 2 - a factor: a. 1235 – no b. 1235 – no c. 5000 – yes d. 5005 – no
ii. 3 - a factor: a. 27 – yes b. 31 – n c. 121 – no d. 123 – yes e. 1234 – no
f. 1232 – no
iii. 5 and 3 factors: a. 275 – only 5 b. 285 – both 5 and 3 c. 1230 – both 5 and 3
d. 1235 – only 5

iv.

Number	2	3	4	5	6	7	8	9	10	11
a. 362880	√	√	√	√	√	√	√	√	√	X
b. 3628800	√	√	√	√	√	√	√	√	√	X
c. 399168	√	√	X	√	√	√	√	√	X	X
d. 420	√	√	√	√	√	√	√	X	√	X

a.
10 | 362880
2 | 36288
4 | 18144
9 | 4556
7 | 504
4 | 72
9 | 18
2

10 x 2 x 4 x 9 x7 x 4 x 9 x 2

Some write this as:
2 x 2 x 2 x 2 x5 x 3 x 3 7 x 2 x 2 x 3 x 3 x 2
= 2^7 x 3^4 x 5 x 7

b. b = 10 x a

c.
11 | 399168
8 | 36288
8 | 4536
7 | 567
9 | 81
9

399168 = 11 x 8 x 8 x7 x 9 x 9

Can also be = 2^6 x 3^2 x 7 x 11

d.
10 | 420
2 | 42
7 | 21
3

420 = 10 x 2 x 7 x3

5.9.7.1 Extra a. 6 = 2 x 3 b. 62 = 2 x 31 c. 98 = 2 x 7 x 7 d. 256 = 2^8
e. 22 = 2 x 11 f. 198 = 2 x 9 x 11 g. 106 = 2 x 53 h. 162 = 2 x 9 x 9
i. 298 = 2 x 149 j. 10256 = 2^4 x 641 k. 15 = 5 x 3 l. 47 = 1 x 47
m. 141 = 1 x 141 n. 255 = 5 x 5 x 17 o. 189 = 9 x 3 x 7 p. 115 = 5 x 23
q. 147 = 7 x 7 x 3 r. 241=1 x 241 s. 55 = 5 x 11 t. 199 = 1 x 199

5.9.8 a. $\frac{6860}{7}$ =980 Numerators <7000 Therefore Ans: 1000 Approx 150 less

Therefore 1000 – 20 = 980

b. $\frac{788}{8}$ = 98.5 Numerator < 800 Therefore Ans: 100 12 less

Therefore 1 less = 100 – 1 ≈ 99

c. $\frac{12423}{123}$ = 101 Numerator > 12300 Therefore Ans: 100 130 more

Therefore 1 more = 100 + 1 ≈ 101

d. $\frac{1678}{17}$ = 98 Numerator < 1700 Therefore Ans: 100 20 less

Therefore 2 less = 100 - 2 ≈ 98

e. $\frac{1768}{17}$ = 104 Numerator > 1700 Therefore Ans: 100 70 more

Therefore 4 more = 100 + 4 ≈ 104

f. $\frac{525252}{51}$ = 10305 Numerator > 510000 Therefore Ans: 10000 15000 more

$\frac{15000}{50}$ = 300; Therefore Ans: 10300

g. $\frac{1222382}{1202}$ = 1016 Numerator > 1200000 Therefore Ans: 1000 22000 more

$\frac{22000}{1200}$ = 20; Therefore Ans: 1020

h. $\frac{122230}{1201}$ = 102 Numerator > 120000 Therefore Ans: 100 2200 more

Therefore 2 more Ans: 102

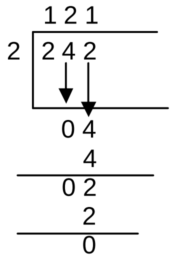
i. $\frac{998}{19}$ = 52 Denominator ≈ 20 Ans ≈ 50

j. $\frac{1028}{19}$ = 54 Denominator ≈ 20 Ans ≈ 53

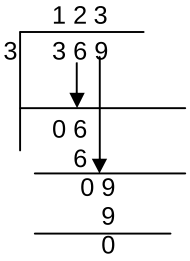
Ex. V.1 a. 9 b. 3 c. 3 d. 9 e. 3 f. 5 g. 2 h. 3 dozens i. Rs. 150
j. Approx Rs. 4 k. 120 l. 3 each.

Ex. V.2

a.



b.



2 1 2
c. 2 | 8 4 8
8
0 4
4
0 8
8
0

1 2 3
d. 5 | 1 0 5 5
1 0
0 5
5
0 5
5
0

1 1 0
e. 6 | 6 6 0
6
0 6
6
0 0
0
0

1 0 1
f. 7 | 7 0 7
7
0 0 7
7
0
0
0

1 1 1
g. 8 | 8 8 8
8
0 8
8
0 8
8
0

1 0 1
h. 9 | 9 0 9
9
0 0 9
9
0
0
0

1 2 1 2 1 2 1
i. 4 | 4 8 4 8 4 8 4
4
0 8
8
0 4
4
0 8
8
0 4
4
0 8
8
0 4
4
0

j same as i

k same as i

l similar to i

Ex. V.3

1 0 9
a. 3 | 3 2 7
3
0 2 7
2 7
0

1 0 3
b. 9 | 9 2 7
9
0 2 7
2 7
0

2 0 3
c. 4 | 8 1 2
8
0 1 2
1 2
0

1 0 9
d. 4 | 4 3 6
4
0 3 6
3 6
0

```
           2 0 3
e.   5  | 1 0 1 5
        | 1 0 ↓ ↓
        |---------
        |  0 1 5
             1 5
        ---------
               0
        ---------
```

f, g, h, i, j to be done in the same way.

Ans: f. 105 g. 102
h. 105 (4 remaining) i. 109
j. 107

Ex. V.4 Writing method is important

```
         0 1 1                   0 6 0 6
a.  11 | 1 2 1       i.  16 | 9 6 9 6
       | 1 1 ↓              | 9 6 ↓ ↓
       |------              |---------
       | 0 1 1              |   0 9 6
           1 1              |     9 6
       -------              |---------
           0                      0
       -------               ---------
```

Similarly all others

Ans: a. 11 b. 506 c. 706 d. 304 e. 403
f. 705 g. 306 h. 313 i. 606 j. 502
k. 20303 (16 remaining) l. 204 m. 503 n. 509

===

Chapter – 6

6.1.2	a. 15 ÷ 3	15/3	Numerator=15	Denominator = 3
	b. 1501 ÷ 501	1501/501	Numerator=1501	Denominator = 501
	c. 999 ÷ 9	999/9	Numerator=999	Denominator = 9
	d. 101 ÷ 101	101/101	Numerator = 101	Denominator = 101
	e. 1010 ÷10	1010/10	Numerator = 1010	Denominator = 10

6.1.3 a. $\frac{15}{5} \rightarrow 15 \div 5$ b. $\frac{10}{3} \rightarrow 10 \div 3$ c. $\frac{123123}{123} \rightarrow 123123 \div 123$

6.1.4 a. $\frac{9}{2} = 4\frac{1}{2}$ b. $\frac{10}{3} = 3\frac{1}{3}$ c. $\frac{17}{4} = 4\frac{1}{4}$ d. $\frac{19}{5} = 3\frac{4}{5}$

e. $\frac{15}{6} = 2\frac{3}{6}$ f. $\frac{20}{7} = 2\frac{6}{7}$ g. $\frac{31}{8} = 3\frac{7}{8}$ h. $\frac{17}{9} = 1\frac{8}{9}$

6.2 a. 5 b. $3\frac{2}{3}$ c. $2\frac{1}{2}$ d. 3 e. 2 f. $2\frac{1}{5}$

g. 2 h. $2\frac{1}{6}$ i. 2 j. $2\frac{1}{7}$ k. $2\frac{7}{8}$ l. 2

m. $1\frac{8}{9}$ n. 2 o. 1 p. $1\frac{1}{9}$ q. $1\frac{1}{4}$ r. $1\frac{3}{7}$

s. $1\frac{2}{3}$

6.4.1 a. $12\frac{3}{10}$ b. $13\frac{1}{5}$ c. $100\frac{1}{10}$ d. $1010\frac{1}{10}$ e. $908070\frac{1}{10}$ f. 123

g. $123\frac{1}{10}$ h. 132 i. 1001 j. $10010\frac{1}{2}$

k. 10101 l. $10102\frac{3}{5}$ m. 90807 n. $908070\frac{1}{10}$

6.4.2 d1) 2 d2) 4 d3) 5 d4) 100 d5) 101 d6) 617
d7) 4444 d8) 4999 d9) 505051

e1) $2\frac{1}{20}$ e2) $4\frac{1}{20}$ e3) $4\frac{19}{20}$ e4) $100\frac{9}{20}$ e5) $101\frac{1}{4}$ e6) $617\frac{8}{20}$

e7) $4444\frac{8}{20}$ e8) $4999\frac{9}{20}$

[see (d1) write (e1), see (d7) write (e7) etc].

6.4.3 a1. 4 (+ remainder 10) a2. 1 (+ remainder 10) a3. 49 (+ remainder 10)

b1. 4 (+ remainder 18) b2. 1 (+ remainder 16) b3. 49 (+ remainder 18)

6.6.1 (Steps are important)

a. $\frac{205}{5} = \frac{200+5}{5}$ Ans: 41 b. $\frac{2025}{5} = \frac{2000+25}{5}$ Ans: 405

c. $\frac{5555}{11} = \frac{5500+55}{11}$ Ans: 105 d. $\frac{2821}{7} = \frac{2800+21}{7}$ Ans: 403

e. $\frac{333378}{3} = \frac{330000+3300+78}{3}$ Ans: 111126

f. $\frac{17171734}{17} = \frac{17000000+170000+1700+34}{17}$ Ans: 1010102

g. $\frac{34516817}{17} = \frac{34000000+510000+6800+17}{17}$ Ans: 2030401

6.6.2 a. $\frac{30021}{3} = \frac{30000+21}{3}$ Ans: 10007

b. $\frac{1339}{13} = \frac{1300+39}{13}$ Ans: 103

c. $\frac{2200121}{11} = \frac{220000+121}{11}$ Ans: 20011

6.6.3 (Step is important)

a. $\frac{156}{12} = \frac{144+12}{12}$ Ans : 13 b. $\frac{143}{11} = \frac{110+33}{11}$ Ans : 14

c. $\frac{169}{13} = \frac{130+39}{13}$ Ans : 13 d. $\frac{209}{19} = \frac{190+19}{19}$ Ans : 11

e. $\frac{544}{17} = \frac{510+34}{17}$ Ans: 32

6.7.4 Answers are not important, steps are important.

a. $224\frac{1}{11}$ b. $373\frac{4}{11}$ c. 103 d. 343 e. 49 f. 49

6.8.3 Steps are important: 2 methods for each.

a. 2 b. 25 c. 14 d. $373\frac{4}{11}$ e. 1919 f. 19 g. 101

Ex. VI.1 a. 114195 b. 152260 c. 91356 d. 22839 e. 38060
f. 30452 g. 7613 h. 7613 (see f and write h)

Ex. VI.2 (More than one answer is possible)
1. +, - 2. +, ÷ 3. +, X 4. -, X 5. X, X 6. X, - 7. +, +
8. X, + 9. ÷, +

Chapter - 7

Ex. VII.1 a. a b. b c. c d. d e. Own

Ex. VII.2 a. 3, 3 b. 5, 3 c. 7, 7 d. 5, 11 e. 299, 1 f. 2, 4 g. Own

Ex. VII.3 a. 2, 9 or 3, 6 b. 6, 10 or 2, 30 or 5, 12 c. 5, 33 etc., d. 5, 279 etc., f. 2, 8 etc., g. Own

Ex. VII. 4 Method important, not the answer
a. 4 x 20 Ans: 76 b. 14 x 10 Ans : 126 c. 100 x 13 ans : 1287
d. 10 x 1 Ans : 120 e. 8 x 150 Ans : 1208 f. 20 x 15 Ans 270

Ex. VII.5 a. 779 – 41 = 748 b. 1800 + 54 = 1054

Ex. VII.6 a. 7 b. 19 c. 41 e. 19 f. 19000 g. 103 h. 1854 ÷ 103 = 18; 18 ÷ 2 = 9 Ans : 9 i. [Ans of h] ÷ 3 = 3

Ex. VII. 7 a. 6 x 7 x 8 b. 6 x 7 c. 6 d. 8 x 9 e. 6 x 9 f. 7 x 8

Ex. VII.8 a. Rs. 5 (30-10) + Rs. 3 x (20) = Rs. 160 b. 50 – 40 = Rs. 10
c. Rs. 27 per person d. 5 each; later 4 each e. 500 f. Rs. 6
g. Rs. 30

Chapter – 8

8.1 Rs. 6.60

8.2 Rs. 60

8.3 Rs. 50

8.4 Assume 40 guntas = 1 acre. Rs. 12 ½ lakhs

8.6 a. 30 cm b. 42 km c. 50 mph d. Own e. Assume 1 kg = 202 pounds Ans = 88 lbs
f. Rs. 4500 g. $222 h. Parle Rs. 9 per kg

8.7 1. Re. 1 2. Rs. 33 3. 5 4. 8 units 5. 2880 or 2920 6. Rs. 3 lakhs 7. Rs. 2 lakhs 8. Son - Rs. 2 lakhs Daughter – Rs. 2 Lakhs 9. 1.5 Lakhs

8.8 1. Rs. 48 2. Rs. 1 Lakh or 83000 3. Yes 4. No 5. OK 6. No loss No Profit.

Chapter – 9

9.1.2 1. 32^{o}C 2. By maths 34^{o}. By guess 32^{o} c or less 3. 40 km/liter

9.1.3 d. 123478

9.1.4 a. 123518

9.1.6 a1) 4 a2) 1.5 a3) 45 b1) 10 lakhs b2) 10000 b3) 10500

9.2.5 a. ~61 b. 210 c. 360 d. Assume 75% Ans: 450 e. 38 f. 32

Ex. IX.1 15

Ex. IX.2 15.7

Ex. IX.3 3 years, 6 months

Ex. IX.4 a. 40 b. 2 c. 1 d. 5 e. 12.2 out of 25 or 50%

Ex. IX.5 Examples : Class- Interval, Frequency, Standard Deviation

Chapter – 10

10.5 a. 1 b. 1 c. 1 d. 5 e. 8 f. 16

10.8 a. $\frac{2}{6}$ or $\frac{1}{3}$ b. $\frac{29}{30}$ c. 1 d. 2 e. $\frac{2}{7}$ f. $\frac{6}{7}$ g. 1 h. 1 i. 1 j.1 k. 2
l. 1 m. 2 n. $\frac{2}{986543}$ o. 1

10.10 a. $\frac{2}{3}$ b. $\frac{1}{3}$ c. $\frac{5}{6}$ d. $\frac{1}{6}$ e. $\frac{3}{4}$ f. 1 g. $\frac{15}{16}$ h. $\frac{1}{16}$ i. $\frac{13}{8}$ j. $\frac{5}{8}$

Chapter – 11

11.1.1 a. >1 b. =1 c. <1 d. <1 f. >1 g. <1 h. >1

11.2.2 a. 1 b. $1\frac{1}{8}$ c. $111\frac{1}{8}$ e. 12361 f. $1\frac{1}{12}$ g. 11 h. $111\frac{1}{6}$ i. $1028\frac{5}{6}$
k. $9\frac{1}{111}$

11.2.3 See 11.2.2 and use the results.
a. $27\frac{1}{2}$ b. $2060\frac{1}{8}$ c. $2\frac{1}{6}$ d. See g of 11.2.2 and do Ans: 11
e. (j) of above $9\frac{1}{111}$ f. $\approx 3\frac{1}{3}$

11.5.4 a. $\frac{1}{3}$ b. $\frac{1}{8}$ c. $\frac{1}{16}$ d. $\frac{3}{7}$ e. $\frac{111}{9}$ f. $\frac{8}{111}$ g. $\frac{2}{9}$ h. $\frac{1}{4}$ i. 4

11.5.5 a. $84\frac{1}{5}$ b. $16\frac{21}{25}$ c. $8\frac{2}{5}$ d. $\frac{24}{25}$ e. 22

Chapter – 12

12.1	A.	1) 1	2) $\frac{4}{3}$	3) 2	4) 3
	B.	1) $\frac{1}{2}$	2) 1	3) 2	4) 3
	C.	1) 1	2) 1	3) 2	4) 3
	D.	1) 1	2) 1	3) 2	4) 3
12.2	A.	1) $\frac{1}{3}$	2) 0	3) $\frac{4}{3}$	4) $\frac{5}{3}$
	B.	1) $\frac{1}{6}$	2) $\frac{5}{6}$	3) $\frac{2}{3}$	4) $\frac{2}{3}$

C. 1) 1 2) $\frac{1}{7}$ 3) 1 4) 1

D. 1) $\frac{16}{17}$ 2) $\frac{3}{17}$ 3) 1 4) 1

12.4.3 1. a. $2\frac{2}{5}$ b. $1\frac{5}{7}$ c. $1\frac{1}{2}$ d. $1\frac{1}{3}$ e. $3\frac{1}{4}$ f.$4\frac{1}{3}$ g. $6\frac{1}{2}$ h. $30\frac{1}{4}$ i. $30\frac{1}{2}$

j. $40\frac{1}{3}$ k. $13\frac{4}{9}$

2. a. $\frac{12}{5}$ b. $\frac{12}{7}$ c. $\frac{3}{2}$ d. $\frac{12}{8}$ e. $\frac{4}{3}$ f. $\frac{12}{9}$ g. $\frac{13}{4}$ h. $\frac{13}{3}$

i. $\frac{13}{2}$ j. $\frac{121}{4}$ k. $\frac{121}{3}$ l. $\frac{131}{3}$ m. $\frac{12001}{3}$ n. $\frac{1201}{4}$

12.7.1 2 methods for the students.

a. $2\frac{1}{2}$ b. $2\frac{2}{7}$ c. $14\frac{1}{5}$ d. $4\frac{1}{3}$ e. $18\frac{24}{35}$

f. $6\frac{3}{10}$ g. $2\frac{2}{3}$ h. $2\frac{2}{5}$ i. $14\frac{2}{5}$

Ex. XII.1 a. $10\frac{1}{4}$ b. $10\frac{1}{4}$ c. $10\frac{2}{3}$ d. $10\frac{1}{4}$ e. $9\frac{2}{3}$ f. $10\frac{1}{5}$

Ex. XII.2 a. $10\frac{1}{4}$ b. $10\frac{1}{4}$ c. $7\frac{1}{3}$ d. $8\frac{1}{4}$ e. $9\frac{2}{3}$ f. $10\frac{1}{5}$

Ex. XII.3 a. 11 b. 11 c. 11 d. $9\frac{1}{2}$ e. $10\frac{1}{3}$ f. $10\frac{3}{5}$

Ex. XII.4 a. $5\frac{3}{4}$ b. $7\frac{3}{4}$ c. $1\frac{1}{3}$ d. 1 e. $\frac{1}{2}$ f. $1\frac{1}{3}$

Ex. XII.5 a. $5\frac{3}{4}$ b. $7\frac{3}{4}$ c. $1\frac{1}{3}$ d. 1 e. $\frac{1}{2}$ f. $1\frac{1}{3}$ (See 4 & 5 above are similar)

Ex. XII.6 a. $\frac{3}{4}$ b. $\frac{3}{4}$ c. $\frac{6}{11}$

==

Chapter – 13

13.5 a. X b. X c. X d. √ e. X f. X g. X h. √ i. X j. √ k. X

l. Own

13.10 a. 6 b. 10 c. 14 d. 18 e. 22 f. 12 g. 15 h. 150 i. 30 j. 30 k. 50

l. 12 m. 12 n. 12 o. 24

13.11 a. $\frac{4}{5}$ b. $\frac{4}{5}$ c. $\frac{1}{2}$ d. $\frac{4}{5}$ e. $\frac{11}{12}$ f. $\frac{5}{6}$ g. $\frac{4}{5}$

13.13 a. $\frac{9}{12} = \frac{3}{4}$ b. $\frac{1}{4}$ c. $\frac{8}{10} = \frac{4}{5}$ d. $\frac{2}{10} = \frac{1}{5}$ e. $\frac{15}{10} = 1\frac{1}{2}$ f. $\frac{1}{10}$ g. $\frac{29}{20} = 1\frac{9}{20}$ h. $\frac{1}{20}$

13.14 Two methods needed

a. $\frac{3}{4}$ b. $\frac{1}{4}$ c. $\frac{8}{10} = \frac{4}{5}$ d. $\frac{8}{10} = \frac{4}{5}$ e. $\frac{15}{10} = 1\frac{1}{2}$ f. $\frac{1}{10}$ g. $1\frac{9}{10}$

h. $\frac{1}{20}$ i. $\frac{33}{36} = \frac{11}{12}$ j. $\frac{1}{36}$ [see 13.14 a to h are the same as 13.13 a to h]

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Chapter – 14

14.2.1 a. 12345 = 1 x 10000 + 2 x 1000 + 3 x 100 + 4 x 10 + 5 x 1
b. 10234 = 1 x 10000 + 2 x 100 + 3 x 10 + 1 x 1
c. 10023 = 1 x 10000 + 2 x 10 + 3 x 1
d. 10002 = 1 x 10000 + 2 x 1
e. 908040 = 9 x 100000 + 8 x 1000 + 4 x 10

14.2.2 a. 100001 > 98231 b. 9024 > 8924 c. 190 > 88

14.2.3.1 a. 88 < 190 < 8924 < 10002< 10010 < 10023 < 10234 < 12345 < 50403 < 98231< 100001 < 908040

14.3.2 a. $\frac{1}{10}+\frac{2}{100}+\frac{3}{1000}+\frac{4}{10000}+\frac{5}{100000}$

b. $\frac{1}{10}+\frac{2}{10000}+\frac{3}{100000}$

c. $\frac{1}{10}+\frac{2}{10000}+\frac{2}{100000}$

d. $\frac{1}{100}+\frac{2}{100000}$

e. $\frac{9}{10}+\frac{8}{10000}+\frac{4}{1000000}$

14.3.3 a. .982 b. .98231 c. .9 d. .9 e. .91 f. .9 g. .88 h. 88

14.3.4 a. .10023< .10234 < .12345
b. .01002 < .090804 < .10023
c. .01002 < .090804 < .10023 < .10234 < .12345
d. .01002 < .090804 < .1001 < .10023 < .10234 < .12345 < .50403
e. .100001 < .101 < .190 < .199 < .8 < .81 < .88 < .89 < .893 < .90 < .91 < .982 < .98231

14.5.2 a. yes b. yes c. yes d. yes e. yes f. no g. no h. no i. No j. yes k. yes

14.6.1 a. .123 b. .10002 c. .5403 d. .504 e. .053 f. .0078 g. .0708 h. .708

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Chapter – 15

15.6 a. .5 b. 1.0 c. 1.5 d. 3.5 e. 27.5 f. .2 g. .2 h. .6 i. 8 j. 1.0 k. 11.0 l. 11.2 m. 11.8

15.8.2 a. 0.6 b. 1.2 (Actually 1.33) c. 1 d. 1.25 e. 1.75 f. 1.2 g. 1.4
h. 1.6 i. 1.8 j. 3.8 k. 0.80 (actually 0.08) l. 0.28 m. 0.42 n. 0.70
o. 0.84 p. 2.4 q. 1.88 r. 1.88 s. 0.375 t. 1.625 u. 0.77 v. 1.88

15.9.1 Ans already given.

15.10.1 2 methods by the students
a. 3.5 b. 2.857 c. 11.8 d. 2.77 e. 2.77 f. 277.7

15.11.1 a. 0.22 b. 0.233 c. 0.2404 d. 0.515 e. 0.5001 f. 0.5617 g. 6.0617 h. 0.665
i. 0.66606 j. 6.565 k. 2.1 l. 0.21 m. 0.021 n. 1.0021

15.12.1 a. $\frac{1}{6}$ = 0.166; $\frac{1}{7}$=0.1428571; $\frac{1}{9}$ = 0.11 b. $\frac{22}{7}$ = 3.1428

==

Chapter – 16

16.2 a. a. 0.95 b. 0.91 c. 0.34 c. 0.59
b. I class - a and b; Failed – c
c. a. 30% b. 60% c. 75% d. 90%
d. a. 30% b. 50% c. 92% d. 72%

16.4 a. 55% b. 60% c. 60.25% d. 1% e. 2% f. 5% g. 10% h. 99%

Chapter – 17

17.3.1 a. No Profit No Loss b. No Profit No Loss c. Profit Rs. 200 d. profit Rs. 100
e. Loss 10000 f. Profit 2 lakhs

17.4.1 a. Profit 44% b. Profit 50% c. Profit 55% d. Loss 4% sell at Rs. 100 each.

Chapter - 18

18.5 a. Rs. 10 b. Rs. 20 c. Rs. 18

18.6 a. 50 b. 150 c. 300 d. 1000 e. 5 f.1

18.7 a. 50 b. 100 c. 250 d.5 e. 5 f. < Re 1

18.7.1 g. Rs. 2 h. ~ Rs. 2

18.8.1 a. 250% b. 240% c. 200%

18.10.2 a. 400 b. 1000 c. 25% d. Bank is better

18.12.4 a. Rs. 400 b. 180%

Chapter – 19

19.11 1. 290 2. 295 3. Compound interest 4. Simple Interest 5. Yes

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Chapter – 20

1. 1, 3, 5, 7, 9, 11, 13, 15, 17, 19
   0, 2, 4, 6, 8, 10, 12, 14 ,16, 18, 20

2. Place value is “thousand”

3. 6

4. a. A b. A

5. 12, 6, 4, 3

6. a. .5 = 50% b. .75 = 75% c. .3 = 30% d. .66 = 66% e. .8 = 80%

7. a. $\frac{2}{3} = \frac{4}{6} = \frac{10}{15}$ b. $\frac{4}{3} = \frac{8}{6} = \frac{20}{15}$ c. $\frac{3}{5} = \frac{9}{15} = \frac{30}{50}$

8. a. 10005 b. 12345

9. $\frac{6}{2}$ x $\frac{6}{3}$ = 6

10. a. 100 b. 0.001 c. 3600 d. Refer (nor recommended for memory) e. 12

11. a. 2.5 b. False c. True d. d

12. a. 4 b. 1 c. 10, 20, 30 d. 0 e. $\frac{9}{3} = 3$

13. a. 111105 b. 123450 + 12345 = 135785

14. a. 70 b. 112.5

15. a. $\frac{1}{4},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{3}{4}$ b. $\frac{1}{260},\frac{1}{250},\frac{1}{25},\frac{1}{22},\frac{1}{2}$ c. $\frac{4}{240},\frac{3}{160},\frac{5}{255},\frac{1}{50},\frac{2}{97}$

16. a. $\frac{5}{2}$ b. $\frac{5}{4}$ c. $\frac{7}{4}$ d. $\frac{19}{4}$

17. a. 4 (2 rem) b. 2 (6 rem) c. 3 (1 rem) d. 3 (1 rem) e. 0 (1 rem)

18. a. 204 b. 204

19. a. (5 x 2) + 4 = 14 b. 5 x (2 + 4) = 30 c. (5 x 2) + 4 –2 =12
d. 5 x (2 + 4) – 2 = 28

20. a. $\frac{1}{3}$ b. 1 c. $\frac{2}{3}$ d. 1 e.1

21. a. $\frac{1}{2},\frac{2}{3},\frac{3}{4}$ b. $\frac{1}{6},\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{3}{4}$ c. $\frac{7}{12},\frac{3}{5},\frac{3}{4},\frac{5}{6}$

22. a. $\frac{1}{12}$ b. $\frac{1}{2}$ c. $\frac{1}{4}$ d. $\frac{1}{4}$

23. a. $\frac{2}{3}$ b. $\frac{5}{6}$ c. $\frac{4}{5}$

24. a. $\frac{1}{3}$ b. $\frac{1}{2}$ c. $1\frac{1}{12}$

25. a. 24 b. 30 c. 30

26. Loss Rs. 16000. Nothing. Try to sell 100 more good ones.

27. Rs. 250

28. Rs. 32

29. a. 60% b. 55% c. 375 marks

30. No profit no loss 0%

31. a. Profit 25% b. Profit 50% c. Loss 40% d. Loss 40% e. Loss 25% f. Profit 100%

32. 6.6 km

33. Rs. 50

34. Rs. 50

35. 50

36. 4

37. 8

38. 30 small size; 70 adult size; profit 55%

39. Profit 39.5%

40.

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

41. See text

42. Activity A, B, C

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Chapter – 21

21.5.1 a. Rs. 10 b. Rs. 5 c. Rs. 2.50 d. Yes e. Yes; Condition: sets intact

21.5.3 Do

21.6.1 Do

21.7.1 A. é =4 œ=3 â =2 ∆=1
B. ! =4 ?=3 %=2 œ=1

21.9.1 a. Addition and multiplication: 2 methods to be done; 16, 24, 712
b. All 16

21.9.2 a. Put y = 5 LHS= 3y + 2y = 15 + 10 = 25
RHS = 5 x 5 = 25

Do Similarly b and c.

21.10.2 A. Do it by own B. 199, 1 C: 1. 100 2. 80 3. 81 4. 99 D: 1. 100 2. 80

21.11.2 4, 25

21.11.4 Activity

===

Chapter – 22

22.3.1 1. 200 2. 59 3. 1600 4. Rs. 310

22.3.2 1. 1 2. –35 3. Rs. 800 4: a. Rs.40 b. Rs. 300 c. Rs. 144

22.4 1. d 2. d 3. d

22.6 a. 1 b. 0 c. –1 d. 4 e. –4 f. 2 g. 0 h. –1 i. 6 j. –6

22.7 c, e, h, j

22.8.2 Pictures to make

22.11.1 (a) a. 7 b. –3 c. 1 d. 7 e. –3 f. –7 g. 7 h. +3 i. –1
j. 7 k. –7 l. –7 m. 7

(b) a. 12350 b. 12340 c. –12340 d. –12340 e. 12350
f. –12340 g. 12340 h. -12350

22.11.2

-	**1**	**2**	**3**	**4**	**5**	**5**	**7**	**8**	**9**
1	0	1	2	3	4	5	6	7	8
2	- 1	0	1	2	3	4	5	6	7
3	- 2	- 1	0	1	2	3	4	5	6
4	- 3	-2	- 1	0	1	2	3	4	5
5	- 4	-3	- 2	- 1	0	1	2	3	4
6	- 5	- 4	- 3	- 2	- 1	0	1	2	3
7	- 6	- 5	- 4	- 3	- 2	-1	0	1	2
8	- 7	- 6	- 5	- 4	- 3	- 2	-1	0	1
9	- 8	- 7	- 6	- 5	- 4	- 3	- 2	- 1	0

Chapter – 23

23.5 a. 85 b. 100 c. 2 d. 1 e. 11 f. 2

23.6 a. 4 b. 1 c. 2 d. 4 e. 1 f. 1 g. 1 h. 1

23.7.1 a. 30 b. 10 c. –10 d. 2 e. $\frac{1}{2}$ f. 3

23.7.2 a. 50 b. 30 c. – 60 d. 4 e. $\frac{1}{8}$ f. $\frac{5}{3}$ g. $\frac{5}{4}$

23.9.1 c. A = T + 5 where A = Akka’s age, T = Thangi’s age
T = 10
A = 65

23.9.2 A1. 10 hrs A2. 75 kmph C. 0.1mm

Chapter – 24

24.1 1. Yes 2. No 3. No 4. No 5. Yes 6. No 7. Yes 8. Yes 9. Yes 10. Yes
11. Yes

24.2 See 24.1

24.5 a. √ b. X c. X d. X e. √ f. √ g. √ h. √ i. √

24.6.1 a) 5a + 5b + 5c b) 5a + 3a + 2a (=10a) c) 5a + 3a d) 5a + ab e) a^2 + ab + ac
f. 5 x 1 + 5 x 2 + 5 x 3 + 5 x 4 + 5 x 5 (=75) g. 15a h. 2x + 2y + 2z I. ax + ay + az

24.6.2 a. $\frac{4}{5}+\frac{1}{5}=1$ b. $\frac{a+b}{5}=2$ c. $\frac{8}{a}=\frac{8}{4}=2$ d. $\frac{7}{a}+\frac{8}{b}=\frac{7}{7}+\frac{8}{8}=2$

24.8.1 a. 4a + 5b b. 2a – 5b c. –4a + 5b d. a2 +b2 +c2 +a + b +c e. 6a + 8b + 11c
f. 37 g. –19 h. +13 i. 98 j. 135 k. 83 l. 15 m. 225 n. M>K

24.12.1 a. 0 b. y c. -x

24.12.2 a. 0 b. y c. –a d. a e. a

Chapter – 25

25.1.1 a. √ b. X c. X d. X e. √ f. √ g. √ h. X i. √

25.1.2 a. X b. X c. X d. X e. √ f. √ g. √ h. X i. √

25.1.3 Activity

25.3 1. 30x 2. 30x 3. 30x 4. 30x 5. 90x 6. 60x 7. 60x 8. 15x + 60 9. 15x + 60

25.4.1 1. 2x 2. 2x 3. 15x 4. x + 4 5. 3 (x + 4) 6. x 7. 1

25.4.2 a. $\frac{2}{x}$ b. 2x c. 3 d. 1 e. $\frac{1}{2x}$ f. 1

25.5.1 a. X b. X c. √ d. X e. √

25.5.3 a. $\frac{5}{x}$ b. 1 c. $\frac{6}{n}$ d. 1 e. $\frac{45}{d}$ f. 3 g. 9 h.15 i.1 j. $\frac{a+b+c+d+e}{5}$

k. 2 l. $\frac{a+b+c+d+e}{n}$

25.8 B: 1) 6 x 2) 18 x^2 3) 6 x 4) 9 x 5) 8 x^2 6) y^{10} 7) 10d 8) b^n 9) x^m

25.9 a. $20a^3$ b. $39a^2$ c. 296a

25.10.1 a. $\frac{1}{5}$ b. $\frac{1}{50}$ c. $\frac{1}{500}$ d. 1 e. 100 f. 10000 g. $\frac{1}{8}$ h. $\frac{8}{1}$ i. $\frac{3}{8}$ j. $\frac{8}{3}$

k. $\frac{100}{8}$ l. $\frac{125}{100}$

25.10.2 a. 30 b. 0.480 c. 0.003 x 66 x 99 = 0.3 x 66 = 20.0

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Chapter – 26

26.3 a. 50 b. 42 c. 10 & 40 d. (5, 25), (6, 20), (7, 15), (8, 10) etc e. 8, 10

26.6.1 A:

1. Money in my account = m
2. Money I owe you = m_1
3. 2d = x d= distance between points
x = total distance / day

4. $I = 3I_1$ I = my eating, I_1=your eating

5. $T_N = \frac{1}{2}T1$ T1 = your time T_N – Nirmala's time

6. B_p = 4 BD B_p = Paradise bill; D_p = Darshni bill

7. S = L – 10% L S = Sale Price, L = Label Price

B: 1) 4500 2) 110 3) 100 4) 6, 2 5) 2 days, 3 man days 6) 60 7) Rs. 68.60

C: 1) 200 to N, 100 to K 2) 20 / day to N, 10 / day to K

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Chapter – 27

27.2.1 Writing steps is important

a. 720

b. Product = 10 x x x 3 x y x 4
= 30 x x x 3 x y x 4
= 120 x x x y
= 120xy

c. Result = $2(5+x)+3(y+4)$
= $10+2x+3y+12$
= $2x+3y+22$

d. $5x+22$ e. $x+2$ f. $-x-2$

27.3.2 Step by step

a. i) 5 x 38 = 190 ii) $\frac{38}{1000}$ x 300=11.4 = Rs. 11.40

b. Loose oil

27.4.1 a. √ b. X c. X d. √ e. √ f. X g. √ h. √ i. X j. √

27.4.2 a. √ b. X c. √ d. x – 2 = Y – 5 - 3 Wrong; x – 2 = y – 5 + 3 True
e. √ f. X g. √ h. √ i. X j. √

27.6.1 i) LHS = x + a + b
RHS = x – (b – a)
= x – b + a
= x + a – b LHS = RHS Therefore True

ii) √ iii) X iv) √ v) √ vi) √

27.8.1 Activity

27.10 i) x = $5\frac{1}{2}$ ii) Rs. 5.50 iii) Rs. 15 iv) Rs. 200, Rs. 100 v) x = 2 vi) 5 vii) 35

27.11.3 a. 10 b. 1000 c. 8 d. 2 e. 2 f. 8 g. 30 h. $\frac{1}{2}x - \frac{1}{3}x = 25$ Ans: $x = 150$
i. a=20 j. 20 k. 55

27.14 a. (My age) -5 b. 250 c. Rs. 5 d. Rs. 600; Rs. 6/kg profit 40%

===

Chapter – 28

28.6.3 3 methods

a. 225 b. 2.25 c. 0.0225 d. 240 e. 440 f. 4.4 g. 4.48 h. $\frac{9}{64}$ i. 6.25 j. 0.32

28.7.2 (Activity)

28.9.1 a. 10 b. 100 c. 100 d. 25 e. 15 f. 60

28.9.2 a. 10a b. 10a c. 100a d. $25a^2$ e. a f. 5a g. $15a^2$ h. $15a^2$

28.12.2 a. 2700 b. 2700x c. 6 d. 6ab e. 18 f. 18 (a + b) g. 30 h. 30a (x – y) i. 26
j. $\frac{26}{a}$ k. 56 l. 51 m. 12.8 n. 4.4 o. 44 p. $56\sqrt{a}$ q. $4.4\,a\sqrt{b}$ r. $\frac{4.4}{a}$
s. $44\frac{a}{b}$ t. $56\frac{\sqrt{a}}{b}$

28.14 i. √ ii. x iii. x iv. √ v. √ vi. x vii. √ viii. x ix. x x. √

28.15.1 Only for advanced students.

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Chapter – 29

29.2.1
$(a \pm b)^2 = a^2 \pm 2ab + b^2$ $(p \pm q)^2 = p^2 \pm 2pq + q^2$ $(u \pm v)^2 = u^2 \pm 2uv + v^2$
$(a + b)^2 = a^2 + 2ab + b^2$ $(p + q)^2 = p^2 + 2pq + q^2$ $(u + v)^2 = u^2 + 2uv + v^2$
$(a - b)^2 = a^2 - 2ab + b^2$ $(p - q)^2 = p^2 - 2pq + q^2$ $(u - v)^2 = u^2 - 2uv + v^2$

29.3.1
xa + xb
xa – ab
$a^2x – cx$
20 + 40

21368 - 4936
100 - 80
2xy + 6x
4abx + 8ax

$ax^2y – 4ax^2$

29.4 1. $5x + 10x + 20 + 40 = 15x + 60$ Do this way.
2. $5342x – 1234x + 21368 – 4936 = 4108x - 16432$
3. $2xy + 6x + y + 3$
4. $4abx + 8ax + b + 2$
5. $a^2x – cx + a^2 - c$
6. $400 - 80 – 75 + 60 = 355$
7. $ax^2y – 4ax^2 + 4y - 16$

29.5 1. $x^2 – x(a + b) + b^2$ 2. $x^2 – 2ax + a^2$ 3. $x^2 – 2xy + y^2$ 4. $a^2 – 2ab + b^2$
5. $25 – 5 (a + b) + ab$ 6. $25 – 10a + a^2$ 7. $25 – 10y + y^2$
8. $25 –15 –15 + 9 = 25 –30 + 9 = 25 –30 +9 = -5 + 9 = 4$ 9. $36 – 12b + b^2$

29.7 1. $(100 – 1)^2$ 2. $(100= 1)^2$ 3. $(50-1)^2$ 4. $(50 + 1)^2$
5. $(1000 +2)^2$ 6. $(500 + 5)^2$ 7. $(20 + 2)^2$ 8. $(20 – 2)^2$
9. $(100 + 12)^2$ 10. $(100 – 12)^2$

29.8.1 a. $(100 –1) (100 + 1) = 9999$ b. $(100 – 5) (100 + 5) = 9975$
c. $(100 – 8) (100 + 8) = 9936$ d. $(50 – 8) (50 +2) = 2436$
e. $(10000 + 5) (10000 – 5) = 100000075$

29.10 Activity – Do.
29.11 Activity – Do.
29.12 Activity – Do.

===

Chapter – 30

30.1 a. 110 b. 110 c. 110 d. 121a – 11 e. 110a f. 0 g. a h. 0 i. a+1 j. a

30.2 a. 11, 13, 15, 17, 19, 21, 23, 25 b. 10, 12, 14, 16, 18, 20, 22, 22, 24,
c. 100, 95, 90, 85, 80, 75, 70, d. 60, 55, 50, 45, 40, 35, 30,
e. x, 2x, 3x, 4x, 5x, 6x, 7x, f. 2x, 4x, 6x, 8x, 10x, 12x, 14x,
g. 50x, 45x, 40x, 35x, 30x, 25x, 20x h. 18x, 15x, 12x, 9x, 6x, 3x, 0
i. 2, 4, 8, 16, 32, 64, j. $2x, 4x^2, 8x^3, 16x^4, 32x^5$

30.3 a. $\frac{2}{3} = \frac{4}{6} = \frac{10}{15}$ b. $\frac{4}{3} = \frac{8}{6} = \frac{20}{15}$ c. $\frac{2x}{3} = \frac{4x}{6} = \frac{10x}{15}$ d. $\frac{2a}{5x} = \frac{4a}{10x} = \frac{10a}{15x}$
e. $\frac{2}{3} = \frac{?}{3x} = \frac{22}{33} = \frac{10y}{15y}$ f. $\frac{a}{a^2} = \frac{a2}{a^3} = \frac{10a4}{10a^5} = \frac{ka^3}{Ka^4}$ g. $\frac{a(a+1)}{2} = \frac{4a\,(a+1)}{2 \text{ x } 4}$
h. $\frac{a}{b} = \frac{a \text{ x } 4}{b \text{ x } 4} = \frac{a \text{ x } (b+1)}{b \text{ x } (b+1)}$

30.4 a. $\frac{3}{4}$ b. $\frac{1}{15}$ c. $\frac{3}{2}$ d. $\frac{15}{4}$ d. $\frac{c}{bd}$

30.5 a. $\frac{2}{3} = \frac{4}{6} = \frac{10}{15}$ b. $\frac{4}{3} = \frac{8}{6} = \frac{20}{15}$ c. $\frac{3}{5} = \frac{9}{15} = \frac{30}{50}$ d. $\frac{2a}{3} = \frac{4a}{6} = \frac{10a}{15}$
e. $\frac{3}{5x} = \frac{9}{15x} = \frac{10}{50x}$ f. $\frac{2a}{5x} = \frac{4a}{10x} = \frac{10a}{25x}$ g. $\frac{2}{3} = \frac{2x}{3x} = \frac{20}{30} = \frac{10a}{15a}$
h. $\frac{a}{a^2} = \frac{a2}{a^3} = \frac{10a^4}{10a^5} = \frac{ka^3}{ka^4}$ i. $\frac{a\,(a+1)}{1 \text{ x } 2} = \frac{4a\,(a+1)}{2 \text{ x } 4}$ j. $\frac{a}{b} = \frac{a \text{ x } 4}{b \text{ x } 4} = \frac{a \text{ x } (b+1)}{b \text{ x } (b+1)}$

30.6 a. $\frac{5}{6}$ b. $\frac{10}{21}$ c. $\frac{41}{42}$ d. $\frac{179}{84}$ e. $\frac{1}{6}$ f. $\frac{13}{42}$ g. $-\frac{59}{84}$ h. $3\frac{1}{21}$ i. $\frac{179x}{84}$
j. $\frac{179}{84x}$

30.7 a. 76 b. 126 c. 1287 d. 120 e. 1208 f. 1620 g. 76x h. 126x i.$1287x^2$
j. 120y k. 1208xy l. 1620xy

30.8 1. x, 3x, 5x, 8x 2. x, 4x, 6x, 9x 3. x, 4x, 7x, 9x 4. x, 2x, x^2, x^3 5. x^3, x^2, x, 2x
6. Same 7. Same 8. 9, 99, 101, 309, 310 9. Same as 8 10. Same

30.9 a. $\frac{x}{3}$ b. $\frac{9}{2}$ c. $\frac{b}{6}$ d. $\frac{2d}{3}$ e. $\frac{2e}{5}$ f. $\frac{f}{3}$
g. $\frac{7g}{12}$ h. $\frac{9h}{99}$ i. $\frac{y}{3}$ j. $\frac{a}{13}$

30.10 a. 738 b. 1854 c. 18 d. 190 e. 19 f. 3 g. 3a h. 190y

30.11 a. 2 b. 2 c. 12 d. 0 e. 4 f. 9 g. 3 h. 2

30.12 a. 82 b. 2 c. 2 d. 1 e. $\frac{21}{20}$ f. 2 g. $\frac{82}{39}$ f. $\frac{9}{4}$ g. 5

h. 1 i. 1 j. 1 k. 9 l. 1 m. 1 n. 1 o. 1 or 0 p. 1 1 1

q. 8 r. 16

30.13 1) 54 2) 4 3) 26 4) 3 5) 0 6) 3 7) 49 8) 9 9) 48 10) 10

30.14 1. $6a^2 - 2$ 2. $20a^2 + 20b^2$ 3. $6a^2 + 6b^2 + 6ab + 6a + 6b$ 4. $4a^2 + 14ab – 2a + 2b$
5. a – 3b + 3c

30.15 1. $15a^2$ 2. $90a^2b^2$ 3. $–15a^2$ 4. $–15a^2$ 5. $90a^2b^2$ 6. $90a^2b^2$ 7. $a^2b^2c^2$
8.$8a^2b^2c^2$ 9.1 10. –1

30.16 1. $x^2 + 8x + 16$ 2. $x^2 - 8x + 16$ 3. $a^2 + 4ab + b^2$ 4. $4b^2 – 8ab + a^2$
5. $a^2 + 8ab + 4b^2$ 6. 102.04 7. 98.04 8. ≈10000 9. ≈1000000
10. a^4 – b4 11. $(x^2 + Y^2) (x + y) (x + y)$ 12. (2a + b) (2a – b)
13. (3x + 2Y) (3x – 2y) 14. $x^2 + \frac{1}{x^2} + 2$ 15. $x^2 + \frac{1}{x^2} - 2$

30.17 1. Rs. 236 2. 5,10 3. 1, 2, 6, 5 4. 122, 66 boys, 56 girls 5. 15, 75 6. 3, 27
7. 14 8. 10, 20 9. 25 10. Yes 30, 40 11. 10, 100; 5, 25; 1, 1
12. Eg. Let age be 14. Ans. 128 (given by the students) Age = $\frac{\text{Last 2 digits}}{2}$

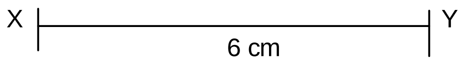
30.18 1. a. 200 b. 1200
2. a. $\frac{1}{4}$ b. 250 mill amperes
3. a. 5 b. 1000
4. a. 10 b. 12
5. a. 50 b. 10 (Units/Sec)
6. a. 5.58 b. 0.04
7. 1) 1 2) 4 3) 2 4) 5 5) 1 6) -1
8. 1) 0 2) 1 3) 9 4) 16 5) 1 6) 16
9. a. 154 sq. cm cm^2 or (square centimeter) b. 154 sq. m^2 m^2 or (square meter)
10. a. 1540 cm^3 (cubic centimeter) b. 1540 m^3 (cubic meter)
11. $A_1 = 2500$ $A_2 = 2400$ $A_1 > A_2$

30.19 1) 2 2) 2 3) x = 3, y= 2 4) 5 5) 3 6) ±2 7) ±2 8) ±a
9) ±(a+5) 10) 5 11) 2a – 5 12) 9 13) x = 4, y = 2 (can be – 4, -2 also)
14) 5 15) 4 16) $\sqrt{5}$ 17) 4

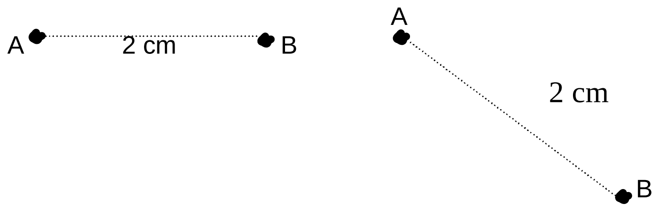
==

Chapter – 31

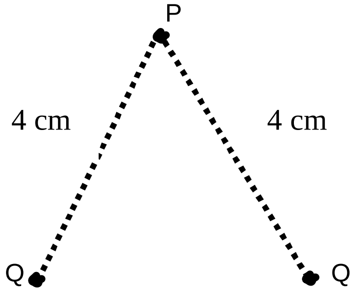
31.9 a)



c)

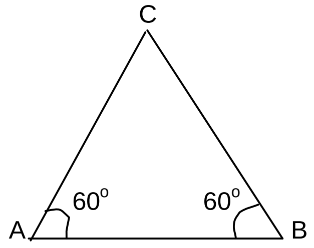


d)



e)

31.18
1) Doing activity
2) a) infinite b) infinite
3) Parallel lines, Intersecting, Perpendicular
4) Thick, Thin, Dotted, Dot – Dash
5) 1. Right 2. Acute 3. Obtuse 4. 180 deg or Obtuse 5. Obtuse 6. Acute
7. 90 deg or Right
6) (0 – 89^0): 2, 6; (90^0): 1, 7; (91 – 180^0): 3, 4, 5
7) a) d b) 8 c) 8
8) a) d b) 4 c) 6
9)



CA = CB (also = AB)
$\angle ACB = 60^o$

10) Activity
11) Square – 1; Rectangle – 18; Triangle – 12; Trapezium – 8;
12) Activity – Area = 150000m^2
13) Δ ABC = 16 Sq units; Δ DEF = 20 Sq units; Δ PQR = 20 Sq units
14) ABCD = 50 Sq units; IJKL = 20 Sq units; PQRS = 50 Sq units
15) 100 Sq units; 100 Sq units

==

Chapter – 32

32.4
a) 1.7 b) 2.2 c) 2.8 d) 3.6 e) 9.5 f) 17 g) 22
h) 28 i) 36 j) 30 k) 54 l) 72 m) 81 n) 11.4
o) 99

32.7.2 a) 40 m b) 26 m c) Rs. 2000 and Rs. 1300 d) Rs. 50 x 56 = Rs. 2800

32.8
a) and b) Figures c) Rs. 20 x 200 = Rs. 4000; same
d) Use 1 feet = 30 cm e) Activity

==

Chapter – 33

33.2 a1. 24 a2. 24 a3.16 **b, c, and d activity**

33.4 I. Isosceles: A, B, F Equilateral: C, E Rightangles: D, G
II. A. 60 deg B. 30 deg C. 70 deg D. 60 deg E. 45 deg
F. 120 deg G. 100 deg

33.5 1) 5 1b) 3 1c) 4 2) $10\sqrt{2}$, $10\sqrt{2}$ 3) 10 4) ~7

==

Chapter – 34

34.3.2 1) 45 deg 2) 60 deg sector 3) 24; 15 deg sector

34.5 1) 154 sq. cm 2) 3 m 3) 14 cm, 6 m 4) 9 times

34.6 1) Large difference in circumferences 2a) Activity 2b) Activity
3) Activity 4) ≈ 990 inches 5) 4times 6) Second Child 7) ~ 0.92 m

==

Chapter – 35

35.4 1) 8000 liters 2) 8000 liters 3) No

35.7 1) 40 liters 2) ≈same as 1 3) Total volume = 10^6 liters. One method is a cylinder of 1 m dia and 0.6 m height (many answers possible).

35.8 1) ≈180 m^3 / hr ≈180000 liters / hr 2) 80 cc / min a) 800 cc b) ≈5 liters

35.11 Student's Work.

==

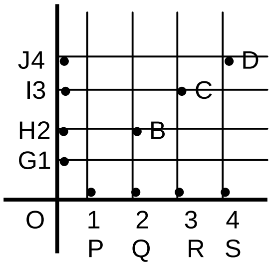
Chapter – 36

36.6 1) Own 2) 1 m = 100 cm; 1m = 1000 mm 3) Yes 4) Use verniers 5) Activity

==

Chapter – 37

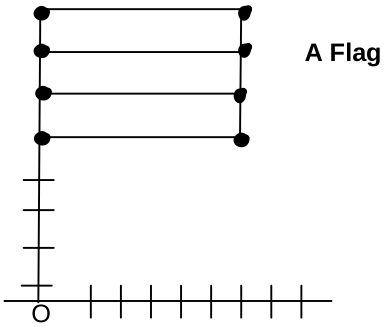
37.4.1



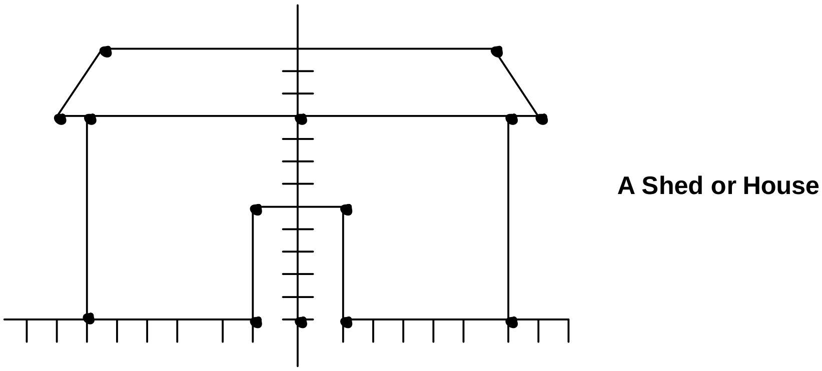
37.5 a. B, C, D and O straight line
b. G, H, I, J and O straight line
c. P, Q, R, S and O straight line
d. a is a line of slope or angle of 45^o
b is a vertical line, y axis
c is a horizontal line, x axis
e. 4 points will be (X, Y_1), (X, Y_2), (X, Y_3), (X, Y_4) Where X will be the same. Y different values.

37.7.3 (1) to (11) activity.

37.8 I.



II.



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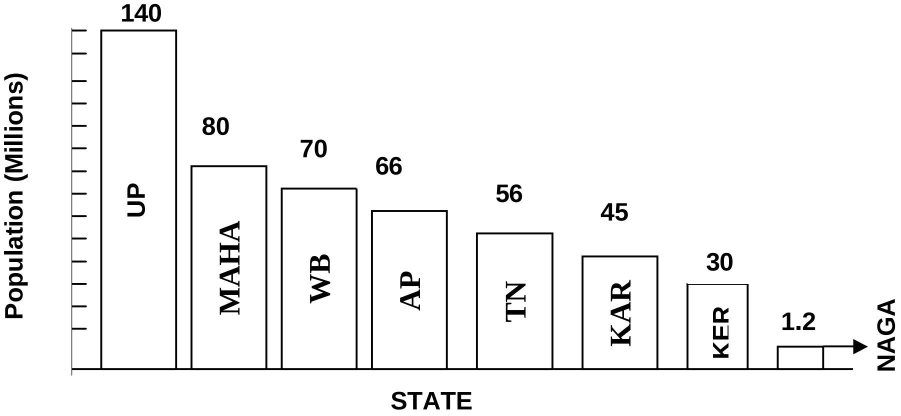
Chapter – 38

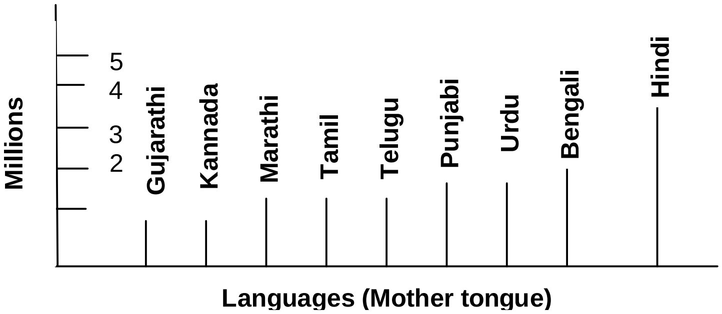
38.4 Activity to do.

38.5 Activity to do.

38.6 I.

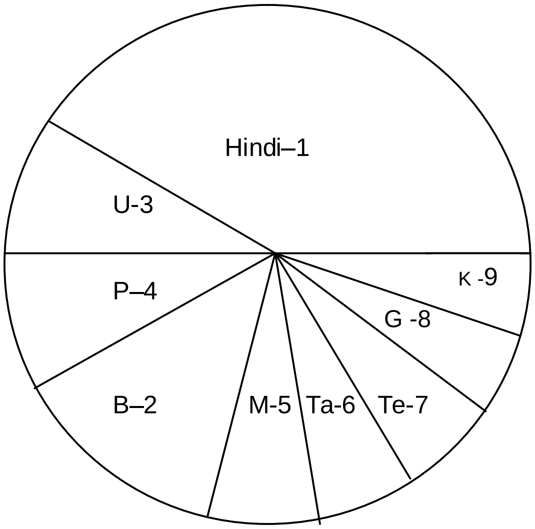


II.

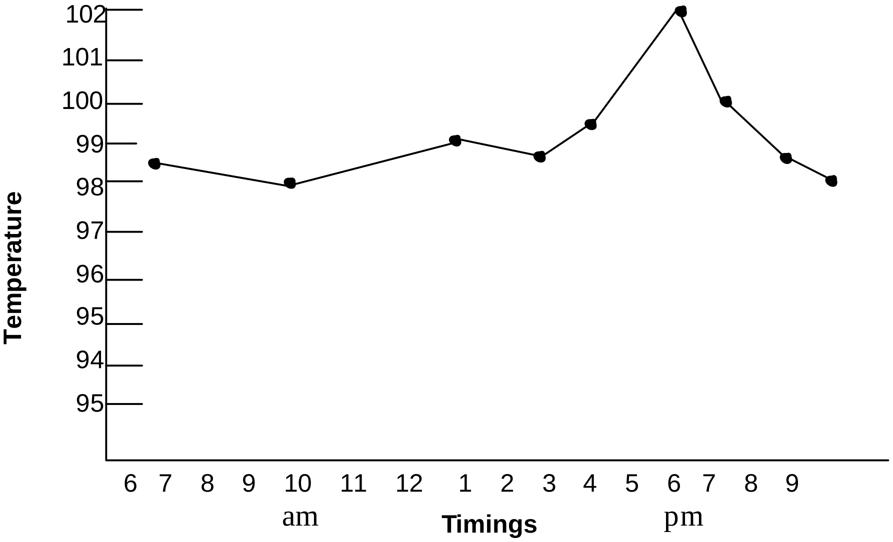


1.	500	150
2.	200	60
3.	100	30
4.	100	30
5.	75	22.5
6.	75	22.5
7.	75	22.5
8.	50	15
9.	50	15
	--------	-------
	1225	367.5 ≈ 360

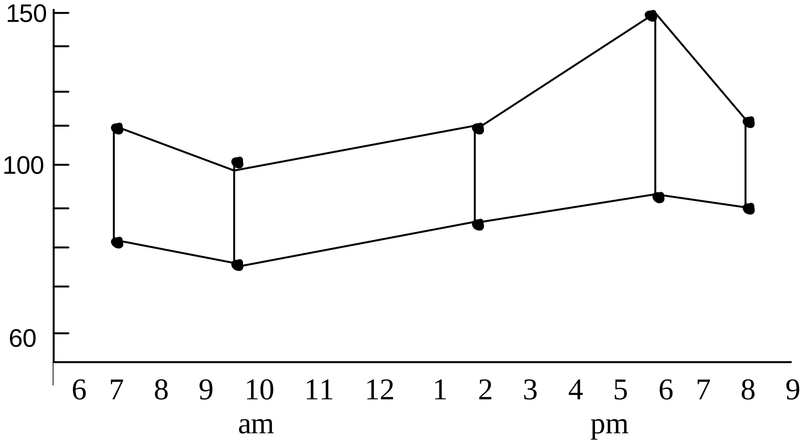
$$\frac{360}{1225} = .294 \approx .3$$



III.

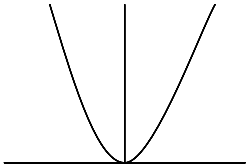


IV.

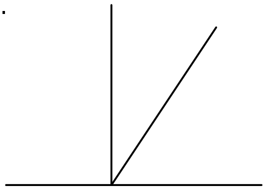


V. Own

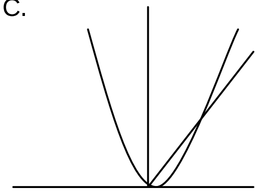
38.7 A.



B.



C.



E. $x + y = 3$ $y = 3 - x$ (a)

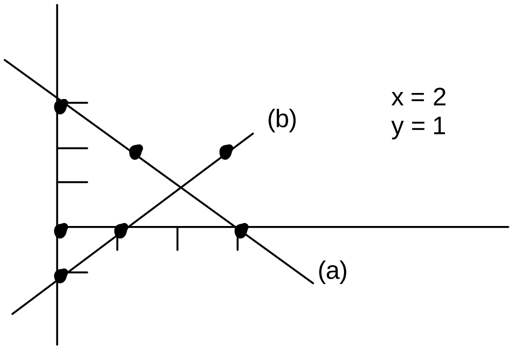
$x - y = 1$ $y = x - 1$ (b)

(a) $y = 3 - x$

x=	0	1	3
y=	3	2	0
(x, y)	(0,3)	(1,2)	(3,0)

(b) $y = x - 1$

x=	0	1	3
y=	-1	0	2
(x, y)	(0,-1)	(1,0)	(3,2)



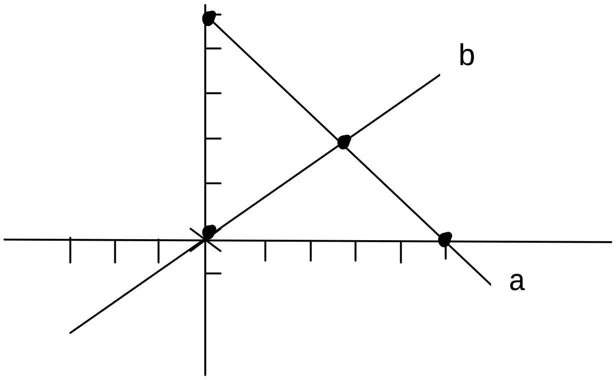
$x = 2$
$y = 1$

F. a a. $x = y = 5$ and $2x - 3y = 0$

$y = 5 - x$ (a) $y = \frac{2}{3}x$ (b)

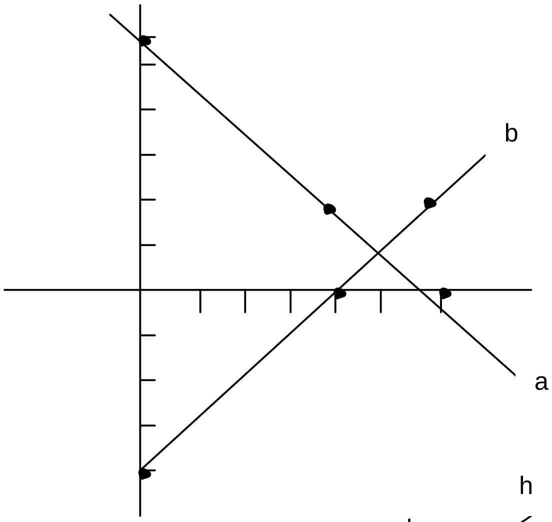
x=	0	5
y=	5	0
(x, y)	(0,5)	(5,0)

x=	3	0	-3
y=	2	0	-2
(x, y)	(3,2)	(0,0)	(-3, -2)



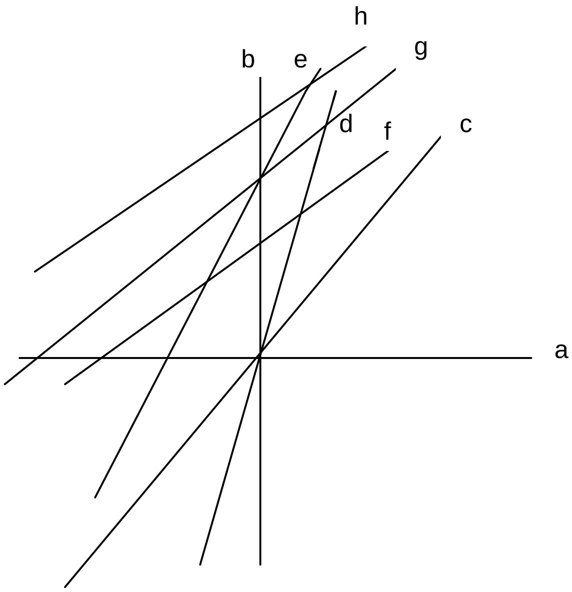
$x = 3$
$y = 2$

F. b x + y = 6 x – y = 4



x = 5
y = 1

G.

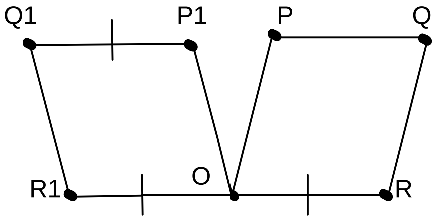


h. 1. x axis is y =0

2. y axis is x = 0

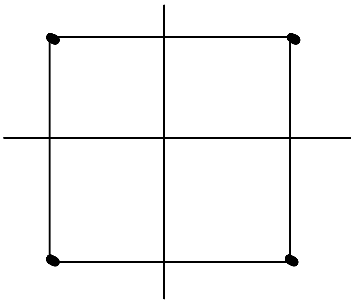
3. y = 10

38.8 1. Work
2. Work
3. A and B:



Barfi or rhombus

C:



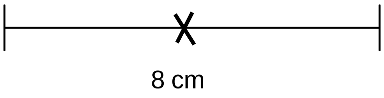
Square

D: Areas of all equal!

==

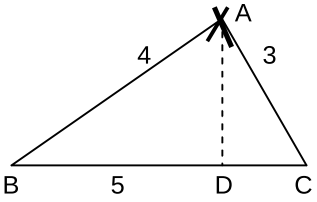
Chapter – 39

39.1.1



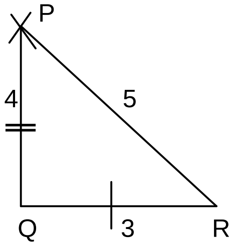
No triangle is possible

39.1.2



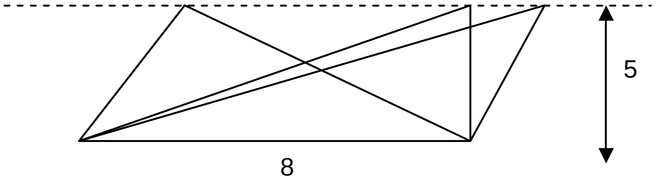
$$\text{Area} = \frac{1}{2} \text{ BC x AD}$$

$$= \frac{1}{2} \text{ x 5 x } \underset{\substack{\downarrow \\ \text{(to be measured)}}}{\text{AD}}$$

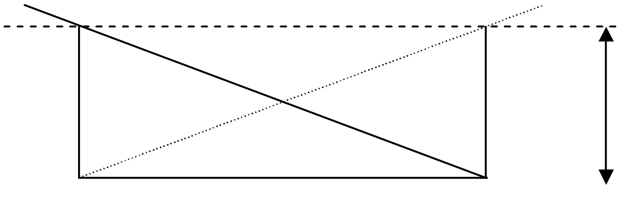


$$\text{Area} = \frac{1}{2} \text{ x 3 x 4} = 6 \text{ Sq. Units}$$

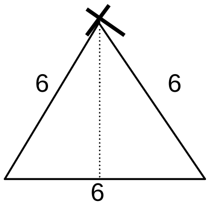
39.1.3



39.1.4



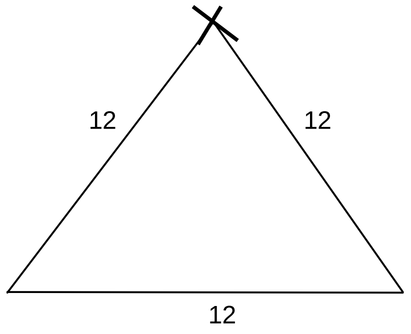
39.1.5



$$(\text{Area})_1 = \frac{1}{2} \text{ x 6 x 6 x } \frac{\sqrt{3}}{2}$$

$$= 9\sqrt{3}$$

$$(\text{Area})_2 = 4 \text{ x } (\text{Area})_1$$

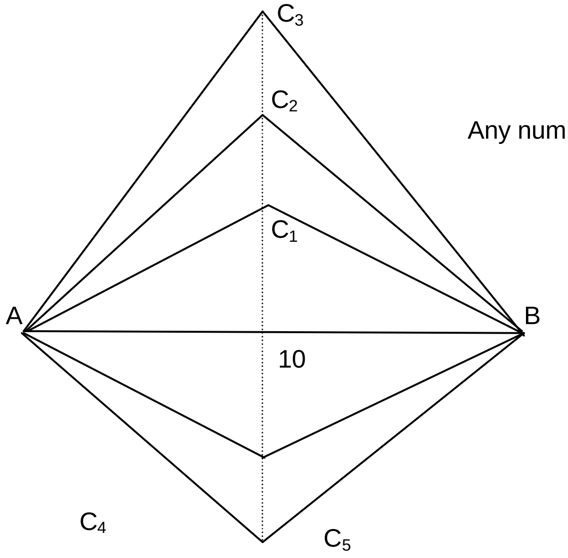


$$(\text{Area})_2 = \frac{1}{2} \text{ x } 12 \text{ x } 12 \text{ x } \frac{\sqrt{3}}{2}$$
$$= 36\sqrt{3}$$

[Can show this by drawing on graph sheet]

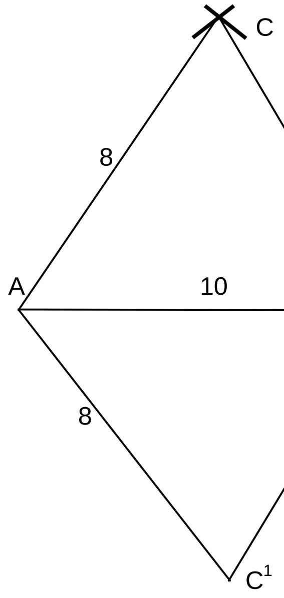
39.1.6

a.

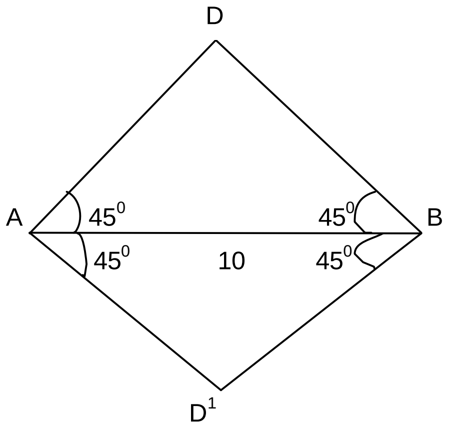


Any number of Isosceles Triangles!

b.

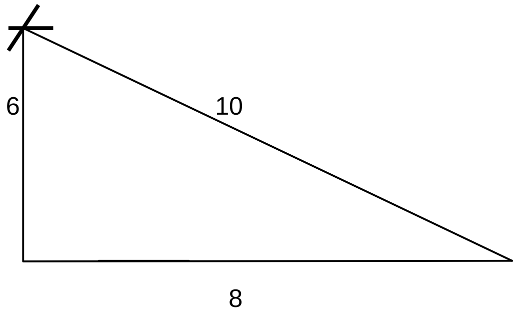


c.

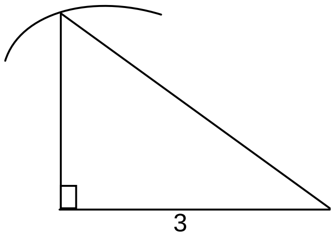


39.1.7

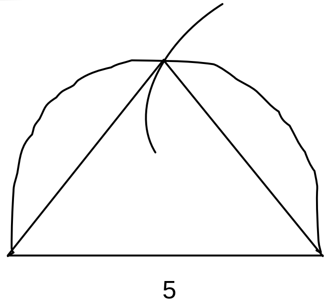
a.



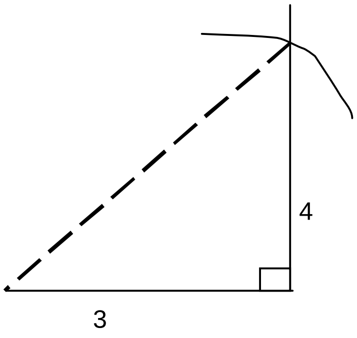
b.



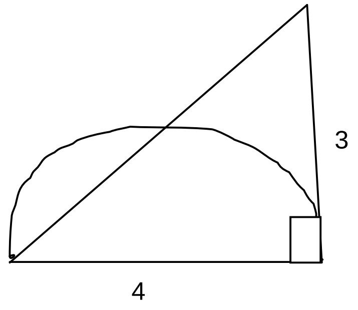
or



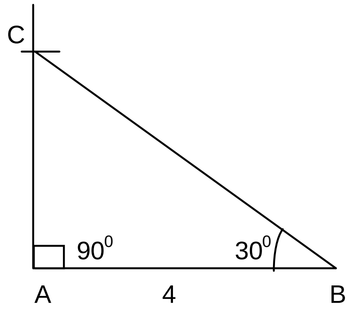
c.



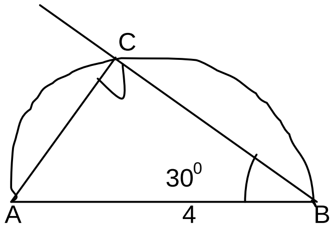
or



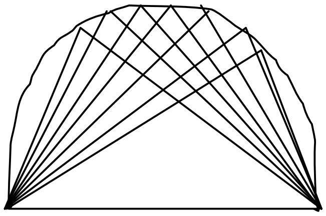
d.



or



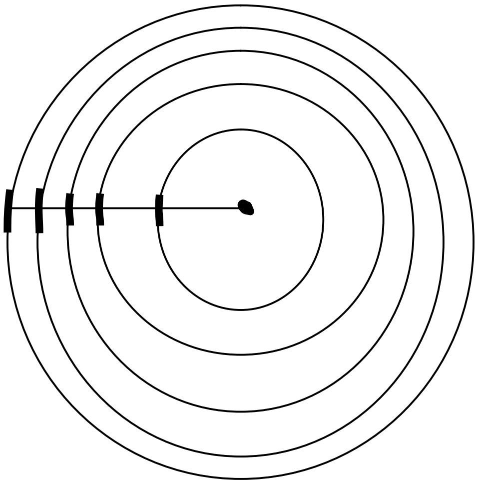
e.



Many Triangles!

39.2.2 a. Yes b. Many c. Only one

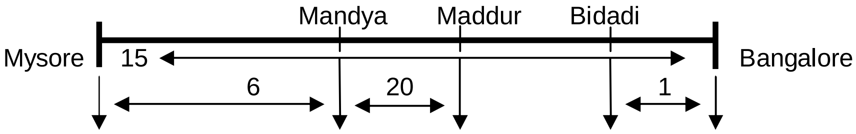
39.2.3



5 Concentric Circles

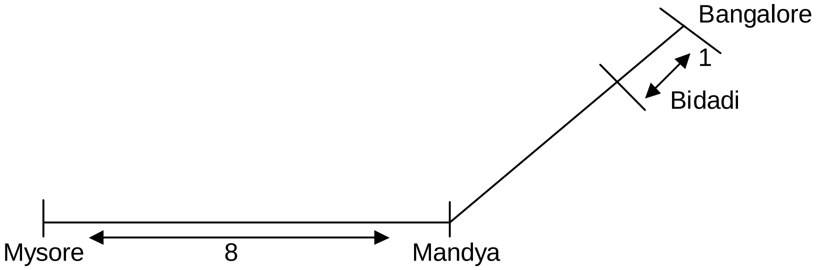
39.3.1

a.

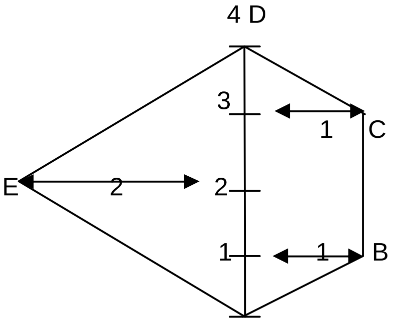


Scale: 1 cm = 10 km

b.



c.



Scale:
1 Cm = 100 m
For 1 cm = 10 cm
[40 cm long paper is needed]

0 A

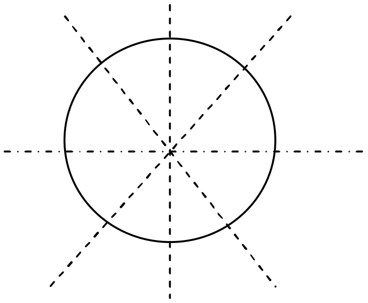
$$\text{Area} = \frac{1}{2} \times 1 \times 1 + 1 \times 2 + \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 4 \times 2$$

$$= \frac{1}{2} + 2 + \frac{1}{2} + 4 = 7 \text{ cm}^2 = 7 \times (100)^2 = 7 \times 10^4 \text{ m}^2$$

$$\text{Cost} = 7 \times 1000 \times 10^4 = 7 \times 10^7 = 7 \text{ crores}$$

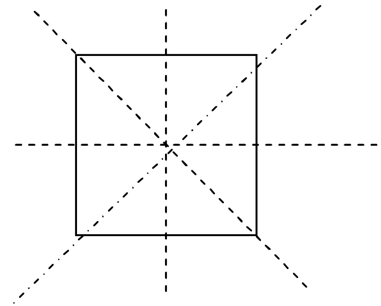
39.4

A. Circle

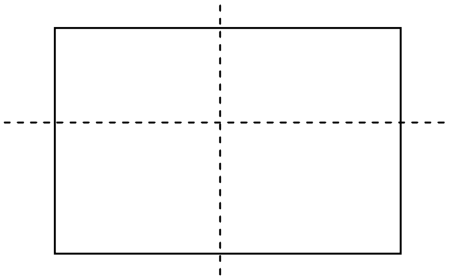


Any diameter

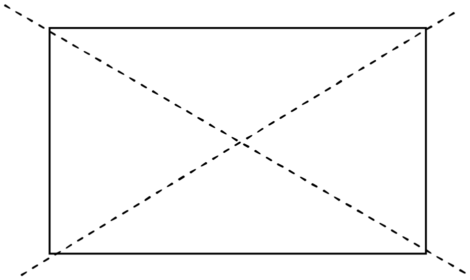
Square



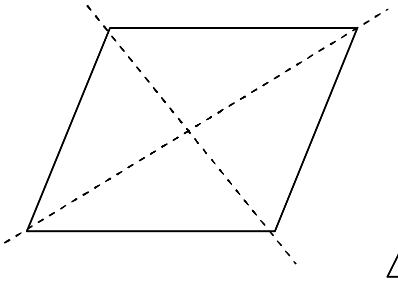
4 Lines

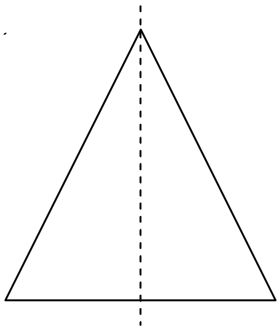


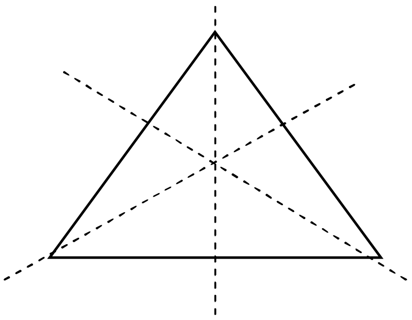
Strictly 2

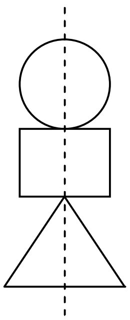


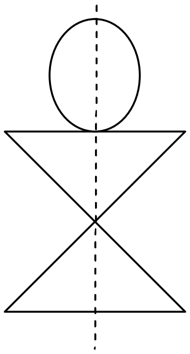
These also if separable

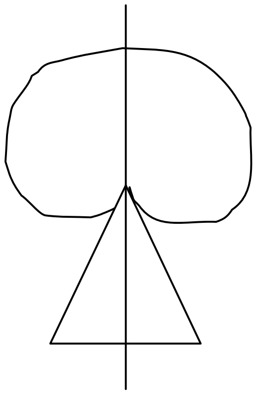


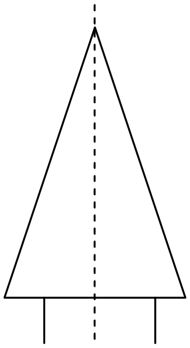




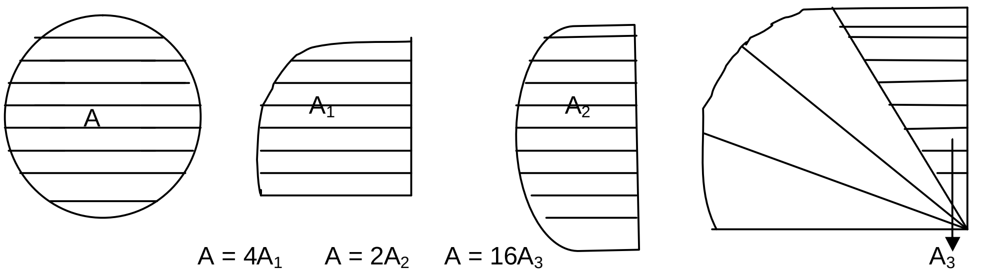








B.



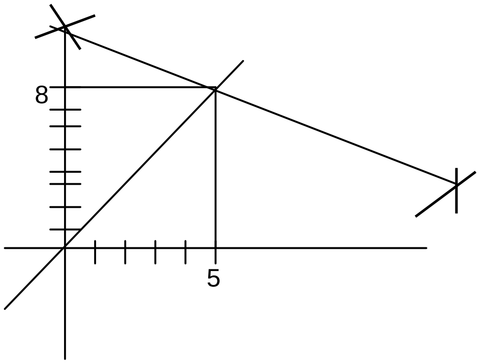
$A = 4A_1$ $A = 2A_2$ $A = 16A_3$

39.5

B. $2X - Y = 2$ $Y = 2X - 2$

$X + 2y = 21$ $Y = \frac{21 - X}{2}$

$X = 5$ $Y = 8$



For Graph:

C.

$X + 10 = X^2 - 10$

$X^2 - X - 20 = 0$

$(X - 5)(X + 4) = 0$

$X - 5 = 0$ therefore $X = 5$

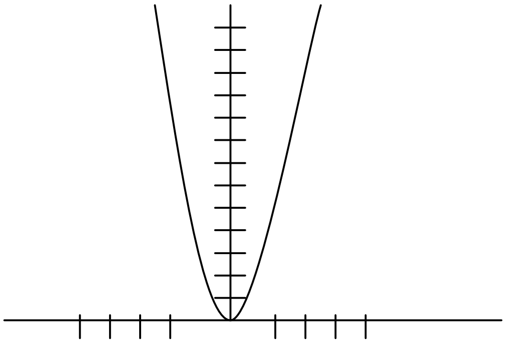
$X + 4 = 0$ therefore $X = -4$

$X + 10 = x^2 - 10$

$X + 20 = X^2$

Draw $y = X^2$

And Draw $Y = X + 20$



Therefore make it number plus 1 is equal to square of the number minus 1. find the number.

$X + 1 = x^2 - 1$

$X + 2 = X^2$

$Y = X^2$

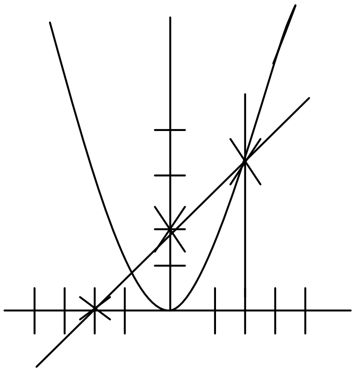
$Y = X + 2$

$X = 2$ and $X = -1$

$4 - 1 = 2 + 1$

$(-1)^2 - 1 = (-1) + 1$

$0 = 0$



39.6 A. North East B. South West (-4, -4) C. (-4, 4)

39.8 A. $8m^3$; just enough for 1m height – one lorry Ok
B. $1m^3$ = 1000 liters – 5 days
C. ≈ $.9m^3$
D. B is bigger

39.9 A. 2.4 m for 3m-dia cone B. ≈ 1.8 m

C. $V_1 = \pi \frac{d^2}{4} \text{ x } 2d + \frac{1}{3} \pi \frac{d^2}{4} \text{ x } \frac{d}{2}$

V_1 = (only) $V^2 - V^1 = \frac{1}{3} \frac{\pi d^2}{8} = \frac{\pi d3}{24}$

$$V^2 - V^1 = \frac{\pi d^3}{24} / \frac{2}{\pi d^3} = \frac{1}{12}$$

He makes 8% profits

===

Chapter – 40

This is an example of a final test. Solutions are not attached to this book. Students should ask if they need it.

****

Appendix – 1

Basic Operations in Algebra:

The four basic operations +, -, x, ÷ are used in this book in the traditional manner. However, efforts have been made to explain every step. The idea of negative numbers is not given in the usual way of 'continuum of quantities'. Instead illustrative examples are use. Thus one can see 'hole and solid' concept. This helps to explain addition, subtraction (of both +ve and –ve numbers).

When it comes to multiplication this analogy fails. In this book, we have attempted a concept, which we hope will help in teaching at school level.

Let us analyse addition of negative numbers. When we write 5 + 4 it is really +5 + 4 = +9. It is just that 5 units and 4 units of the same type (viz +ve) of numbers coming together (adding).

Similarly –5 – 4 = -9. It is just 5 units (= items) and 4 units of the same kind (viz -ve) come together. It ADDS to –9. 5 – 4 is the same as (+5 – 4). Here, for each negative unit, one positive unit annuals the effect (like repaying loan). Thus +5 – 4 = 1 i.e., 4 debtors cancelled by 4 creditors and remaining is 1 credit (i.e, + 1). Subtraction term itself is not necessary here.

Consider –5 + 4. By a similar reasoning, we get 1 debit (i.e., -1).

Multiplication is many times addition.
Thus (+3) (+n) = 3(+n) = + n + n + n = 3n
Similarly (+3) (-n) = 3(-n)
= -n – n – n
= -3n

Here the sign (+) in the multiplying factor is not the same as the + in the bracket. If we understand this we can explain (-)(-) and (-)(+) results.

The rule (-) x (-) = + appears to be strange and explanations are not convincing to a logical thinker. Teachers, I am sure, would have experienced this difficulty. More so, if they try to explain this on the basis of debtors for negative, and creditor for positive. Here is a concept.

The basic assumption of the following explanation is that in the ideal number system (continuum) both positive and negative exist; and the symbols used (+) or (-) indicate something else. More clearly, when you say (-) x (-x) the second belongs to the number continuum and therefore is the quality (or nature, or attribute or SIGN) of the number x. The first (-) is an operational symbol. Here comes our explanation. In the usual coming together (=joining, addition) of numbers (or quantities) (-) indicates subtraction or removal (of a quantity). It is quantitative and therefore requires a quantity following it.

The first (-) symbol is an operator and works on a quantity. It has no quantity on its own. The function of this operator is reversal (of sign). By the same argument (+) of some quantity means conserving (i.e, keeping as such) (of sign).

Example:
(-4) means (you) owe someone (4). Thus –4 is a quantity i.e., Dr. of 4. Now (-) (-4) means reversing the nature. i.e., Dr. becomes creditor. (This is because number has only 2 states). Thus (-) (-4) = +4.

Explanation:

(+) (+1) means conserving (=keeping the nature) + status of number 1.
Ans: +1, i.e. 1.

(-) (+) means reversing (= opposing the nature) + status of number 1.
Ans: Opposite of +1= -1

(+) (-1) means conserving the _ve nature of –1. Therefore –1.

Now,

(-1)(-1) means reversing the –ve nature of 1. That is (-1) = +1
Hence, the rule (+) x (+) = +
(+) x (-) = -
(-) x (+) = -
(-1) x (-) = +

Thus the right way of doing (-5) x (-4)
(-5) x (-4) = (-) (5) x (-4) = (-) (-20) = +20.

But in practice, any which way works.
(-5) (-4) = (-)(-) (5) (4) = +20 OK.

===

Appendix – 2

Common Conventions in Geometry:

• P	Point, capital alphabet
▬ (AB)	Line, capital alphabets
├─┤ a	Length, small alphabet (no units)
∠ Or ∡ Ø	Angle, Greek alphabet (small)
△	Triangle
⊥	Perpendicular Lines
○	Circle
r, d	Radius, diameter (of circle)
A	Area (in square units of length) (Capital A)
V	Volume (in cubic units of length) (Capital V)
x,y,z etc	Variables used in algebra
P (x_1,y_1)	x_1 and y_1 - numbers. P is a point in coordinate geometry (i.e. on a graph sheet)

===

Appendix – 3

Exponents:

$a \times a = a^2$

$a \times a \times a = a^3$

$a \times a \times a \times \ldots\ldots$ n times $= a^n$

$a^m \times a^n = (a \times \ldots\ldots \text{ m times})(a_x \ldots\ldots \text{n times}) = a^{(m+n)}$

Therefore $a^m \times a^n = a^{(m+n)}$

$$\frac{a^m}{a^n} = \frac{(a \times \cancel{a} \times \cancel{.}\ldots \text{m times})}{(a \times \cancel{a} \times \cancel{.}\ldots \text{n times})} \quad \text{Say } m > n$$

$$= (a \times a \ldots\ldots (m-n) \text{ times}) = a^{(m-n)}$$

Therefore $\frac{a^m}{a^n} = a^{(m-n)}$

$\frac{a^m}{a^m} = a^{(m-m)} = a^0$ LHS = 1; $a^o = 1$

(Anything)^o = 1

$\frac{1}{a} = \frac{a^0}{a} = a^{(0-1)} = a^{-1}$

$\frac{1}{a^n} = \frac{a^0}{a^n} = a^{(0-1)} = a^{-n}$

Thus $a^{-1} = \frac{1}{a}$

$a^{-n} = \frac{1}{a^n}$

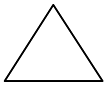
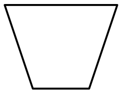
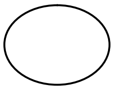
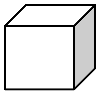
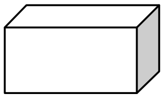
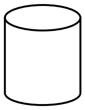
$\sqrt{a} \times \sqrt{a} = a.\ a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^1 = a$

Therefore $\sqrt{a} = a^{\frac{1}{2}}$

Appendix – 4

Collection Of Important Formulas.

Geometrical figures:

	Name	Shape (= diagram)	Quantity to Calculate
a.	Triangle		area, perimeter
b.	Square		area, perimeter
c.	Rectangle		area, perimeter
d.	Trapezium		area, perimeter
e.	Circle		area, perimeter
f.	Cube		surface area, volume
g.	Box		surface area, volume
h.	Sphere		surface area, volume
i.	Cylinder		surface area, volume
j.	Any others		surface area, volume

Let the students see some book and fill up.

Appendix – 5

Tips for Making Maths Easy

1. Understand – Symbols – Use them.

+, -, X, ÷, =, ±, >, <, ≤, ≥, ≠, ≈

$\boxed{\therefore}$ Sometimes $\boxed{n}$ $\boxed{\sqrt{\ } \quad \sqrt[3]{\ }\ , \quad (\ldots\ldots)\ (\ )}$

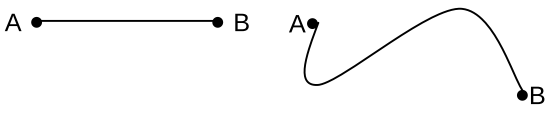
2. Learn substitution

3. Use your own notations; Use $\boxed{\text{Let x be}}$

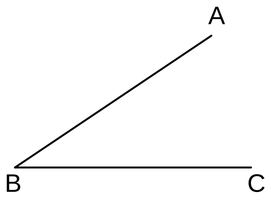
4. Use LHS = RHS etc liberally

5. Always give names (A, P. R etc) to points (geometry)

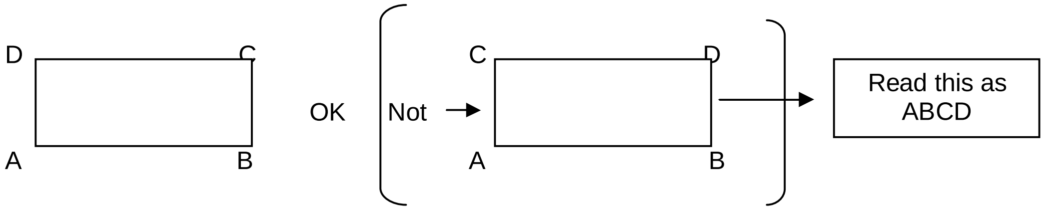
6. Learn to give names to all geometric shapes



Lines



$\angle ABC = \angle CBA = A\hat{B}C = C\hat{B}A\ldots\ldots$



7. Know the special like horizontal, vertical, perpendicular, right angle, parallel, annular, cone, prism, cube, parallelepiped, pipe, cylinder, segment, sector, hemisphere, hexagonal, tangential.

8. Learn to make your own graph: x axis, y axis etc. Learn how to join points or not to join.

9. Decimals, percent are very important in engineering.

10. Learn formulas along with meanings of symbols and units of the quantities.

11. Apply approximation methods to verify the order of values.

Subject Index

{Numbers refer to paragraphs. Ch = chapter]

ACTIVITIES

1.2.1, 1.2.3, 1.2.7, 2.1, 2.4.2, 2.5, E.III.6, E.III.7, E.III. 3 [5],[6], 18.3, 18.4.1, 17.7, 17.8, 16.5, 16.8, 15.2, 13.5, 13.8.3, 7.5, CH 7, 806[D], 9.1.2[C] [D], 10.3, 10.10, 13.8.2, 13.8.3, 14.5.3, 15.2, 17.7, 17.9, 21.2, 21.3, 21.4, 21.6.2, 21.7.1, 21.5.3, 21.10.2[A], 21.11.1, 22.11.2, 22.12, 24.13, 25.1.3, 25.2.1, 25.4.2[A], 26.2, 26.3, 26.4, 27.8, 27.12, 28.3.1, 28.4.1, 28.5.2, 28.7.2, 29.10, 29.11, 31.1, 31.2, 31.3.5, 31.4.2, 31.6.2, 31.8.2, 31.10, 31.11, 31.12, 31.13, 31.16, 31.17, 32.1, 32.3, .2.5, 32.6, 33.3, 34.3, 34.4, 34.1, 35.2, 35.5, 35.7, 35.8, 35.9, 35.10, 36.6, 37.2, 37.3, 37.4, 37.8, 37.9, 38.1, 38.2, 38.3, 38.4, 38.5, 38.6[V], 38.10, 39.2,